MIT LIBRARIES
3 9080 02762 5836

VAIL LIBRARY

Gift of American Telephone
and Telegraph Company
to the
Massachusetts Institute
of Technology 1912

ENGINEERING ECONOMICS

McGraw-Hill Book Co. Inc.
PUBLISHERS OF BOOKS FOR

Electrical World ▿ Engineering News-Record
Power ▿ Engineering and Mining Journal-Press
Chemical and Metallurgical Engineering
Electric Railway Journal ▿ Coal Age
American Machinist ▿ Ingenieria Internacional
Electrical Merchandising ▿ Bus Transportation
Journal of Electricity and Western Industry
Industrial Engineer

620.05
F53

ENGINEERING ECONOMICS

FIRST PRINCIPLES

BY

JOHN CHARLES LOUNSBURY FISH

MEMBER AMERICAN SOCIETY OF CIVIL ENGINEERS; MEMBER AMERICAN RAILWAY ENGINEERING ASSOCIATION; SOMETIME DIVISION ENGINEER LAKE SHORE AND MICHIGAN SOUTHERN RAILWAY (NEW YORK CENTRAL LINE WEST); PROFESSOR OF RAILROAD ENGINEERING STANFORD UNIVERSITY; AUTHOR OF "EARTHWORK HAUL AND OVERHAUL," ETC.

SECOND EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1923

Copyright, 1923, by the
McGraw-Hill Book Company, Inc.

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS - YORK PA

PREFACE TO THE SECOND EDITION

The first edition of "Engineering Economics" dealt with economic choice of structure. Experience has shown that there were omitted from that edition some pertinent factors which are at times important. A revision of the text was undertaken to remedy the omission. It was found, however, that in the plan of the old book there was no logical place for the missing factors. This difficulty suggested that the fundamental problem of engineering economics is not choice of structure, but, rather, choice of investment. The idea was put to test. The factors disclosed by analyzing the problem of investment choice were found to include those omitted from, as well as those included in, the first edition. Thus it became clear that the new edition in order to be satisfactory must be, not a revision of the old in the usual sense, but a new writing that should begin with the problem of investment and follow the path indicated by the analysis.

As a result, this is more than a new edition: it is a new text.

The present book opens with a picture of the engineer in the work of the world. The picture shows that engineering knowledge, unlike medical and legal knowledge, can be converted into community service only through the medium of business, and that consequently the engineer's professional success and contribution to public welfare depend in a great measure on his understanding of the business side of engineering—of engineering economics.

The text proper sets out with the analysis of the investment problem, which analysis soon discloses two classes of data that influence choice of investment: (1) data from which yield or profit can be calculated, and (2) data which cannot be incorporated in the calculations: in short, reducible and irreducible data.

The calculation of yield or profit involves the elements of compound interest and sinking funds. For this reason, a practical method of making the calculation is merely outlined at

3122

first; next the principles of compound interest and sinking funds are explained; and then the method of calculation is presented in detail.

The work of evaluating the reducible data, commonly called "estimating," being no less important than the reduction of such data, receives extended treatment.

Among the more important irreducible data of investment are the characteristics of business units. Business units, therefore, including private and public corporations, together with their organization, control, management, and ways of raising capital, are sketched with some particularity. Other important classes of irreducible data are also discussed.

First principles of financial statements and cost keeping are developed from common experience. There is included an elementary treatment of valuation of public service corporations, followed by the derivation and comparison of several depreciation formulas.

The text closes with a chapter on engineering reports, and is followed by appendices containing specimen incorporation papers, numerous tables of formulas and of values, a bibliography, and an extensive collection of questions and problems.

For the convenience of the reader certain changes in the mechanical features have been introduced in this second edition, as follows: (1) chap. I begins with section §101, chap. II with §201, chap. III with §301, and so on, so that the chapter in which the section lies is indicated by the number of hundreds in the section number; (2) each equation, table, and figure is designated by the number of the section in which it makes its first appearance; (3) key words in the running text are set in bold-faced type, in lieu of formal side-heads; (4) numerous diagrams summarize the results of analyses; and (5) minor paragraphs are set off by smaller type.

Other differences between the two editions may be discerned by comparing the present table of contents with the following list of chapter heads of the old edition: (I) the problem of economic selection, (II) interest, (III) sinking funds, (IV) first cost, (V) salvage value, (VI) elements of yearly cost of service, (VII) estimating, (VIII) basis of economic comparison, (IX) procedure for economic selection, and (X) examples of economic selection.

This edition, like the former, is designed to serve as a text for study and as a manual for the office.

While the book is written primarily for student and practitioner of engineering, it should be of value to the student of business and to the business man, because it deals with a fundamental business problem, the problem of investment choice.

For the rest, the new text must speak for itself.

Finally, I take pleasure in introducing the reader to a number of individuals who have given a hand to the work. For critical examination of manuscript I am indebted to my colleagues: J. B. Canning, assistant professor of Economics; T. J. Hoover, dean, School of Mining and Metallurgy; C. A. Huston, dean, School of Law; S. S. Seward, Jr., associate professor of English; and A. C. Whittaker, professor of Economics. I have to acknowledge a similar indebtedness to J. B. Cox, civil engineer, Paia, Maui, T. H.; Williston Fish, general manager, Chicago Surface Lines; Job Fish, Jr., general works manager, Otis Elevator Company; H. H. Hall, chief engineer, Standard Oil Company (of California); Mrs. Dare Stark McMullin, San Rafael, California; and Col. H. H. Whitney, United States Army. Valuable suggestions have been received also from A. E. Berriman, chief engineer, The Daimler Company, Ltd., Coventry, England; C. T. Brady, Jr., Buenos Ayres, Argentine; E. H. Freeman, professor of Electrical Engineering, Armour Institute of Technology; E. B. Wilson, professor of Mathematics, Massachusetts Institute of Technology; and from my students: Fred Coffman, U. B. Gilroy, R. C. Ingebritson, and K. R. Jenkins, who in various ways have assisted in the preparation of the manuscript. Sherman Kimball, of Bradford, Kimball & Co., San Francisco, kindly supplied the specimen papers shown in Appendix A; and McGraw-Hill Book Co., Inc., and H. P. Gillette and R. T. Dana generously permitted the insertion of Table K taken from their "Cost Data for Mechanical and Electrical Engineers."

J. C. L. Fish.

Stanford University, California,
October, 1922.

PREFACE TO THE FIRST EDITION

Every engineering structure, with few exceptions, is first suggested by economic requirements; and the design of every part, excepting few, and of the whole is finally judged from the economic standpoint.

It is therefore apparent that the so-called principles of design are subordinate to the principles which underlie economic judgment.

This important fact usually escapes the student of engineering because, while he may have seen hundreds of books on the principles of design and his time is largely employed in studying these principles and their application, he has seen not one book devoted to the principles which underlie economic judgment, and his books and his instructors merely mention these in passing.

It is true that during the past dozen years, chiefly as a result of the writings of such men as H. P. Gillette and F. W. Taylor, interest in the economic side of engineering has greatly widened; so that we now see the word "economics" in titles of technical articles, chapters, and even books.

Most of these works, however, assume the reader to be familiar with fundamental economic principles.

The present work was undertaken with the belief that to the engineer a working knowledge of first principles is as essential in the economics as in the mechanics of structures; and that special study and drill in the application of principles are as advantageous in the one case as in the other.

The book is intended to meet the first needs of the student, and to render effective service in the office. It is hoped that it will facilitate the introduction of formal instruction in engineering economics in the engineering schools, and assist sound engineering practice.

J. C. L. Fish.

Stanford University, California,
March, 1915.

CONTENTS

Page

Preface to Second Edition v
Preface to First Edition. viii

Chapter

I. INTRODUCTION 1

II. ANALYSIS OF PROBLEM OF INVESTMENT 6
  Introduction 6
    Analysis 6
    Choice. 6
  Analysis of problem of investment 8
    General Case 9
    Irreducible data 15
    Special cases 16
  Prerequisite to further analysis 19

III. INTEREST 20
  Time value of money 20
  Simple interest 20
  Compound interest 22
    Elements. 22
    Equivalence 28
    Uniform series of payments. 30
    Comparisons 34
  Sinking funds, annuities, etc. 34

IV. ANALYSIS OF PROBLEM OF INVESTMENT (continued) 36
  Annual depreciation cost 36
  Operation cost 39
  Interest on capital 39
  Income 40
  Summary of analysis. 40
  Review and preview. 41

V. PRELIMINARY ANALYSIS OF FIRST COST AND OF OPERATION COST 44
  First cost. 44
  Operation cost 54

VI. PRACTICAL ANALYSIS OF FIRST COST 56
  First cost. 56
  Materials cost 60
  Labor cost. 62
  Unit cost 63
  Other analyses of first cost 65

PAGE
VII. PRACTICAL ANALYSIS OF FIRST COST (CONTINUED) 66
PROCEDURE IN ESTIMATING 66
In general 66
Typical procedure for estimating in detail 68
Abbreviated procedures 71
Use of analysis diagram 76
VIII. PRACTICAL ANALYSIS OF FIRST COST (CONCLUDED). 78
PROCEDURE IN ESTIMATING (CONCLUDED) 78
Prices, wage rates, man-hours 78
Bringing old cost data up to date 80
Aids in estimating 83
IX. BUSINESS UNITS 86
USE OF BORROWED CAPITAL 86
THE INDIVIDUAL BUSINESS 89
THE PARTNERSHIP 91
X. BUSINESS UNITS (CONTINUED) 94
PRIVATE CORPORATION 94
Creation and organization 94
Ownership 97
Control and management 99
How the private corporation raises capital 102
Financial condition 106
XI. BUSINESS UNITS (CONCLUDED) 108
PUBLIC SERVICE CORPORATION 108
PUBLIC CORPORATION 110
PROMOTION 115
XII. IRREDUCIBLE DATA OF PROBLEM OF INVESTMENT 121
IRREDUCIBLE QUALITIES OF INVESTMENT 121
CONDITIONS AFFECTING IRREDUCIBLE QUALITIES 122
Investor's financial relation to business unit 122
Type of business unit 123
Individuality of business unit 125
Type of opportunity 125
Trend of conditions 126
XIII. BUSINESS STATISTICS 129
FINANCIAL STATEMENTS 129
Balance sheet 129
Income sheet 136
Supplementary statements 139
Account 140
Ledger 145
XIV. BUSINESS STATISTICS (CONCLUDED) 149
COST KEEPING 149
COMPARISONS 158

PAGE
XV. MISCELLANEOUS FORECASTS 162
  OUTPUT, LIFE, PERIOD OF SERVICE 162
  INTEREST RATE 164
  POPULATION 166
  EARNINGS 168
  PRICES AND WAGE RATES 169

XVI. VALUATION 171
  FAIR VALUE 171
    Salvage, scrap, and wearing value. Depreciation 171
    Price fixed by buyer and seller 175
    Fair price fixed by third party chosen by buyer and seller . 175
    Fair price fixed by court or commission 175

XVII. VALUATION (CONCLUDED) 180
  FORMULAS FOR DEPRECIATION AND SALVAGE VALUE 180
    Derivation of formulas 180
    Comparison of formulas 191

XVIII. ERRORS IN ESTIMATING 199
  SIMPLE AND COMPOUND ESTIMATES 199
  COMPENSATION OF ERRORS 201
  ERROR IN FORECASTING FUTURE COST 203

XIX. ENGINEERING REPORTS 207
  INTRODUCTION 207
  THE WRITTEN REPORT 210
    General principles 210
    Specifications for a report 213
    Procedure for preparing a report 216
  SUMMARY 217

APPENDICES

APPENDIX
A. SPECIMEN PAPERS 219
  ARTICLES OF INCORPORATION 219
  CERTIFICATE OF INCORPORATION 221
  BY-LAWS 222

B. TABLES OF FORMULAS AND OF VALUES 228

C. BIBLIOGRAPHY 269

D. QUESTIONS AND PROBLEMS 278

INDEX 299

ENGINEERING ECONOMICS

CHAPTER I

INTRODUCTION

101. The person who has more money or its equivalent than suffices for his current needs seeks to employ the excess or **capital** in remunerative ways.

Such a person is a **capitalist.** The boy with a dollar beyond his wants is a capitalist. He may deposit the dollar in a savings bank with an eye to interest, or buy a dollar's worth of marbles with the hope of selling them at a profit.

102. The opportunities for the remunerative employment of capital lie in the field of rendering **service** satisfying people's wants.[1]

The student's mental picture of the "field of rendering service" may be made more vivid by considering that when a man buys a pound of nails, for example, he is paying for a series of services all rendered to satisfy his need of nails at a certain time and place. These services include mining the ore from which the nails are made, transporting and reducing the ore, conveying the resulting metal to the factory, making the nails, carrying them first to the wholesaler and thence to the retailer, and finally delivering them to the consumer. The list includes only the more obvious classes of service. Every machine that helps in making the nails performs some service to the consumer, and each service of the series of services which transforms ores or other materials into the nail machines contributes to the direct service of making the nails. As far as the consumer is concerned, all of these services are equally important, for if any one of them were not rendered his need of the nails would remain unsatisfied.

---

[1] Some of these wants are of questionable character, as all know. But only a small proportion of the total capital of the country is employed in "anti-social enterprises," and such enterprises are excluded from our consideration.

103. The field of service is the field of **business enterprise.** To convert capital into service is a function of private or public **business units,** the most common of which are (1) the individual business, carried on by the "sole proprietor;" (2) the partnership; (3) the private corporation (or "company"); (4) the public corporation.

104. The true and legitimate purpose of any business unit, private or public, is the rendering of that service which offers the most advantageous employment of capital available to such business unit.

The people make the greatest demand for the service they most want. Satisfying a great demand means a great volume of sales. A great volume of sales goes toward making a maximum·profit. Thus the foregoing paragraph is only an indirect way of saying, "The true and legitimate purpose of any business unit, private or public, is the rendering of that service which the people most want."

105. To render any proposed service the business unit may require some means, such as a machine, building, road, bridge, vehicle, shaft, plot of ground, etc., or a "plant," that is, a combination of some of the means just named. In the following pages the word **structure** will be used as a general name for every such means, from a nail to the most complex plant.

A structure may be simple or complex, but these terms are merely relative. A railroad yard is simple compared with the railroad as a whole, but highly complex compared with one of its component parts, such as a rail or tie or even one of its tracks.

Each part of a complex structure is itself a structure, and in operation performs some contributory service in furtherance of the total service rendered by the complex structure.

106. Many structures are such as to require engineering knowledge to create or operate them. These, whether simple or complex, are called **engineering structures.**

A conspicuous example of a complex engineering structure is the railroad, of which many of the component parts are only less complex engineering structures, as bridges, trestles, tunnels, embankments, track, shops, terminals. Among other well known examples of highly complex engineering structures are water works, gas works, power plants, street railways, sewer systems, mining plants, and various kinds of industrial plants. The creation and operation of such structures, though undertaken by the owners with an eye to profit, either direct or indirect, are nevertheless part of the cooperative effort which has produced our civilization.

107. Service ready for sale is the resultant of a **series of acts,** the first of which is the act of some person's brain in conceiving the idea of the enterprise. In the typical case the acts of the entire series, in order of time, are as follows:

1. Conception of the enterprise.
2. Investigation to determine the volume of service which will probably be required; the feasibility of rendering the service; the cost of rendering it; and the probable return.
3. Organization of the business unit to own, control, and manage the enterprise.
4. Obtaining the capital.
5. Designing, and preparing plans and specifications of any structures necessary for producing the service.
6. Building the structures.
7. Operating the structures and carrying on the production of service.
8. Selling the service.

Considering the acts of the series in their relations to one another, we see that each is rationally a prerequisite of succeeding acts.

108. The **engineer,** as an engineer, takes a part (leading or subordinate according to circumstances and the nature of the enterprise) in one or more of the foregoing acts, as follows: in the investigation (2); in obtaining capital (4); in designing (5); in building (6); and in operating (7); and, where engineering knowledge is requisite for the appreciation or intelligent use of the service, the engineer plays some part also in selling service (8).

The engineer, then, is an important contributor to the public welfare; and, as will soon appear, the business unit is the only **channel** through which he can make his contribution.

109. Investigation, organization, and obtaining capital (acts 2, 3, 4) are the work of the promoter, who may or may not have conceived the idea of the enterprise. Promotion is, itself, a business carried on by a business unit, the owner of which is the promoter. (The promoter may be an individual, or partnership, or other business unit.)

Once the enterprise has passed the stage of organization and capitalization, the new-born business unit initiates and directs the succeeding acts.

Hence it is that although the engineer's work is a necessary and conspicuous part of the service rendered by an engineering structure, his work is initiated, controlled, directed, and judged by the business unit.

110. In fact, the more widely we reflect upon it, the more inclined we are to believe that the business unit is practically the only available medium or agency through which the engineer can convert his professional knowledge into public benefit. Hence the engineer should know not only engineering technic, but also the relation which his technical work bears to the non-engineering work of the enterprise; and this means that he should know something of the principles and methods employed by the non-engineering departments.

The engineer in an enterprise may be likened to the pitcher, for example, in a ball game. The pitcher is distinguished from the other players by his special skill in pitching. But pitching, however skilful, will not of itself win the game. The pitcher must know team work also, and the better he knows it, the greater his value to the team. To be good at teamwork he must know the fundamental principles which control the work of each of the other positions on the team and the methods by which the principles are usually applied.

The engineer should understand the objects (social as well as financial) of the business unit, how it is organized and controlled, how it obtains capital, and how it functions through all its parts in its self-imposed work of transforming capital into service.

The young engineer-draftsman of the engineering department of a railroad spends his working hours applying engineering technology. On promotion his time is given to analyzing problems assigned to him; distributing the disclosed sub-problems among his subordinates; directing, supervising, and checking the work of the subordinates; collecting and amalgamating the solutions of the sub-problems to effect the solutions of the original problems; and reporting the results to his superior. Thus, after promotion, his work is partly technical, partly executive.

At each succeeding promotion the problems assigned to him become more comprehensive and their correct solution requires more knowledge of the work of the non-engineering departments of the road.

When (if ever) he becomes chief engineer, his hours are devoted chiefly to executive duties. He has been singled out for promotion not because he is more skilful in engineering technology than his fellows, but because he has shown (1) a good understanding of the relation of the work of the engineering department to the work of the other departments of the enterprise; (2) loyalty to the enterprise and his superiors; and (3) ability to bring himself and his subordinates to cooperate harmoniously and effectively with the other working units of the enterprise.

If the foregoing is typical of the engineer's work as he advances to greater and greater responsibilities—and it is typical—it is plain that the duties of the engineer require for their performance, at each succeeding stage of his advancement, a greater knowledge of the enterprise as a whole.

The chief engineer must base his hope of promotion to the office of manager, or president, not upon his knowledge of engineering technology, but on his grasp of the larger problems of the enterprise and his ability to coordinate the efforts of the various departments.

111. Engineering Economics deals with those business problems which require more or less knowledge of engineering for their solution. Thus Engineering Economics may be said to deal with the elements of cost of service performed by the engineering structure, the elements of revenue, the inter-relations of these elements, and their resultant effect upon profit.

112. The fundamental problem of Engineering Economics is the problem of investment, and may be stated thus: given two or more opportunities for the employment of capital, to determine which is the most desirable.

Since the problem is essentially the same whether there are only two opportunities or three or more, it will serve every purpose of exposition to consider it as involving only two. To save words let us limit the problem to two opportunities.

The engineer will recognize as particular forms of the foregoing problem of investment the following: (1) to choose between two structures proposed for a specified service; (2) to choose between two proposed ways of operating a given structure to produce a specified service; and (3) to determine which of two proposed rates of production by a given structure will result in the greater net profit.

113. The problem of investment is thus one continually presented to the engineer. It is by analyzing this problem that the principles of Engineering Economics are to be discovered.

CHAPTER II

ANALYSIS OF PROBLEM OF INVESTMENT

INTRODUCTION

201. Through his own inquiries and the solicitations of those who seek capital, the capitalist learns of a number of opportunities for the employment of his available funds. No two opportunities are of precisely equal promise, and it is for the capitalist to choose which among them he will accept. Thus arises the **problem** of making choice between two or more opportunities for investment.

It often happens that only one opportunity is holding the attention of the capitalist; and the question then before him is simply whether or not he shall accept it. Even then, though he is considering apparently only the one specific opportunity, he does, nevertheless, bring to his aid in reaching a decision such knowledge as he possesses of opportunities in general.

As analyzing a problem is a necessary preliminary to solving it, our first efforts must be given to

Analysis

202. To **analyze** a problem is to take it apart, and discover how the parts are related to one another.

As soon as a problem is divided into its major parts, two or more, each major part stands forth as an individual problem.

If the answers to these major problems are ready at hand, the analysis need not be carried further; but if the major problems are, one or more, yet to be solved, then each unsolved major problem calls for analysis. Thus the analysis is carried forward from stage to stage.

203. Of course there can be only one set of ultimate or simple elements in a given problem. As we can divide a township[1] into sections by first separating it into six equal north-and-south strips and then, by east-and-west lines, subdividing each strip into six equal parts; or by first separating the township into six equal east-and-west strips and then subdividing each strip by north-and-south lines into six equal parts; or by otherwise dividing and subdividing; so we can divide the usual problem into its ultimate elements by drawing one set of lines or another. Thus, while the ultimate elements must be the same, regardless of the direction in which the lines are drawn, the compound elements resulting from practical or incomplete analysis will vary according to the directions that may be given to the division lines.

---

[1] The township referred to is approximately square, and contains thirty-six sections each approximately one mile square, bounded by meridians and parallels.

204. In practice the analysis is not usually continued until all of the simple elements have been discovered, but on the contrary is halted in each direction as soon as the result of a step in the analysis discloses factors of known value, whether these factors be simple or compound elements.

Again, the directions in which the lines are drawn in analyzing a problem are often determined more or less by the purpose and circumstances of the analysis.

Chap. v and vi are good examples of two or more analyses of one problem.

205. If the answer to a problem has been found by making the computations indicated by an analysis, this answer can often be more reliably checked by making another analysis on different lines and carrying out the resulting new set of computations; for this process gives a test not only of the accuracy of the first set of computations, but also of the logic of the first analysis. Thus it is frequently desirable to make more than one analysis of a given problem.

We shall not carry forward the analysis of the problem of choice of investment in all detail in natural order, because this would thrust upon the attention of the student in too rapid succession a number of elements beyond his experience, and result in confusing him. The plan adopted is to sketch, in the rough, the first steps in the analysis, thus bringing to light the major component problems, to complete certain parts of the analysis in convenient order, and finally to return to enter more thoroughly into the first steps of the analysis.

A complete analysis of the problem of investment would lead us into several fields: (1) the mathematics of investment; (2) estimating; (3) irreducible data; (4) financial judgment. In this book the analysis is carried into the first three fields, beginning with the mathematics of investment (chap. ii and iv).

Choice

206. In solving a problem of selection between two given opportunities of employing capital, the last step is to make the **choice**; and before entering upon the analysis of the given problem, it may be well to consider some of the elements of choice in general.

207. Intelligent choice between two objects is necessarily preceded by a **comparison** of the objects.

208. Merely to compare any two objects is to place them actually or mentally side by side, and transfer the attention alternately from one or more **qualities** of one to the corresponding quality or qualities of the other, noting **likeness** or **difference,** and if difference, as nearly as may be, its relative or absolute amount.

209. Some qualities have greater influence on choice than others, that is have greater **weight.**

210. The weight that should be given to a particular quality is determined by the **purpose** to which the object is to be put.

Appearance may have much greater weight than cost or durability when pleasure is the purpose of buying an automobile; but if the machine be bought to render service in a business enterprise, appearance may receive little consideration as compared with capacity, power, reliability, efficiency, first cost, or cost of operation.

The weights of many qualities may be zero in any given case. Seldom if ever are all of the qualities of one object compared with the corresponding qualities of another object.

211. Quality comparison is more or less difficult when the quality is not expressed numerically.

We know, let us say, that one automobile is decidedly more pleasing to the eye than another; but we have no unit of appearance, cannot express appearance quantitatively, and consequently cannot express numerically the difference between two appearances that give different degrees of pleasure. (Nevertheless, we are conscious of the difference, and, when called upon to do so, we base a decision on this consciousness with considerable confidence.)

212. Quality comparison is not difficult when the quality is expressed numerically.

For example, if one automobile weighs 3000 lb., and another 2500 lb., the comparison of weights means a subtraction: one weighs 500 lb. more than the other; or, at most, a subtraction and a division: one weighs one-fifth more than the other. So with other qualities capable of numerical expression, such as power, size, speed, life, and cost.

213. However, among qualities which are readily expressed numerically, there are some whose effect on choice cannot be expressed numerically.

For instance, we know that delivery of car A cannot be made inside of 3 mo. and of car B inside of 6 mo. The waiting period, a factor in choosing between cars A and B, is numerically known, and yet there is no ready measure of the effect of length of waiting period on choice.

A quality whose effect on choice can be numerically expressed we shall call a **reducible datum;** and we shall give the name **irreducible datum** to a quality whose effect on choice cannot be so expressed (see §228).

ANALYSIS OF PROBLEM OF INVESTMENT

214. Let us begin the analysis of the problem of choosing between opportunities of employing capital by applying the preceding principles of choosing to the following example.

General Case

Example.—We have the option of leasing a certain house at $900 a year for a period of 4 yr.; we can furnish the house at a cost of $4000, sublease it at $2000 a year, and sell the furnishings for $1600 at the expiration of the lease. The cost of collecting rent, of insurance, etc., will be about $100 a year. On the other hand, for the same period, we can loan $6000 at 8% with good security. Which opportunity shall we choose?

First Solution (by General Principles and Natural Procedure)

215. Among the data of the example we recognize each of the named sums as a pertinent quality, a quality affecting choice. No other qualities are specifically named. Does this mean that there are no other pertinent qualities? Let us consider the matter.

By visualizing, for each of the proposed investments, the probable pertinent events and circumstances of the 4 yr. period, and calling past experience to our aid, we discover a number of points germane to the question, as follows:

Leasing and furnishing the house, subletting, collecting rent, and selling the furnishings would give us a certain amount of trouble, care, and responsibility; none of which would attend the loan.

In the house opportunity the risk of losing periodic income or part of capital invested in furnishings would be considerable; but in the case of the loan, let us say, these risks would be nearly negligible. Finally, if the capital were once put into furnishings, the possibility of withdrawing it without sacrifice before the end of the 4 yr. period would be slight; while the note probably could be sold for cash at any time quite readily at little or no sacrifice; and this is a consideration of some importance unless we are sure that we shall have no need to withdraw the capital before the end of the 4 yr.

216. It is obvious that all of the foregoing characteristics or qualities (including those expressed numerically) of the two opportunities, should influence the choice that is before us. It is through a comparison of these qualities that we must judge the relative desirabilities of the two opportunities.

217. To make the work as easy as possible, let us tabulate the foregoing data. See Table 217.

TABLE 217.—COMPARATIVE STATEMENT

	Quality	Opportunity		Difference in favor of	
		I (House)	II (Loan)	I	II
	Income:				
1	Rent or interest.......	$2000 per annum	8% ($480) per annum	$1520 per annum	
2	Capital recovered...	$1600 lump	$6000 lump	....	$4400 lump
	Outgo:				
3	Capital invested.....	$4000 lump	$6000 lump	$2000 lump	
4	Rent paid..........	$900 per annum	None	....	$900 per annum
5	Expense paid.......	$100 per annum	None	....	$100 per annum
6	Trouble, care and responsibility.......	Considerable	None	....	x
7	Risk of loss.........	Considerable	Negligible	....	y
8	Marketability of investment.........	Slight	Good	....	z

218. As we look at Table 217 and compare, quality by quality, we see that the data \$2000 and 8%, opposite quality 1, are not directly comparable; and to make them comparable, we must either find the per cent ratio of 2000 to 4000 (50% it is), or we must take 8% of 6000 (giving us \$480). We have adopted the latter procedure, and have written \$480 in parenthesis after "8%." Now we have a direct comparison of \$2000 and \$480; and we note a difference of 2000 − 480 = \$1520 per annum in favor of opportunity I.

On the basis of quality 2, opportunity II is the more advantageous by the lump sum \$4400.

On the basis of quality 3, opportunity I is better than opportunity II by \$2000 lump.

On the basis of quality 4, opportunity II has the advantage by \$900 per annum.

On the basis of quality 5, also, opportunity II has the advantage by \$100 per annum.

219. The comparison of the two opportunities is now completed, as far as the numerical data are concerned, and the differences discovered have been entered under "Difference" in the fourth and fifth columns of the table.

Looking at the two difference columns, we note that in favor of opportunity I we have \$1520 per annum and \$2000 lump; and in favor of II, \$4400 lump and 900 + 100 = \$1000 per annum. Lump payments cannot be directly compared or combined with annual payments. The best we can do, from entries in the difference columns, is to say that in regard to annual sums, I has the advantage of II by 1520 − (900 + 100) = \$520 per annum; and that in regard to lump sums, II has the advantage by 4400 − 2000 = \$2400 lump. This leaves us too far from a choice.

We can approach nearer by converting the \$2000 lump into a corresponding annual sum: $^{2000}/_{4}$ = \$500. (This \$500 is only approximately the annual sum corresponding to \$2000, as will be apparent after reading chap. III.) With the \$2000 lump thus converted into \$500 per annum, we can say that the advantage of opportunity I is 1520 + 500 = \$2020 per annum. In the same way, converting the \$4400 lump, we find the corresponding annual sum is $^{4400}/_{4}$ = \$1100. Thus the advantage of II reduces to 1100 + 900 + 100 = \$2100 per annum.

The numerical advantages of opportunity I being now reduced to the one expression, \$2020 per annum, and those of II to the one expression, $2100 per annum, we note that the net advantage is 2100 − 2020 = $80 per annum, and lies with opportunity II. In other words, so far as numerical qualities are concerned, II is better than I by $80 per annum. However, it must be remembered that the opportunity offering the $80 advantage in yield, requires $2000 more capital. The question then resolves itself into what yield could be secured from the $2000 elsewhere if opportunity I were accepted (see §223).

Note.—Instead of converting lump sums into annual sums as we have done, we might have converted the annual sums in the difference columns into lump sums; and the net advantage of II would have been found to be $320 lump—which is the same (approximately) as $80 per annum.

220. In pursuing the comparison above, the steps taken have been guided solely by the principles of comparison previously stated. We have drawn in no way on business experience. Thus the effectiveness of the principles has been demonstrated.

However, the procedure used in current practice is far more convenient than that used above, while at the same time it conforms to the same principles.

Second Solution (by General Principles and the Procedure of Current Practice)

221. Let us make a new start on the solution of Example 1 (which is repeated below for convenience)—this time conforming to current practice.

Example (repeated).—We have the option of leasing a certain house at $900 a year for a period of 4 yr.; we can furnish the house at a cost of $4000, sublease it at $2000 a year, and sell the furnishings for $1600 at the expiration of the lease. The cost of collecting rent, of insurance, etc., will be about $100 a year. On the other hand, for the same period, we can loan $6000 at 8% with good security. Which opportunity shall we choose?

222. In practice it is usual first to combine numerical qualities of income and outgo into one derived quality, namely, annual yield. The calculations are shown in Table 222.

TABLE 222.—CALCULATION OF YIELD

Opportunity I

Income (annual rental)			$2000	
Capital invested (first cost of furnishings)	$4000			
Capital recovered (salvage from furnishings)	1600			
Total depreciation	$2400			
Annual depreciation cost[1]		$600		
Annual rental paid	900			
Expense	100			
Annual operation cost		1000		
Service[2] cost			1600	
Yield (money)			400	
Yield, per cent $({}^{400}/_{4000})100$				10

Opportunity II

Income (interest on $6000 at 8 %)			$480	
Capital invested	$6000			
Capital recovered	6000			
Annual depreciation cost		0		
Annual operation cost		0		
Service cost			0	
Yield (money)			$480	
Yield, per cent				8

223. The money yields ($400 for opportunity I and $480 for II) are directly comparable, and indicate an advantage for II of $480 − 400 = $80. But, the per cent yields (10% for opportunity I and 8% for II) on the contrary indicate an advantage for opportunity I of 10 − 8 = 2%. That is, the opportunity giving the greater money yield, in this example, gives the smaller per cent yield.

---

[1] This $600, found by dividing the total depreciation by the number of years, is of course the *average* annual depreciation, and is the true annual depreciation cost only on the supposition that the rate of interest on the sinking fund is zero. (The true annual depreciation cost, assuming 4% on sinking fund, is $565.20, for the calculation of which see §401.) Annual depreciation cost must not be confused with the actual annual drop in the value of the furnishings (see §403).

[2] The "service" rendered in opportunity I is that of supplying some one's need of furnished quarters. The service rendered in opportunity II is that of satisfying directly some one's desire for the loan of $6000.

How then shall choice be made in cases such as this, where money yields and per cent yields give opposite indications? The answer is to be discovered by stating the situation a little differently: by employing $4000 in I we can obtain a $400 yield; or by putting in an *extra* $2000 and taking II we can obtain an *extra* $80 in yield. A money yield of $80 from $2000 investment is 100 ($^{80}/_{2000}$) = 4 %.

224. Evidently, as far as yield is concerned, the choice between the two opportunities hinges on whether or not we regard 4% a satisfactory yield on capital. Wellington, forty years ago, said:

> No increase of expenditure [in first cost] over the unavoidable minimum is expedient or justifiable, however great the probable profits and value of an enterprise as a whole, unless the INCREASE can with reasonable certainty be counted on to be, in itself, a profitable investment. Conversely, no saving of expenditure [in first cost] is expedient or justifiable, however doubtful the future of the enterprise as a whole, when it can with certainty be counted on that the additional expenditure at least will, at the cost of the capital to make it, be in itself a paying investment.[1]

225. We have gone as far in the matter of choice as is possible in a comparison limited to the data of income and outgo. We next enter upon a consideration of the last three qualities (of Table 217), namely: (6) trouble, etc., (7) risk of loss, and (8) marketability.

Comparing opportunities I and II on the basis of quality (6), we can be sure that the advantage is on the side of II, but we can assign no numerical value to this advantage. (These two facts are indicated in the table by writing x in the right hand column.) Similarly, comparisons on basis of quality (7) and quality (8), assure the advantages to II, but these advantages are incapable of expression except in general terms; hence y and z in right hand column.

226. The progress so far made toward a choice between the two opportunities described in the example, may be concisely set forth as in Table 226.

---

[1] *Economic Theory of Railway Location* (New York: John Wiley & Sons).

Table 226.—Comparative Statement

	Opportunity		Difference in favor of	
	I (House)	II (Loan)	I	II
Capital required	$4000	$6000	$2000	
Money yield (an $80 yield on $2000 is a 4% yield)	400	480	.....	$80
Trouble, care and responsibility	Considerable	None	.....	x
Risk of loss	Considerable	Negligible	.....	y
Marketability	Slight	Good	.....	z

227. An inspection of Table 226 makes it clear that (1) if we regard 4% a satisfactory return on the additional capital required in opportunity II, the calculated advantage is with II; and therefore all the advantages are with II, since x, y, and z favor II; and hence we must logically decide for II. But (2) if 4% is held to be unsatisfactory, the calculated advantage lies with I; and our final decision rests on whether or not we give to this advantage greater weight than we give to the contrary advantage $x + y + z$. Probably the reader would join the author in calling 4% an unsatisfactory yield; and the degree of unsatisfactoriness might readily be agreed upon by comparing the 4% with current yields on capital employed in various remunerative ways. But no two persons could be expected to give precisely the same weight to $x + y + z$, for this weight is necessarily affected by the personal equation. One person will regard lightly the "trouble, care, and responsibility;" while another, of different temperament, may go so far as to say that he would not shoulder the trouble, care, and responsibility under any circumstances.

Irreducible Data

228. The reader, while following the comparisons above, no doubt noted the fact that in the course of the work all of the qualities of the investments arranged themselves in two sharply defined, mutually exclusive classes: the first embracing the qualities that are, the second those that are not, capable of being reduced to a single numerical value. As we shall have frequent occasion to distinguish the one class from the other or to assign a quality to either one class or the other, it is desirable to give to each class a specific name. The following names have been chosen as being descriptive of the two classes: **reducible data** and **irreducible data.**

SPECIAL CASES

229. The principles and procedure developed in connection with the foregoing general example are applicable to all cases in which choice must be made between various opportunities for the employment of capital. However, among the special cases there are those whose distinguishing features are of interest sufficient to justify some description here.

230. Special Case 1. Equal Capital.—If in the foregoing example, the required capital for opportunity II had been the same as for I, namely, $4000, the reduction of numerical data would have given the results shown in Table 230.

TABLE 230.—COMPARATIVE STATEMENT

	Opportunity		Difference in favor of	
	I (House)	II (Loan)	I	II
Capital required..........	$4000	$4000	$0	$0
Money yield..............	400	320	80	
Per cent yield............	10	8	2	
Irreducible data...........	....	....	....	$x + y + z$

Thus where the opportunities under comparison require equal sums of capital, the comparison of reducible data reduces to only a difference between yields; and the calculable advantage is with the greater yield.

231. Special Case 2. Incomes Unknown but Equal.—Consider the following: A manufacturer finds that his expanding business requires the erection of an additional building. He decides to erect the building of either wood or brick, whichever promises to be the better investment.

If he could make a reasonable estimate of income to be expected from the building, he could choose on the basis of yield and irreducible data. But since the building is to render some auxiliary service furthering the chief service that he sells, he would find it impracticable if not impossible to put any satisfactory evaluation on income from the building. All he can say of the promised income is that it will be the same whether the building be of wood or of brick. Not knowing the income, he cannot know the yield. How then is the manufacturer to choose between the two investments?

Suppose his data to be as follows: Anticipated duration of service is 15 yr. The brick would cost $48,000; cost practically nothing for repairs; and sell at the end of 15 yr. for $3000. The wooden shop would cost $31,000; its upkeep annually $2000; and it would sell for $1000 at the end of the period. Which to build?

Let us apply the procedure of the general case to these data, as in Table 231.

TABLE 231.—CALCULATION AND COMPARISON

	BRICK	WOOD
First cost	$48,000	$31,000
Salvage value	3,000	1,000
Total depreciation	$45,000	$30,000
Annual depreciation cost	3,000[1]	2,000[1]
Annual upkeep, etc.	0	2,000
Annual service cost	3,000	4,000
Annual income (same for both—call it M)	M	M
Annual yield	M – 3000	M – 4000

COMPARISON

	Opportunity		Difference in favor of	
	Brick	Wood	Brick	Wood
Capital required	$48,000	$31,000	....	$17,000
Yield	M-3000	M-4000	$1,000[2]	

Evidently the choice hinges on whether or not the manufacturer regards favorably a yield of $1000 on $17,000, that is, 5.9%. (Very likely 5.9% would not appeal to him, especially if he were borrowing money at 6 or 7%.)

---

[1] These values are approximate, being the result of taking 1/15 of the total depreciation. For the exact method of calculating annual depreciation cost see §§401, 402.

[2] (M-3000) – (M-4000) = 1000. Thus the equal though unknown incomes present no obstacle to our calculating the *difference* between the money yields.

To the reader who has followed the example just above and its solution it is obvious that this special case offers no obstacle to the application of the method used in the solution of the general example. It is plain too that in special cases like this the difference between the two money yields is equal to the difference between the two service costs.

232. It should interest the student of engineering to know that by far the greater proportion of investment problems which engineers are called upon to solve are similar to the one which we have just been considering.

The promoter of a railroad enterprise causes estimates to be made of prospective income as well as of prospective costs. But no estimate is made of income from service rendered the enterprise by a bridge, a tunnel, the roadbed, a freight or passenger station, a fence, and so on.

233. Special Case 3. The Single Opportunity.—Often the capitalist has under consideration a single opportunity for investment, and then his question is: Shall I accept this opportunity?

In each case the capitalist undertakes to answer the question by ascertaining the qualities of the investment, including yield; and comparing these qualities with some imaginary standard set of qualities which he holds in his mind as pertaining to the least advantageous opportunity which he believes acceptable to him. Thus if the offered opportunity lies in the retail grocery business and promises a yield of, say, 15% his decision to accept or reject the offer will be based partly on his idea as to what the minimum yield from that kind of enterprise should be in order to make it inviting.

In this connection, it may be pertinent to add that the minimum acceptable per cent yield varies from business to business. For example, a mining opportunity, which promised a yield no greater than that of an ordinary grocery business, would be declined. This difference in acceptable yield is of course owing to the fact that the yield is the factor which, by competition between capitalists and between opportunities, is adjusted to make all opportunities approximately equal in acceptability when all qualities are considered.

234. Reviewing these special cases, we see that though they differ from the general case, which we solved first, their solution is readily effected by the same principles and the same procedure.

PREREQUISITE TO FURTHER ANALYSIS

235. The progress so far made in the analysis of the problem of choice of opportunities for investment, is shown by the diagram, Fig. 235.

- Choice of investment (based on)
  - Yield (annual) Opportunity I
    - Income
    - (−)
    - Service cost
      - Depreciation
        - First cost
        - Salvage value
        - Life
      - (+)
      - Operation
  - Yield (annual) Opportunity II
    - (Same subdivisions as for opportunity I above.)
  - Irreducible data

FIG. 235.—Analysis of problem of investment.

It may be well to state for the benefit of the student who has never before met a diagram of analysis, that the *diagram is no part of the analysis* itself. The diagram is a memorandum list of disclosed elements, but differs from an ordinary memorandum list in this, that the names of the elements are so arranged that their space relations on the page correspond as nearly as may be to the logical relations between the elements. Thus the diagram is a sort of graphic representation of the results of the analysis. Analysis diagrams have been a material help to me in making the analysis discussed in this book, and such diagrams are freely used in the following pages in the belief that they will be equally helpful to the student in reading the book.

The diagram, on its face, indicates that we could choose between two opportunities were we given all the elements: income, first cost, salvage value, life, operation cost, and irreducible data. But it must be remembered that while in the analysis of service cost we found the *true qualitative* relations between depreciation and its elements: first cost, salvage value, and life; we found only the *approximate quantitative* relations. The *true* quantitative relations will be developed in chap. IV. The development involves compound interest and sinking funds. If the reader has already a working knowledge of the latter he may turn at once to chap. IV and continue the analysis of cost of service; otherwise he must defer reading chap. IV until he has made the fundamental principles of chap. III his own.

CHAPTER III

INTEREST

Time Value of Money, Simple Interest, Compound Interest, Sinking Funds, Annuities, etc.

TIME VALUE OF MONEY

301. It is the universal custom for the lender of money to require the borrower to pay money for the use of the money borrowed. The lender is said to charge **interest** for the use of his money. When, in conformity to some agreement between two persons, A and B, A must pay to B a sum of money on a named date, called the *due date,* the sum is said to be **due** on that date, and if not paid when due, becomes **overdue,** and so remains until paid. If A lets the sum become overdue, it is reasonable that he should pay interest to B on the sum during the time that it remains overdue; because the fact that the demand by borrowers always exceeds the supply of money in the hands of lenders makes it fair to assume that were the sum paid when due, B could loan it at interest. This growth of money in time must be taken into account in all combinations and comparisons of payments. It is the object of this chapter to show the law of this growth and its effects on a number of common business transactions.

SIMPLE INTEREST

302. When the interest is made to be directly proportional to time, it is called **simple** interest. The elements of problems in simple interest are: (1) the principal; (2) the interest (a sum of money); (3) the time; (4) the rate; and (5) the amount.

1. The **principal** is the sum of money, borrowed or otherwise owing, upon which interest is paid or to be paid.

2. The **interest,** as stated above, is the sum paid for the use of the principal.

3. The **time** is that during which the principal is owing by the debtor to the creditor. The dates on which interest falls due are called **interest dates,** and the interval between two

successive dates is called an **interest period.** The interest period is uniform in any one agreement or transaction; but is variously taken as one year, one half-year, one quarter-year, or one month.

A common custom in computing interest is to assume that each month contains 30 da., and each year **360** da. Nevertheless "the courts invariably rule that the year to be used in all interest calculations is the period of **365** da."[1]

4. The **rate** is the ratio between the simple interest earned by the principal in one interest period, and the principal. That is,

$$\text{rate} = \frac{\text{simple interest on principal during one interest period}}{\text{principal}}.$$

The interest period is variously taken as a month, a quarter, a half-year, or a year. However, when a rate is mentioned without specifying the length of the interest period on which it is based, it is always understood that the interest period is 1 yr.

As an example of interest rate: If $16 is paid for the use of $200 for 1 yr., the rate is, by definition, $^{16}/_{200}$ or, as a decimal fraction, 0.08. Again, if $75 is paid for the use of $1000 for one year, the rate is $^{75}/_{1000}$ or 0.075. In practice the rate is not commonly stated directly as we have stated it in the two examples above, but is indirectly expressed by specifying how many units of simple interest one hundred units of principal earn in 1 yr. Thus "$^{16}/_{200}$" becomes "eight per hundred," and "$^{75}/_{1000}$" is the same as "seven and one-half per hundred." However, instead of "per hundred" the Latin equivalent "per centum" (shortened usually to per cent) is universally employed; and the mark, %, is a kind of shorthand symbol for "per cent." So the customary way of expressing the fact of the first example is "$200 at 8%;" and of the second example, "$1000 at 7½%."

It should be noted that the rate as defined above is not 8 in the first example nor 7.5 in the second; but is in the first the ratio $^{8}/_{100}$ or 0.08, and in the second is the ratio $^{7.5}/_{100}$ or 0.075.

5. The **amount** is the sum of principal and interest for any specified period of time.

303. Formulas for the solution of problems in simple interest can be written directly from the definitions given in the preceding sections; and will be found in Table I (see under "Tables" in Index).

---

[1] Watson, James C., *Interest, Discount, and Investment Tables* (Ann Arbor, Michigan: George Wahr).

304. Example.—A note for $280.50, bearing 6% simple interest, is dated Sept. 22, 1905, and falls due June 17, 1907. What is the total simple interest on the note for the full time?

The total interest, I, is the product of the three quantities: principal, P; rate, i; and time, n, expressed in years; that is $I = Pin$ (see Table I).

Solution on the basis of a 365-day year:

Sept. 22, 1905 to Sept. 22, 1906	365 days
Sept. 22 to 30	8 days
October	31 days
November	30 days
December	31 days
January	31 days
February	28 days
March	31 days
April	30 days
May	31 days
June	17 days
Total time	633 days

Therefore $n = {}^{633}/_{365}$,

and the total interest is

$$\begin{aligned} I &= Pin \\ &= 280.50 \times 0.06 \times {}^{633}/_{365} \\ &= \$29.19. \quad \textit{Answer.} \end{aligned}$$

Solution on the basis of a 360-day year:

$$\begin{array}{lclll} \text{June 17, 1907} & = & 1907 & 6 & 17 \\ \text{Sept. 22, 1905} & = & 1905 & 9 & 22 \\ \hline \text{Time} & = & 1 \text{ yr.} & 8 \text{ mo.} & 25 \text{ da.} = 625 \text{ da.} \end{array}$$

Therefore $n = {}^{625}/_{360}$,

and the total interest is

$$\begin{aligned} I &= Pin \\ &= 280.50 \times 0.06 \times {}^{625}/_{360} \\ &= \$29.22. \quad \textit{Answer.} \end{aligned}$$

Of this example, another solution, employing tables, will be found adjacent to Tables G and J (see under "Tables" in Index). Note that the rate in this example is not "6" but "6 %," that is, $^{6}/_{100}$ or 0.06.

COMPOUND INTEREST

Elements

305. In simple interest, as we have seen, interest is computed only on the original principal. In compound interest, interest is computed not only on the principal but also on all unpaid interest which is overdue. The following example is a sort of of moving picture of the **process** of compound interest.

Let the principal be $1, the rate 4%, and the interest period 6 mo. If the interest is paid at the end of each half-year, each payment is $1 \times {}^{0\cdot04}/_{2} = \$0.02$. But if no interest is paid till the end of the year, the interest due at the end of the first half-year (namely $0.02) is added to the principal, $1, forming a new principal, $1.02. The new principal, $1.02, draws interest for the second half-year, and the interest due at the end of the second half-year is therefore $1.02 \times {}^{0\cdot04}/_{2} = \$0.0204$. This interest, $0.0204, is added to the principal $1.02, making a newer principal, $1.0404, which draws interest during the third half-year; and so on.

306. Let us repeat the steps of the compound interest process described above, using letters to represent the several elements.

Let P = the principal;
j = the rate per interest period;
q = the number of interest periods in the time considered;
S = the amount of P at the end of q periods.

During the first interest period the (original) principal P earns the interest Pj and amounts to $P + Pj$ or $P(1 + j)$. That is, the amount for one interest period is equal to the principal that is running during the period, multiplied by one-plus-the-rate. During the second period the new principal $P(1 + j)$ earns interest $P(1 + j)j$ and amounts to $P(1 + j) + P(1 + j)j$, or $P(1 + j)(1 + j)$, or $P(1 + j)^2$; which in turn as a newer principal drawing interest during the third period, amounts to $P(1 + j)^2(1 + j)$ or $P(1 + j)^3$; and so on to the end of the qth period, when the amount will be

$$S = P(1 + j)^q \qquad (306)$$

307. This equation expresses the **law** of compound interest, which in words is: The amount of a sum of money at compound interest during a given number of interest periods is equal to the said sum multiplied by one-plus-the-rate-per-period raised to a power whose exponent is the number of interest periods.

308. Inasmuch as the most commonly specified rate of interest is the rate for which the interest period is the year, it is desirable to express the equation above in terms of the **annual rate.**

Let i = the annual rate (that is the simple interest on one unit of principal during 1 yr.);
h = the number of interest periods in 1 yr.;
n = the number of years.

Then $j = i/h$; $q = nh$; and substituting these values of j and q in eq. 306 we have

$$S = P(1 + i/h)^{hn} \tag{308}$$

Thus, for example, the amount of \$100 for 4 yr. at 6%, payable semi-annually, is $S = \$100(1 + {}^{0.06}\!/_{2})^{2 \times 4} = 100(1.03)^{8}$.

The annual rate, if compounded more than once a year, is often called **nominal rate** in contrast to effective rate discussed below.

309. At times it is desirable to know what rate of interest at annual compounding would have the same cumulative effect as a given annual rate compounding a given number of times—twice or four or twelve times—a year. Such a rate is called the **effective rate.**

For example, I have a chance to loan a sum at 6% per annum payable monthly; and, as I am in the habit of thinking of rates on the basis of annual compounding, I wish to know the rate which compounding annually would give the same return as the present offer.

The formula for calculating the effective rate is obtained by equating for the same time the expression for amount in terms of the given rate and the expression for amount in terms of the effective rate, and solving for the effective rate. See Table II (which may be located by consulting the Index).

For every effective rate there is an equivalent nominal rate compounded two, or four, or any other number of times a year.

310. It is convenient to have a single symbol to represent the amount of one unit of principal for 1 yr. If we let s be this symbol, then its value in terms of i and h must be

$$s = (1 + i/h)^{h} \tag{310}$$

It will be observed that as $(1 + i/h)^{h} = 1 + e$ (e being the effective rate), eq. 310 may be written thus: $s = 1 + e$.

By use of eq. 310, eq. 308 can be abbreviated to

$$S = Ps^{n} \tag{310a}$$

311. The growth of a sum of money under the operation of compound interest is sometimes called **accumulation** of that sum. To **accumulate** a sum to a given date is to find the amount of the sum on that date. The quantity s^n is then referred to as the **accumulation factor.**

Table A (see under "Tables" in Index) gives values of the accumulation factor, s^n, for various values of n and i, based on annual compounding. On the page facing the table will be found examples illustrating the use of the table.

312. A common problem in compound interest is to find the sum which under given interest conditions will accumulate or amount in a given time to a stated sum; or, in customary terms, to find the present value or **present worth** of the stated sum. Obviously the present worth corresponds to principal, P, in eq. 310a ($S = Ps^n$) and the "stated sum" corresponds to amount S therein. Thus present worth in terms of the stated sum is:

$$P = S(1/s^n) \tag{312}$$

For example, the present worth of \$100 due 4 yr. hence, if money is worth 6% per annum payable annually, is $P = 100[1/(1.06)^4]$.

Other expressions sometimes used to signify present worth, are **present justifiable expenditure** and **capitalized value** or capitalized cost (see also § 320).

313. The quantity $1/s$ is so frequently used that it is convenient to represent it by a single letter. Let this letter be p. Then

$$p = 1/s \tag{313}$$

and, of course, $p^n = 1/s^n$, and eq. 312 can be written,

$$P = Sp^n \tag{313a}$$

314. The operation of finding the present worth, for a given date, of a sum payable on a later date is sometimes referred to as **discounting.** For example, instead of saying "Find the present worth, for Jan. 1, 1920 of \$100 payable Jan. 1, 1930," we may say, "Discount to Jan. 1, 1920, \$100, payable Jan. 1, 1930." The quantity p^n is then referred to as the **discounting factor.** Since $p^n = 1/s^n$, or s^{-n}, the discounting factor is the reciprocal of the corresponding accumulation factor.

Table B (consult Index) gives values of the discounting factor, p^n, for various values of n and i on the basis of annual compounding. It will be observed that each value in Table B is the reciprocal of the corresponding value of Table A. The use of the table is illustrated on the page opposite the table.

It should be noted that there are two dates concerned in calculating the present worth of a single sum, namely, the date for which present worth is to be figured, and the due date. If the former date is stated in a problem, the computer must imagine himself, in point of time, on that date; but if the former date is not stated directly or indirectly, he figures the present worth as for the actual date on which he makes the calculation.

315. Formulas for **rate** and for **time,** obtained by solving eq. 308 for i and n respectively, are given in Table II (consult Index).

In solving eq. 308 for n, the first step is to divide both sides by P, and, next, to pass to logarithms, obtaining $\log (S/P) = hn \log(1 + i/h)$.

Time Schedule

316. The discussion of the application of the law of compound interest to various transactions will be much facilitated by the use of the time schedule, which is neither more nor less than a graphic calendar.

The time schedule is no part of the laws of compound interest, nor is it necessary in any way to the understanding of those laws. It is merely a device by means of which time relations between payments can be taken in by the eye at a glance. Such schedules have been time and error savers to me and have been used in the text in the hope that they may be equally helpful to the reader.

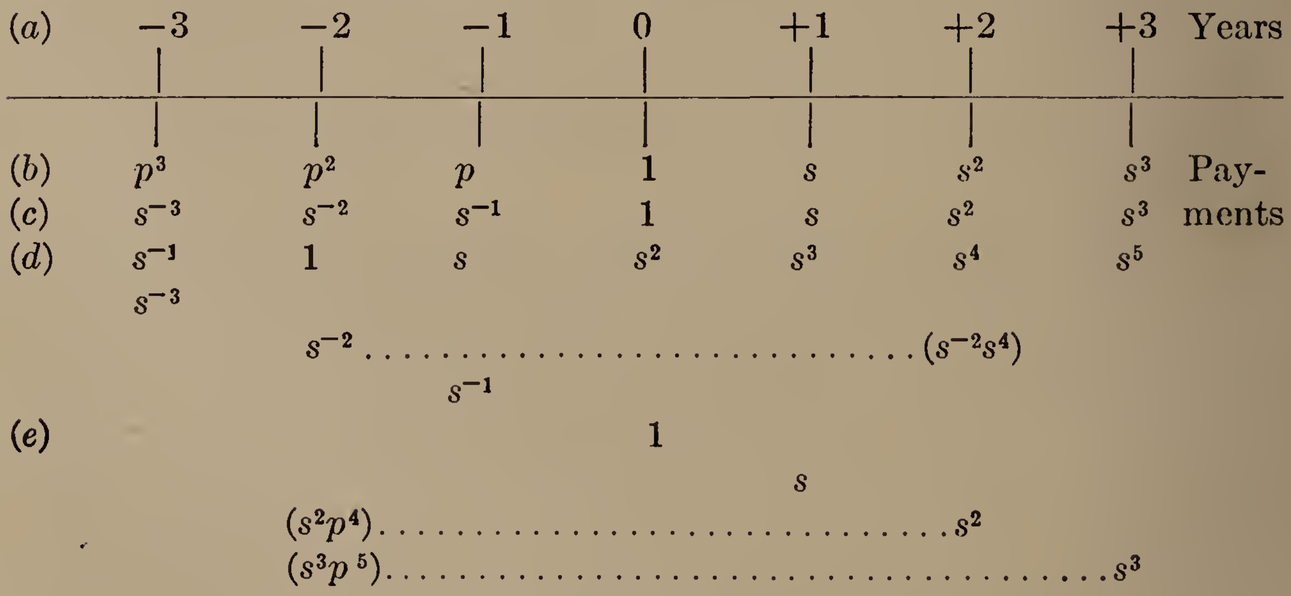
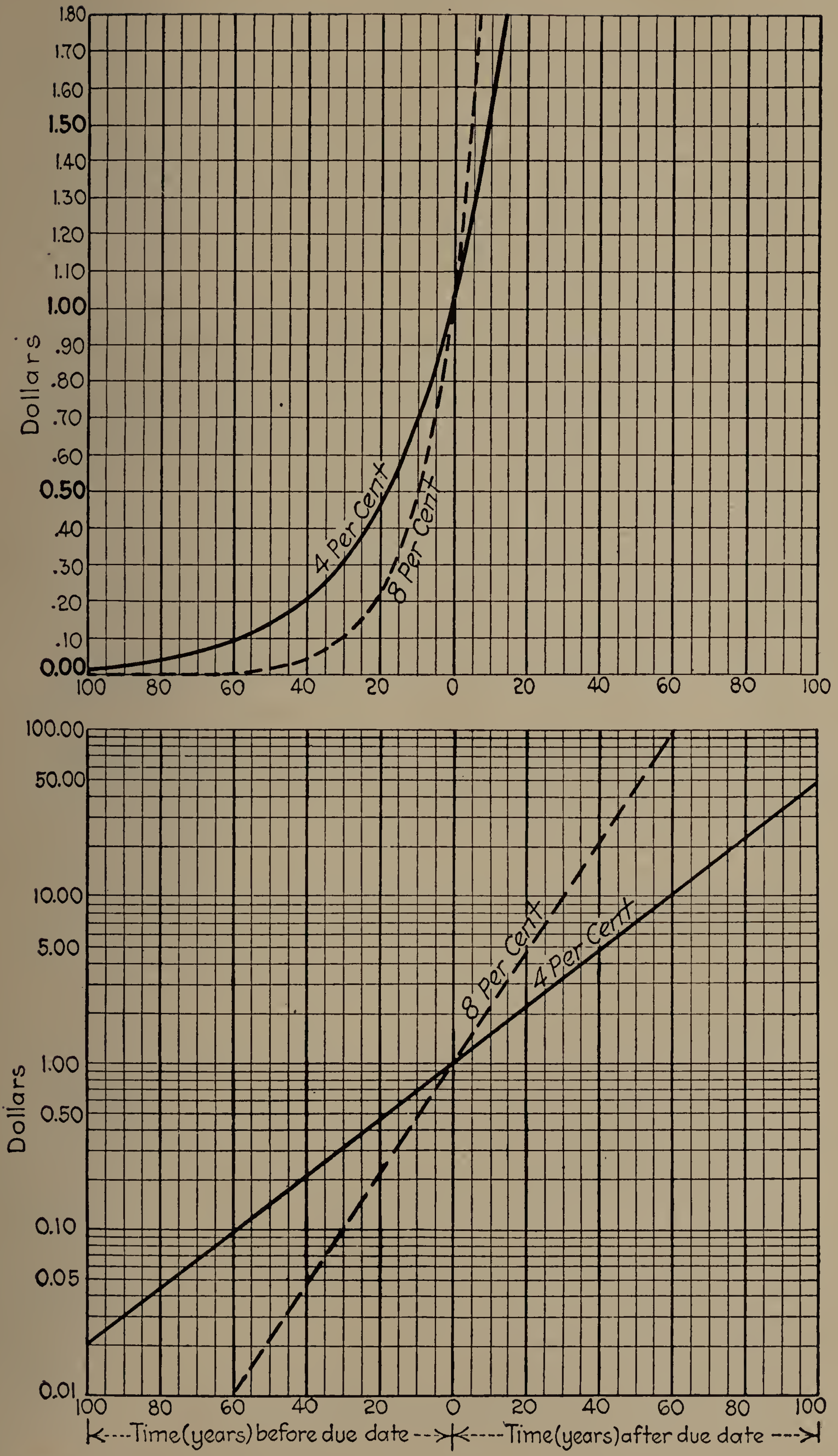

(a) −3 −2 −1 0 +1 +2 +3 Years
(b) p^3 p^2 p 1 s s^2 s^3 Pay-
(c) s^{-3} s^{-2} s^{-1} 1 s s^2 s^3 ments
(d) s^{-1} 1 s s^2 s^3 s^4 s^5
s^{-3}
s^{-2} .... $(s^{-2}s^4)$
s^{-1}
(e) 1
s
(s^2p^4) .... s^2
(s^3p^5) .... s^3

FIG. 316a.—Time schedule.

The paper is divided into equal intervals, right and left, each representing a chosen unit of time, and the intervals, or the division marks, are labeled with appropriate figures or letters; so that the whole forms a time scale, or graphic calendar, or time schedule. The graduated line (*a*) of Fig. 316a, with the figures written above, is such a time schedule.



FIGS. 316b, 316c.—Graphs of compound interest law. [FIG. 316b (upper): arithmetic scales; FIG. 316c (lower): arithmetic and logarithmic scales.]

In this particular schedule the scale is so figured as to measure time both forward and backward from the division point labeled 0. On line (*b*), below each date mark, we have recorded a symbol representing a payment. The payment recorded below the zero date, 0, is 1.

Each of the payments recorded in line (*b*) will be recognized as the accumulation or amount, for 1 yr., of the next preceding payment; and each payment will also be recognized as the present worth of the next succeeding payment.

Line (*c*) is precisely the same as line (*b*) except that in line (*c*) p has been replaced by its equal $1/s$ or s^{-1}. Line (*c*) shows the symmetry of the discount factors and accumulation factors about the 1.

Multiplying all of the factors of line (*c*) by some power of s, as s^2 say, we obtain the series of factors shown on line (*d*). It is to be noted that the only effect of the multiplication has been to shift the whole series to the left by a space representing 2 yr. of time.

By staggering the payment symbols on the schedule, so as to give each symbol a horizontal line to itself, as shown on line (*e*), this decided advantage is gained: the symbol for the result of discounting or accumulating each payment to any date can be written in its correct position on the schedule without confusion. For example, s^{-2} accumulated 4 yr. gives $s^{-2}s^4$ which is shown in parenthesis on its correct date. Similarly, s^2 discounted 4 yr. and s^3 discounted 5 yr. give respectively s^2p^4 and s^3p^5 which are shown in parentheses on the same correct date.

In Figs. 316b and 316c will be found two graphic representations of the law of compound interest. The yearly ordinates for each curve correspond to the series $s^{-n} \ldots 1 \ldots s^n$ of Fig. 316a. The two figures, 316b and 316c are plotted from the same data: the difference in the resulting "curves"—in one case curved, in the other straight—results from the fact that in Fig. 316c the vertical scale is logarithmic.

EQUIVALENCE

317. It should be manifest that each payment of the series s^{-3}, s^{-2}, s^{-1}, 1, s, s^2. . . is the **equivalent** of each of the other payments; that is to say, if A owes B a sum s^{-3} payable on a certain date, it is a matter of indifference to both men, as to whether A shall pay B s^{-3} on the date named, or $(s^{-3})s = s^{-2}$ a year later, or $(s^{-3})s^2 = s$ two years later, and so on—provided the rate used in calculating s is satisfactory to both A and B.

If I deposit on Jan. 1 a sum P in a savings bank paying 4% compounding semi-annually, the bank will give me my choice of: $P(1.02)$ six months later; $P(1.02)^2$ one year later; $P(1.02)^3$ a year and a half later; and so on. And if I should deposit a sum equal to $P(1.02)^{-2}$, they would let me have my choice of: $P(1.02)^{-2}(1.02) = P(1.02)^{-1}$

six months later; $P(1.02)^{-2}(1.02)^2 = P$ one year later; $P(1.02)^{-2}(1.02)^3 = P(1.02)$ a year and a half later; and so on.

To say, then, that a sum P payable on a certain date is equivalent to a sum Q payable n yr. later, is to say that P will amount to Q on Q's date, or to say that Q discounted to P's date will equal P. The equations

$$Ps^n = Q$$
$$P = Qs^{-n}, \text{ or } Qp^n$$

which severally express the equivalence of P to Q, are called **equations of value.**

The relation between P and Q is not changed, of course, by multiplying or dividing either equation by any number. Dividing the first equation by s^n results in the second equation. Dividing the first equation by say s^2, gives $Ps^{n-2} = Qs^{-2}$,—also an equation of value. We see that dividing by s^2 has the effect of accumulating P and discounting Q to a common date which is 2 yr. earlier than Q's date. Obviously, then, if any two payments of different dates reach equality when reduced by accumulation or discount to any common date, the two payments are equivalent; and vice versa.

318. A **common problem** in equivalence is to find a sum P which payable on date t_4 say, shall be equivalent to say three other sums Q_2, Q_5, and Q_6 payable respectively on dates t_2, t_5, and t_6 (see Fig. 318).

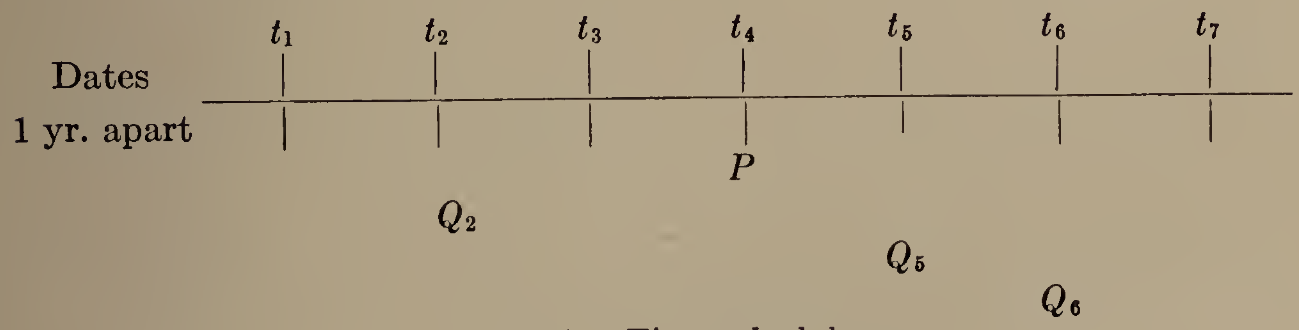


FIG. 318.—Time schedule.

If P is the equivalent of the three other payments, then after all of the payments have been reduced to a common date, for instance t_1, we shall have this equation of value:

$$Pp^3 = Q_2p + Q_5p^4 + Q_6p^5.$$

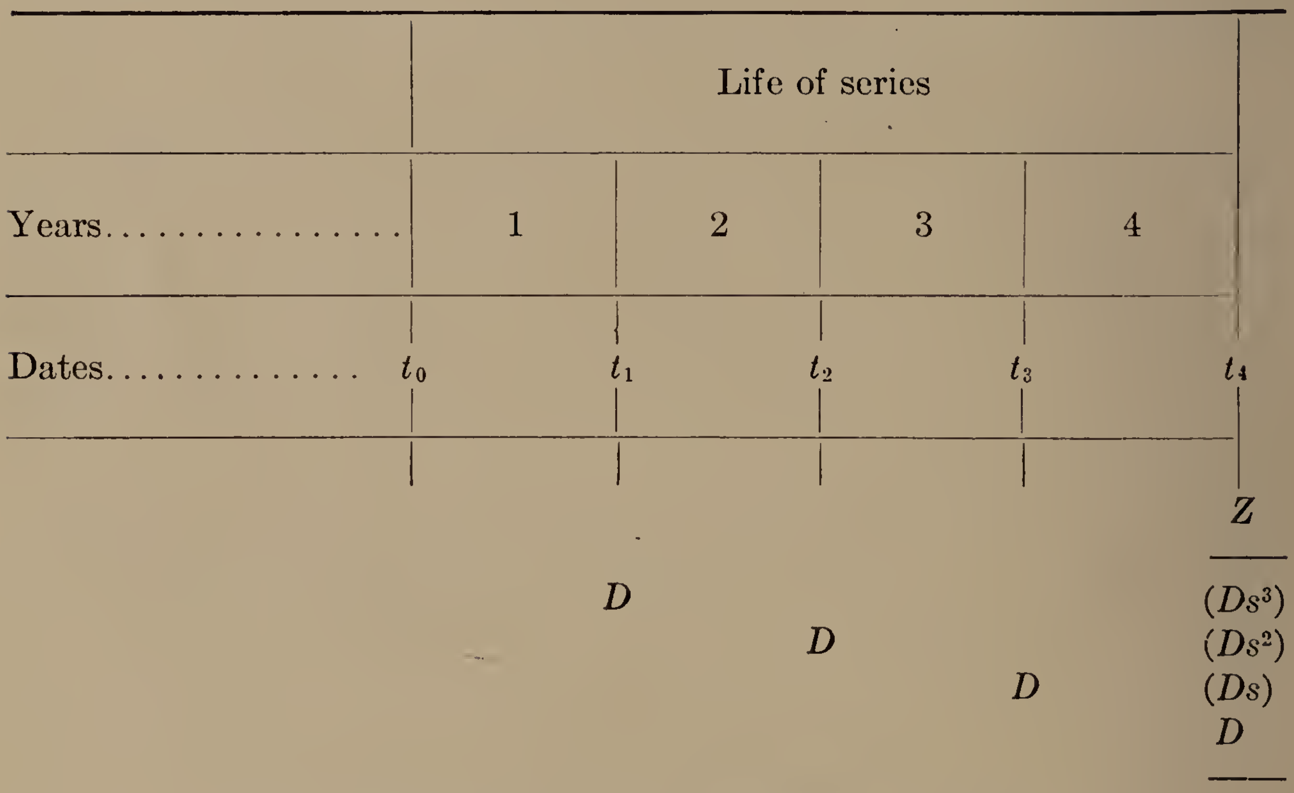
Dividing both sides by p^3, and replacing the resulting p^{-2} by its equal, s^2, we have for P, the equivalent payment sought, the following expression:

$$P = Q_2s^2 + Q_5p + Q_6p^2.$$

It is worthy of note that this last equation is precisely the equation of value which would have been obtained at first if we had reduced all of the payments to *P's* date, namely t_4.

UNIFORM SERIES OF PAYMENTS

319. Another problem of frequent occurrence is this: to find a uniform series of payments, *D, D, D,* . . ., which shall be equivalent to a given single payment, *Z*, at the end of the life of the series. The converse problem is also common. The elements of the problem and its converse are inserted on the time schedule, Fig. 319.



$$Z = D(1 + s + s^2 + s^3) = \text{amount of } D\text{-series.}$$
$$D = Z/(1 + s + s^2 + s^3) = \text{serial payment.}$$

(See Tables IV, V, VI.)

FIG. 319.—Time schedule of uniform series of end-of-year payments *D, D,* . .

The solution of the problem and the solution of the converse problem, both shown in Fig. 319, evidently consist in forming an equation of value between *Z* and the *D*-series, and solving for *Z* and for *D*, respectively.

The common date actually chosen for the equation of value, in Fig. 319, is t_4; but of course the final equations for *Z* and *D* would have been the same as those given, if any other date had been chosen.

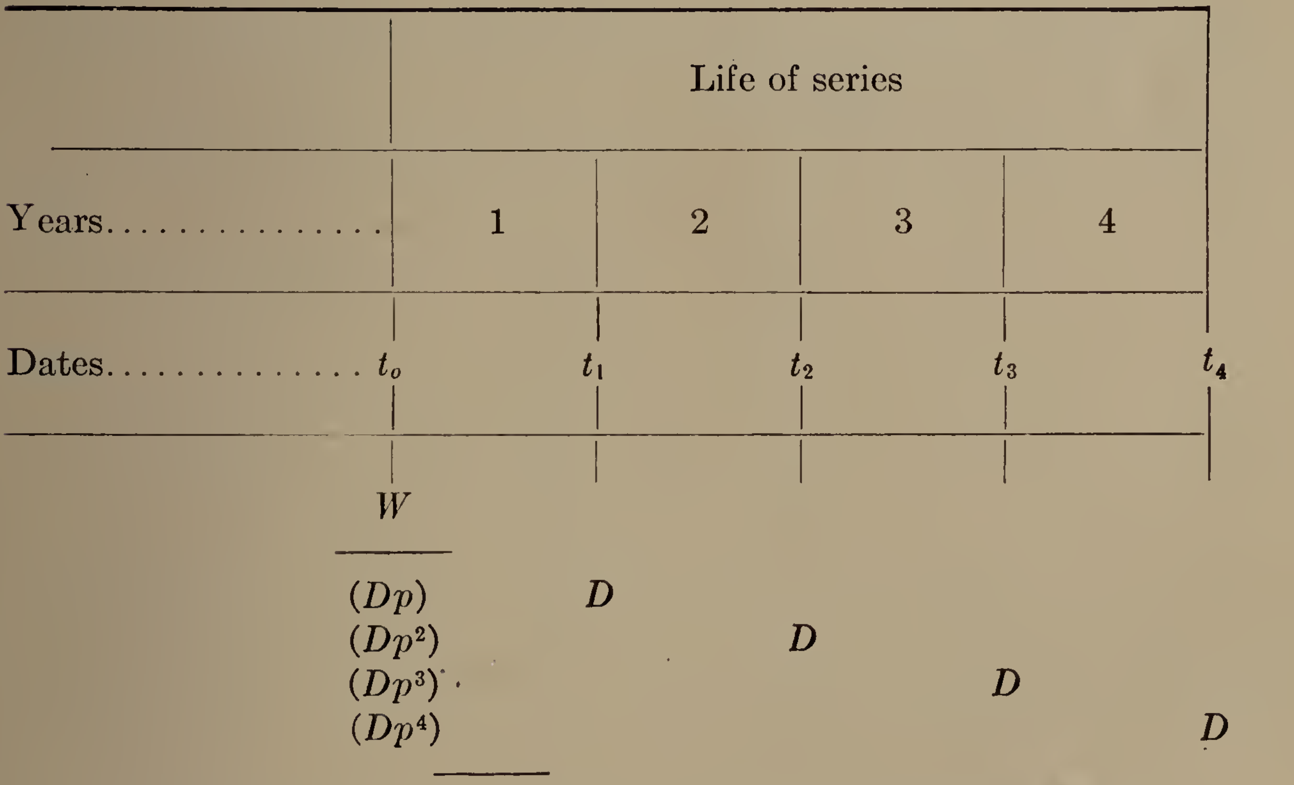
Z, which is evidently the sum of the amounts of the individual *D*'s, is called the **amount** of the series.

As indicated in Fig. 319, it is customary to consider that the D-payments are made at year-ends; and consequently to consider the **life** of the series as beginning 1 yr. before the first payment and closing immediately with the final payment.

320. A third problem, similar to the foregoing, is: to find a uniform series of payments which shall be equivalent to a given single payment, W, made at the beginning of the life of the series; and another problem frequently met is the converse of that stated. This problem and its converse are set up in Fig. 320.

The principle of solution is obviously the same for this pair of problems as for the preceding. The equations expressing W in terms of D, and D in terms of W, are given in Fig. 320.

W is plainly the **present worth** of the series, for it is the sum of the present worths of the individual D's calculated for the initial date.



$W = D(p + p^2 + p^3 + p^4)$ = **present worth** of D-series.
$D = W/(p + p^2 + p^3 + p^4)$ = **serial payment.**

Fig. 320.—Time schedule of uniform series of end-of-year payments, $D, D, \ldots$

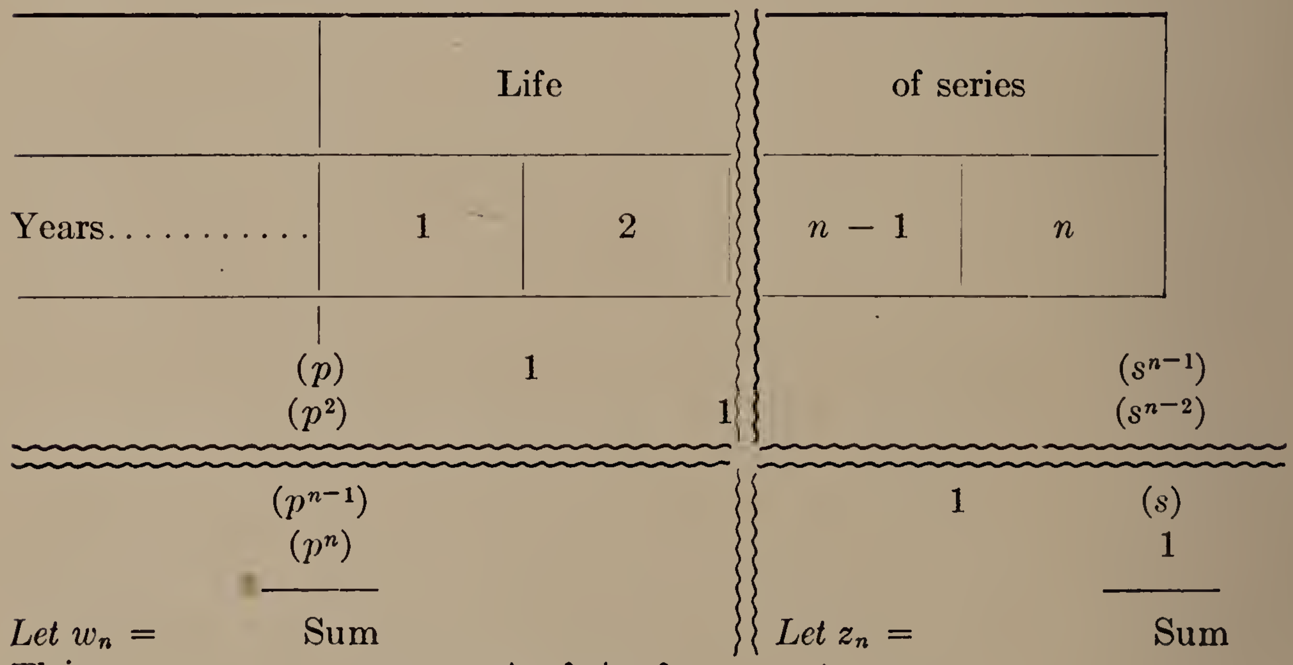
Other terms are sometimes used with the significance of "present worth" of a series. For example: What is the **present justifiable expenditure** to avoid the annual maintenance expense of \$500 per annum for 15 yr.? Again: What is the **capitalized cost** of 15 yr. maintenance, if the annual cost is \$500? And again: What is the capitalized value of an income of \$1200 per annum for 20 yr.? To **capitalize** a structure is to find the present worth of all outlays made on account of the structure, both initial and annual outlays. Some authors use the expression "capitalize a structure" to signify: Find the present worth of all outlays made on an infinite series of like structures. This is another way of saying: Find that sum which placed at interest will just suffice to provide funds for the first cost and annual costs of a structure, funds for the first cost and annual costs of a like succeeding structure, and so on for all time.

321. There are **three series of particular interest** because of their utility, as follows:

1. That series in which one (1) is the annual end-of-year payment;
2. That series of which one (1) is the amount; and
3. That series of which one (1) is the present worth.

These series with their characteristic notation are shown in Figs. 321a, 321b, 321c, respectively.

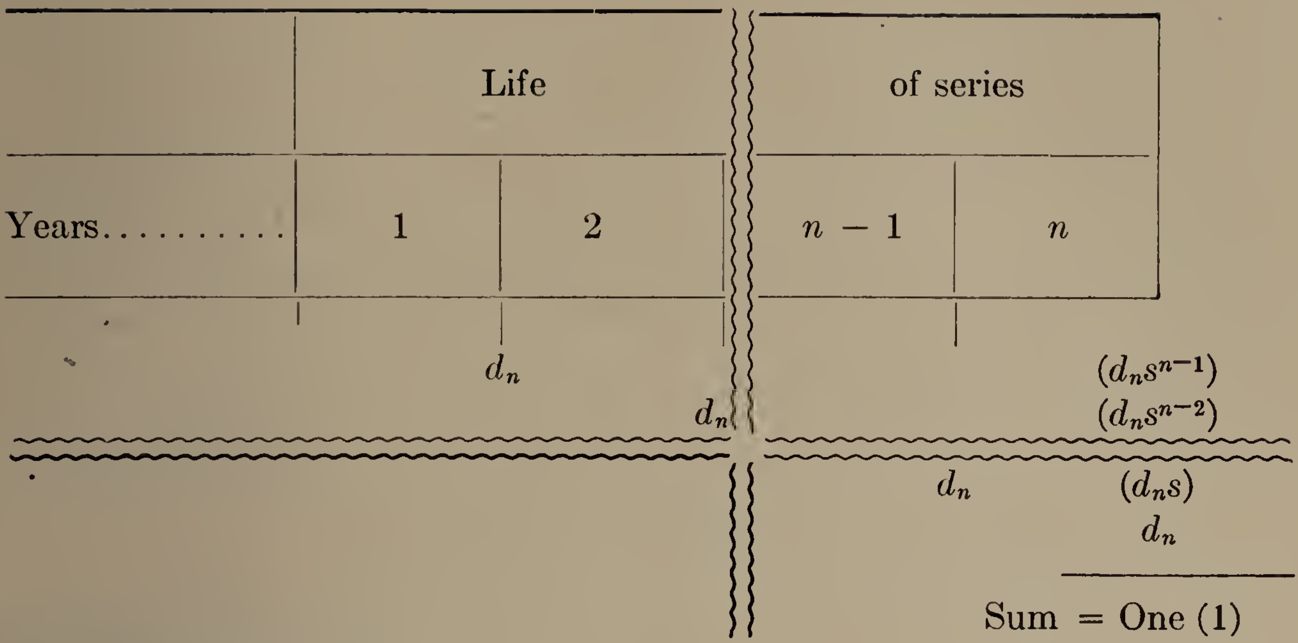
The subscript, $_n$, which appears on each of the symbols w_n, z_n, d_n, and a_n, is simply a tag to indicate the life of the series concerned. For example, the present worth of a 4 yr. 1-series would be represented by w_4; the amount of a 20 yr. 1-series, by z_{20}; and so on.



Then $w_n = p + p^2 + p^3 \ . \ . \ . + p^n$.
= **present worth** of the 1-series.
= **series-discounting factor.**
(See Tables III and D.)

and $z_n = 1 + s + s^2 \ . \ . \ . + s^{n-1}$.
= **amount** of the 1-series.
= **series-accumulating factor.**
(See Tables III and C).

FIG. 321a.—Time schedule of series in which one (1) is the annual end-of-year payment.



Since the sum is one (1):

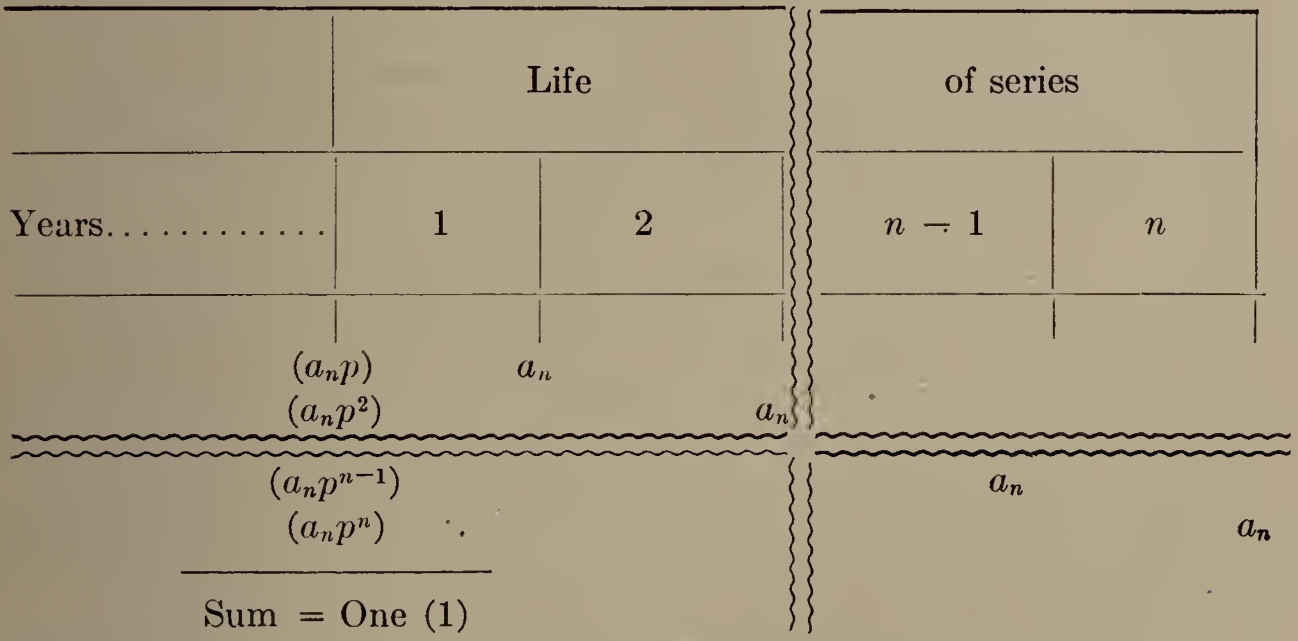
$$d_n = 1/(1 + s + s^2 + s^3 + \ldots + s^{n-1}).$$

= annual **deposit** to accumulate one (1) in n yr.

= the **backward-spreading factor.**

(See Tables III and E.)

FIG. 321b.—Time schedule of a series of payments of which one (1) is the amount.



Therefore:

$$a = 1/(p + p^2 + p^3 + \ldots + p^n).$$

= **present worth** of the 1-series.

= the **forward-spreading factor.**

(See Tables III and F.)

FIG. 321c.—Time schedule of a series of payments of which one (1) is the present worth.

As w_n is the present worth of 1 per annum for n yr., the present worth of D per annum for n yr. is found by multiplying D by w_n. The reason for calling w_n a series-discounting factor is thus apparent. Similar reasons hold for calling z_n a series-accumulating factor, d_n a backward-spreading factor, and a_n a forward-spreading factor.

The values of these factors, z_n, w_n, d_n, and a_n, for various lives and interest rates are given in Tables C, D, E, and F respectively (consult Index).

Formulas for calculating any one of the several elements (except interest rate) of this section are given in Table III (consult Index).

Comparisons

322. The preceding discussion makes it plain that it is only through the indirect means of equivalents that we can compare: (1) two payments on different dates; or (2) two series (regular or irregular) of payments on two different sets of dates.

The equivalents used as intermediaries in a comparison may be either lump sums of a common date, or two uniform series of payments of a common set of dates.

Not infrequently two irregular series of payments, having a common initial date, are compared by the indirect process of comparing their present worths calculated for the common date; and in such cases the present worth is called **capitalized value** (as of incoming payments), or **capitalized cost** (as of outgoing payments).

SINKING FUNDS, ANNUITIES, ETC.

323. One of the commonest uses of a uniform series of payments is to accumulate a specified sum for some anticipated purpose; the payments being made into an interest bearing fund drawing compound interest. Such a fund is a **sinking fund.**

The uniform payments are called sinking fund **deposits;** the life of the series of deposits is the **life** of the sinking fund.

The most common sinking fund problem is this: Given the sum which is to be accumulated by the fund, the life of the fund, the interest rate on deposits; to find the annual deposit. Another problem, less common, is to find the amount of a sinking fund at the end of its life, or at a given age, when the deposit and the rate, and the life, or life and age, are given.

324. Sinking funds sometimes receive specific names corresponding to the purpose of the fund. To **amortize** a debt is to extinguish it by means of a sinking fund; so a sinking fund established with the object of extinguishing a debt is sometimes called an **amortization fund,** and the yearly deposit in the fund may then be referred to as the **annual amortization cost.** To redeem a mortgage is to pay off and take up the mortgage; and the name **redemption fund** is sometimes applied to a sinking fund set up to redeem the obligation at maturity. A sinking fund established for the purpose of providing a lump sum for the cost of a future renewal of a structure, may be called **renewal fund;** and a sinking fund to pay the cost of an anticipated replacement may be referred to as a **replacement fund.** A sinking fund established for the purpose of making good the anticipated fall in value of a structure or piece of land or other property is often referred to as a **depreciation fund,** and the yearly deposit is then called **annual depreciation cost.**

325. Another type of transaction in which the uniform series of payments plays a conspicuous part, may be illustrated by this example: For a present lump payment, W, A agrees to pay to B a uniform sum D annually for n yr. B, in such case, is said to buy an **annuity.** The lump payment is called the **principal** or **investment;** and the annual payment the **yield.** The period of time covered by the annual payments is the **life** of the annuity.

It is readily inferred that the common problem in connection with annuities is to find one of the elements, principal, annual payment, or life, when the other two are known.

It is not uncommon to speak of a uniform series of annual payments as an "annuity," even when there is no thought of an income derived from a lump payment. In such case, the speaker uses the word "annuity" as a short name for a uniform series of annual payments.

Sinking fund and annuity are well contrasted in this statement: Jones established a 20 yr. sinking fund to accumulate a sum with which to buy an annuity. With sinking funds, a regular uniform series of payments is paid *into* an *in*creasing fund drawing compound interest; whereas with annuities, a regular uniform annual sum is drawn *out* of a a *de*creasing fund that draws compound interest.

326. Again, the uniform series of payments appears in connection with the **installment** plan, under which the purchaser agrees to pay so much (some uniform part of the purchase price) per week, or month, or other unit of time, without interest, until the sum of the partial payments is equal to the purchase price and further agrees that the title to the thing paid for shall remain with the seller until the final partial payment is made.

CHAPTER IV

ANALYSIS OF PROBLEM OF INVESTMENT (CONTINUED FROM CHAPTER II)

ANNUAL DEPRECIATION COST

401. It will be recalled that in the general example in chap. II there was an opportunity to invest \$4000 in furnishings which could be sold 4 yr. later for \$1600; and the tabular statement entitled "Calculation of yield" contained the following:

First cost of furnishings (capital invested)........	\$4000	
Salvage value (capital recovered)................	1600	
Total depreciation............................	2400	
Annual depreciation cost $^{2400}/_{4}$.................		\$600

It was anticipated that \$2400 of the original capital, \$4000, would be consumed or converted into service, by the end of the 4 yr. Good business practice, founded on common sense, requires that this shrinkage or total depreciation in the capital be made good out of income from service.

Placing in a safe deposit box \$600 of the income each year would of course pile up the \$2400 needful to make good the total depreciation. However, common sense dictates that the \$2400 be accumulated in a depreciation fund (a sinking fund established to accumulate a sum equal to the total depreciation). The annual deposit required for this depreciation fund is the true annual depreciation cost. If we assume that the deposits draw 4% the true annual depreciation cost is 2400 × 0.2355 = \$565.30. (The 0.2355 is taken from Table E—consult Index—under "4%" and opposite "4.")

402. The error in the approximate depreciation cost is thus 600 − 565.30 = \$34.70. The relative error is $^{34.70}/_{565.30}$ = 0.06, or 6%. The relative error is greater the longer the period. For instance, for a 10 yr. service the true depreciation cost would be 2400 × 0.0833 = \$199.92. The approximate depreciation cost would be $^{2400}/_{10}$ = \$240; and the relative error in this is (240 − 199.92)/199.92 = 0.2, or 20%.

403. Annual depreciation cost must not be confused with the annual depreciation in the value of the furnishings, which probably is not uniform from year to year. The annual depreciation cost is a function of the sum of the yearly depreciations in the furnishings, whereas the annual depreciation in the furnishings is a function of the circumstances. However, both annual depreciation cost and annual depreciation of structure (*e.g.*, furnishings) are often referred to by the single word "depreciation." Depreciation in value of structure is given attention in chap. XVII.

404. We have imagined a sinking fund as established for the purpose of making good the depreciation in the cash value of the furnishings during the 4 yr. service; and accordingly have called it a "depreciation fund." If the capital, $4000, had been borrowed for this enterprise, the sinking fund might have been as appropriately considered as providing the $2400 to help pay, or amortize, the $4000 debt; and have been called, in consequence, an "amortization fund." Thus the terms "depreciation" and "amortization," as applied to costs or funds, are often used interchangeably.

405. By way of preparation for further discussion of depreciation cost certain terms should be defined. If for estimating purposes we imagine a series of like structures, one taking the place of another as that other wears out, we call the first of the series **original structure,** the second of the series we call the **first renewal,** the third we call the **second renewal,** and so on; and this is so whether or not the first structure of the imagined series is considered as following an actual worn-out structure or series of actual worn-out structures. Thus, in the following pages, original structure is the term applied to the first of a particular series of structures held in mind at one time for estimating purposes. **Original life** is the life of such original structure; **original cost** and **original salvage value** are respectively the cost and salvage value of such original structure. **Renewal life, renewal cost,** and **renewal salvage value** are respectively the life, cost, and salvage value of a renewal. In the following pages the several lives of a structure are assumed to be of equal length. When an old or worn structure is superseded by a new structure of the same type, material, and capacity, the old structure is said to be **renewed** and the new structure is called a **renewal,** as stated above. When a structure of some age, whether worn out or not, is superseded by a new structure which differs from the old in type, material, or capacity, the old structure is said to be **replaced,** and the new structure is called a **replacement.**

406. During the years of service of any individual structure, the annual depreciation cost is of course uniform; and it is uniform, also, during any period of service which covers an original life and any number of renewal lives, provided the elements of this cost remain constant from life to life—and, for comparative purposes, this is usually assumed to be the case.

But there are numerous cases in which annual depreciation cost is not uniform over the whole period of service considered. For example, suppose a service is planned for 8 yr., requiring a structure the life of which is only 3 yr. The original structure will be followed by two renewals, but the second renewal will serve only 2 yr. If C be the first cost of each structure; C_3 the salvage value at the end of 3 yr.; and C_2 be the salvage value at age of 2 yr.; and if d_3 and d_2 be the annual deposits required to accumulate $1 in 3 and 2 yr. respectively; then the depreciation cost each year of the first six will be $D_3 = (C - C_3)d_3$; and each year of the last two, $D_2 = (C - C_2)d_2$.

In such cases, it is necessary, for comparative purposes, to calculate and use a hypothetical **equivalent uniform** annual depreciation cost, D.

407. In Fig. 407, D is shown calculated by two different processes. In the upper half of the schedule, D is the result of discounting the three

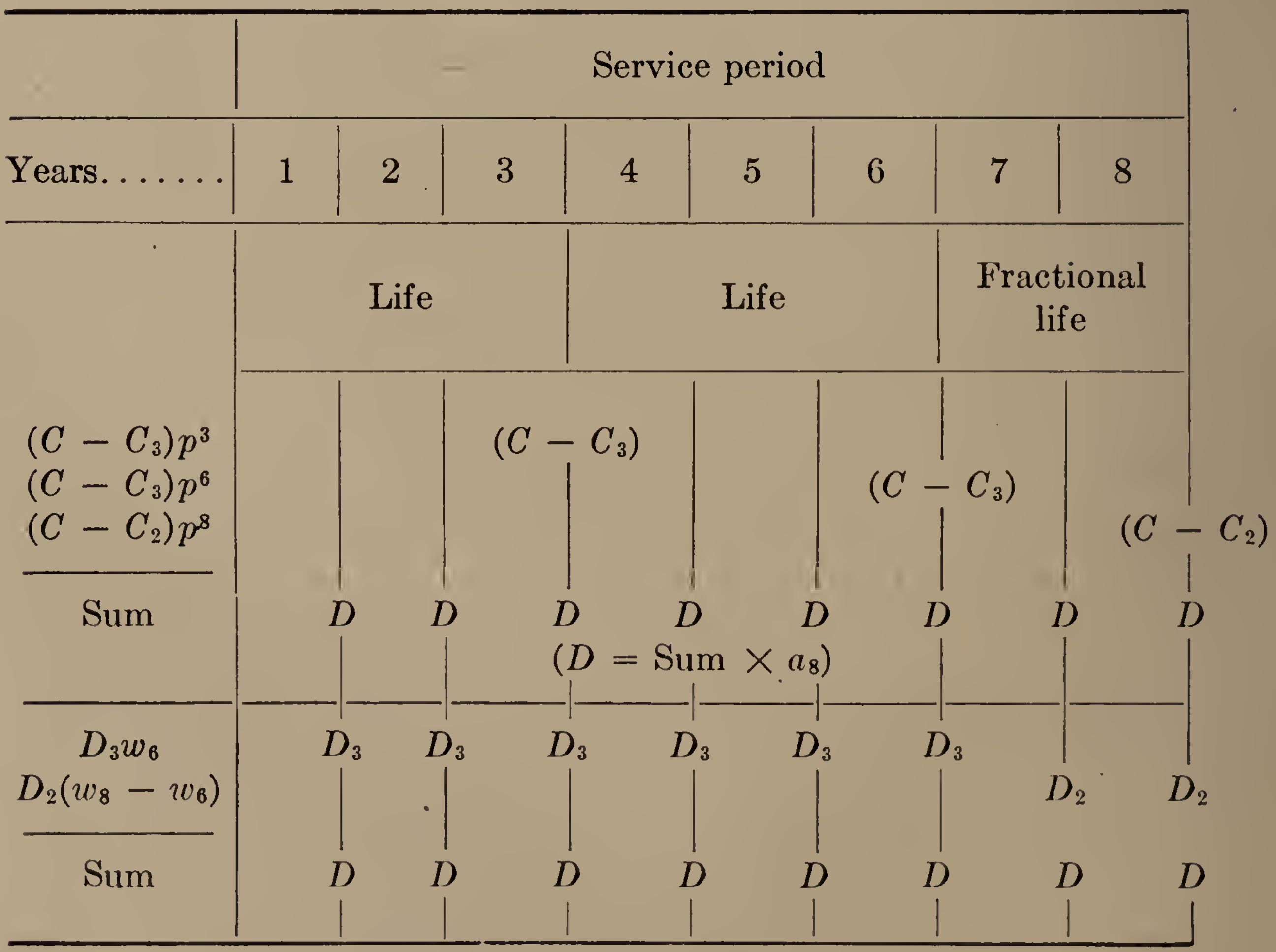


FIG. 407.—Time schedule.

total depreciations, $(C - C_3)$, $(C - C_3)$, $(C - C_2)$ to the initial date of the service, and multiplying the sum of the results by the forward-spreading factor a_8. In the lower half, D is found by discounting the D_3 series and the D_2 series to the initial date, and multiplying the sum of the results by a_8.

The two processes give the same result, of course, because the D_3 series and the D_2 series are the equivalent of the series $(C - C_3)$, $(C - C_3)$, $(C - C_2)$.

The latter process is the shorter if two or more whole lives precede the fractional life; and the formulas for this process are given in Table VII (consult Index).

ANNUAL OPERATION COST

408. All elements of service cost, except depreciation, are in this analysis included under operation cost. Cost of maintenance is therefore a part of the operation cost.

Operation cost usually varies more or less—sometimes widely—from year to year. The only one sum then that can be called the annual operation cost over a series of years is the hypothetical *equivalent uniform* annual operation cost, which is found by reducing the series of actual annual costs to a convenient date, and distributing the sum of the results uniformly over the whole period. The **formulas** for these calculations are given in Table VII.

409. It is often the case that the computer may consistently use the *average*, instead of the equivalent uniform, annual operation cost; as when the period of service is short, or the actual annual costs are nearly uniform.

410. A general analysis of annual operation cost will be found in chap. v.

INTEREST ON CAPITAL

411. If in the general example of chap. II it had been stated that the $4000 had been borrowed, there would have been the necessity of paying the interest each year, and interest may very well be considered to be a part of the cost of service. Some manufacturers reckon this interest as a part of service cost; and others deduct it from yield and call the remainder "profit." We have adopted the latter procedure. When interest is counted a part of service cost, the word "profit" should replace the word "yield" in our analysis diagrams.

412. As to interest on capital invested in an enterprise, it really makes no difference whether we are indebted for the capital to some third person or to our own pockets: the annual interest on the capital is in either case a part of the cost of conducting the enterprise.

INCOME

413. Like operation cost, income from enterprise is likely to be irregular, no two years just the same. Consequently, when making comparisons which involve the incomes for a series of years, it may be necessary to use the hypothetical equivalent uniform annual income. The calculations involved are the same as those indicated under Operation Cost.

SUMMARY OF ANALYSIS

414. In the foregoing discussion of analysis and choice of investment opportunity, many points have been presented in considerable detail for the benefit of the reader who has heretofore given investment no consideration. The minor details, having helped to make clear the major points, have served their purpose and need be considered no further.

The major points—the principles and definitions which suffice for guidance in carrying the analysis of any practical problem of investment choice to the point reached by the diagram, Fig. 414, and in solving the problem—are, with details omitted, here restated:

1. To choose intelligently between two given opportunities for placing capital is to compare the one with the other, with an eye to the purpose of placing the capital.
2. To compare two opportunities is to compare them quality by quality, taking into account only those qualities that affect choice.
3. To compare, quality by quality, requires analysis and a reduction of data to comparable form.
4. To analyze a problem is to separate it into its elements, and ascertain how the elements are related to one another.
5. To solve the problem is to recombine its elements by processes indicated by the ascertained relations.
6. All of the qualities that influence choice of opportunity fall into two mutually exclusive classes: (*a*) reducible data, and (*b*) irreducible data.

7. The diagram (Fig. 414) and Table VII (consult Index) indicate the method of reducing the reducible data.

8. With the following notation:

OPPORTUNITY	CAPITAL INVESTED	YIELD
I.	C	Y
II.	$C + C'$	$Y + Y'$

the rule for the interpretation of the resultant of all reducible data may be stated as follows:

(*a*) If the yield per cent, $100Y'/C'$, *is* satisfactory, the reducible data favor II.

(*b*) If the $100Y'/C'$ is *not* satisfactory, the reducible data favor opportunity I.

Thus opportunity I is positively favored if Y' is either zero or negative; and may or may not be favored if Y' is positive.

9. A reasonable choice between two opportunities of investment is based on a joint consideration of (1) the resultant of the reducible data and (2) all of the irreducible data.

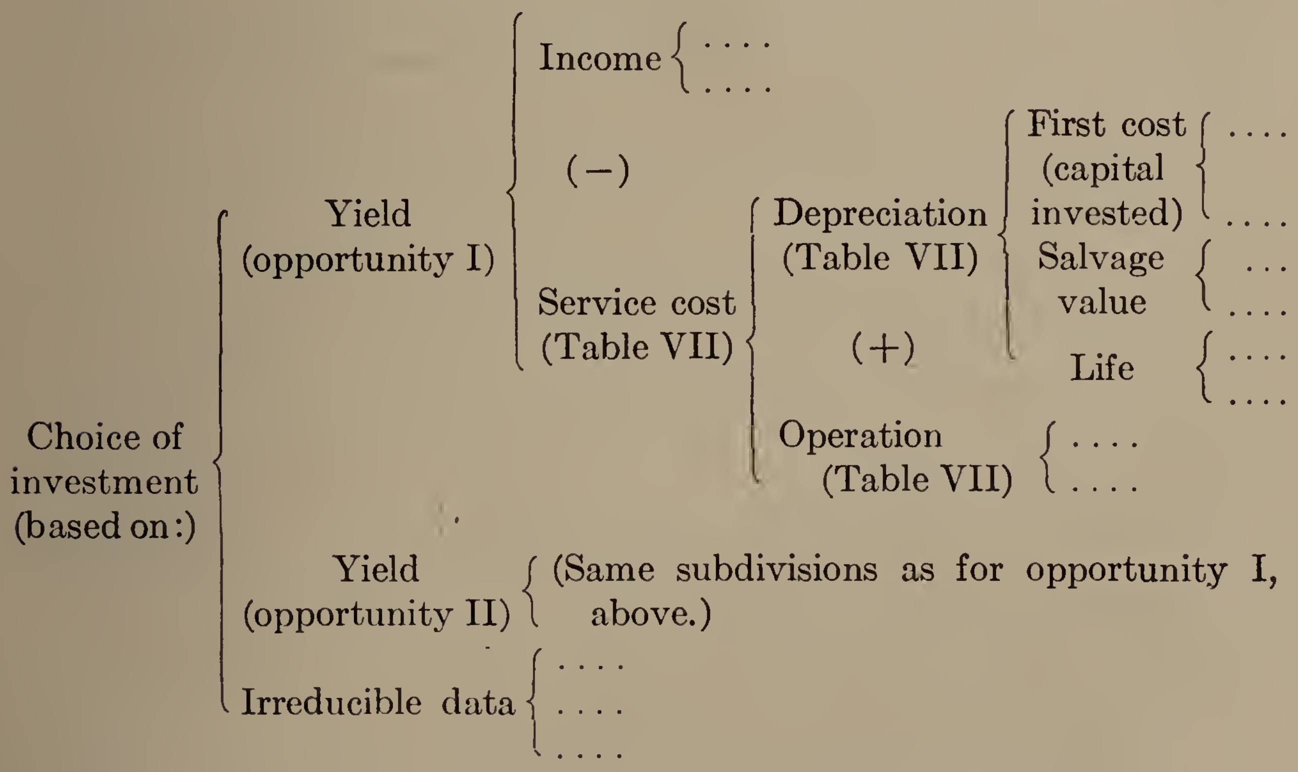


FIG. 414.—Analysis of problem of investment choice.

REVIEW AND PREVIEW

415. We have now traversed one field of the entire ground of the problem of investment choice. At the end of each segment of our path we have sought the known and found instead two or more diverging paths which we must follow in our search.

The diagram, Fig. 414, is the chart of the paths so far traversed. Since leaving the point of entrance into the field, we have been surrounded by the details of the way; but now we have reached a point from which an unobstructed general view of the entire ground can be obtained; and, like all prudent travelers similarly situated, we stop to orient ourselves, that we may continue the exploration with some degree of foresight. And to supplement our own native penetration, we make use of such general information, concerning the fields we have yet to traverse, as has come to us from those who have been there.

1. If income, first cost, salvage value, life, and operation cost are known, we need go no farther forward, but can at once retrace our steps by reducing these elements to yield.

This field, over which we have passed in search of the known and over which we repass in converting known elements into yield, may be called the **arithmetic of investment.** The work of calculating yield from numerical values of the elements named above is characterized by the fact that its intelligent performance requires a knowledge of arithmetic rather than the exercise of judgment based on an understanding of construction, production, distribution, or financing.

The fundamentals of the arithmetic of investment are given in the Summary above.

2. If income, first cost, etc.—the named elements of yield—be unknown, we must analyze each of them. These elements are points at which the paths so far traveled branch into untraveled paths all of which penetrate another field, which we may call **estimating.**

The characteristic of work in this field is the fact that its successful accomplishment requires continual exercise of judgment based on acquaintance with construction or production and distribution (see chap. v, vi, vii, viii).

3. A third field is that of **irreducible data.** To succeed in this field one must have not only more or less knowledge of construction or production and distribution; but also some understanding of the organization, control, and financing of business units, and of the inter-relation of the enterprise and the public, and be possessed of some power of imagination and analytic penetration to discover the various elements that affect probable financial success (see chap. ix, x, xi, xii).

4. On a still higher plane is the work of the remaining field of

the ground covered by the problem. To weigh justly the calculated yields and the irreducible data pertaining to two proposed investments, and arrive at their correct resultant—a reasonable decision as to which of the investments promises to be the more desirable—requires, besides imagination and analytic penetration,.a spontaneous feeling as to the probable quantitative effect of all influential elements and a nice sense of the fitness of things financial.

This, which may be called the field of **financial judgment,** this book does not enter.

CHAPTER V

PRELIMINARY ANALYSIS OF FIRST COST AND OF OPERATION COST

FIRST COST

501. The practical analysis of first cost leads us into a multitude of details, many of which are of like nature but differently related. It seems desirable therefore to make a **preliminary analysis** based on the nature of the items rather than on their relations or immediate purposes. Such an analysis should result in a small number of mutually exclusive classes, to some one of which every possible ultimate item of expense may be appropriately assigned.

502. We learn from the economists that there are three factors of production: land, labor, capital. The cost of producing a thing, ready for service, is therefore the sum of the three costs: cost of land, cost of labor, cost of capital. This is so far satisfactory; but when we undertake to analyze each of these three costs we find that the resulting subdivisions do not answer the present purpose.

503. From §107 we get the suggestion for another classification of the ultimate elements of first cost. This classification, indicated in the diagram, Fig. 503, is often more or less closely followed when compiling the elements of cost of an enterprise from its inception to its materialization ready for service.

$$\text{First cost} \begin{cases} \text{Investigation cost} \\ \quad (+) \\ \text{Organization cost} \\ \quad (+) \\ \text{Capitalization cost} \\ \quad (+) \\ \text{Creation cost} \end{cases}$$

FIG. 503.—Analysis of first cost.

In many cases one or more of these classes of costs are negligible or zero. For instance, if a man builds his factory with his own funds, capitalization cost will be nothing.

It will be found on analyzing each of these four classes, that for the most part the ultimate items in each class are in character very much the same as the ultimate items in each of the other three classes; and for this reason this classification of first cost items does not suit the present purpose.

504. A classification that seems to meet fairly well the specifications of §501, with which we set out, is given in Fig. 504.

First cost {
Labor
(+)
Material things
(+)
Transportation
(+)
Rights, rents, taxes, insurance
(+)
Interest
(+)
Contingencies

FIG. 504.—Analysis of first cost.

Let us consider each of the six classes in some detail.

505. It is intended to include under **labor** all expenditures for labor of all kinds, mental as well as physical. Therefore for the present purpose the cost of labor will be considered to include the cost of the work of the expert and of the professional man as well as of the skilled and the unskilled laborer The cost of labor of domestic animals which are hired or rented is also included here.

Labor is purchased by wages, hire, salaries, fees, commissions, brokerage, premiums, profits, percentage, or lump sum (see also §628).

Wages are paid for skilled and unskilled labor, including the work of foremen, according to the number of hours on duty.

Hire or rent is paid for the labor of hired domestic animals, and will or will not vary with the amount of labor performed, according to the time unit in the hiring agreement.

Salaries are paid to officials, professional assistants, superintendents, draftsmen, bookkeepers, timekeepers, storekeepers, and clerks.

Fees and commissions are paid for such services of architects, doctors, engineers, lawyers, real-estate appraisers, and other experts, as are not covered by salaries.

Commissions or brokerage or premiums may be paid for the work of obtaining the capital needful to meet first cost.

The profit, percentage, or lump sum, which comes to a contractor as a result of fulfilling the terms of a contract, includes the recompense for his labor.

Any condition which lengthens the period of creation of a structure increases the sum paid for salaries, and, usually, also the sum paid for wages and construction plant.

For example, laying new rails on an operating track is comparatively expensive because of the time lost by men and plant, due to passing trains. Again, in erecting a trestle for a railroad not yet in operation, the time of men and forces may be devoted continuously to carrying the work forward; whereas if the trestle is being built for a track in operation, a considerable part of their time may be given over to temporary work for the safe passage of trains and to undoing the temporary work to make ready for continuing the work of construction proper.

506. In the sub-class **material things** is included real estate, materials, manufactured parts, supplies, and domestic animals. If the lands or **real estate** required for the site and operation of the structure is bought outright, the price paid is a part of the first cost of the structure. If existing improvements are bought with the land the price of the improvements is added to that of the land, whether or not the improvements can be used for the purpose in hand. Under **materials** come all materials whatsoever of which the structure is composed, all materials which are used temporarily in the work of construction (for example, scaffolding), and all materials which are used in any necessary preliminary work (such as preparing the site of the structure). Machines, tools, devices, articles of furniture, and the like, which are bought ready made and installed either as aids to creation or as component parts of the structure, are embraced by the term **manufactured parts.** Under the head **supplies,** are included those materials or things which are used up or consumed in furtherance of the creation of the structure; as, for example, water, oil, fuel, power, stationery and other office supplies, etc. Horses, mules, and other **domestic animals** are sometimes bought outright for use in the work of creating a structure.

In the case of real estate, materials, manufactured parts, and domestic animals employed in the work of creating the structure and not used up, the salvage value of such after use should be deducted from their cost; and their depreciation cost, only, entered as a part of first cost of structure.

507. Expenditures may be incurred for the **transportation** of men, domestic animals, materials, supplies, manufactured parts, messages, etc. Cost of transportation is dependent firstly on the distance over which the thing is transported, and secondly on the character of available transportation facilities—whether water, rail, street, highway, mountain road or trail, or a combination of two or more of these. A short trail may equal a long railroad in the matter of exacting transportation charges.

One reason for the use of wood for original trestles and bridges and the later use of steel for replacements, on railroads traversing timbered regions, lies in the change in transportation facilities brought about by opening the railroad.

Cost of transportation is not infrequently a deciding factor in the choice of structure for a proposed service, or in the choice of design or material, or of both, for a given structure.

508. Included in **rights** are royalties on patented articles and processes, cost of charters, franchises, permits, licenses, rights of way over land, etc. **Rents** are paid for the use of grounds and existing structures such as offices, storage buildings, etc., and for leased construction plants, etc. Rental covers interest, taxes, insurance, depreciation, and profits, pertaining to the thing rented. The rental per day or per month is likely to be higher for a shorter than for a longer renting period. **Taxes,** including assessments, may be levied on the structure or parts thereof before the beginning of operation. Yearly taxes vary with time and place, from less than ½% to more than 1½% of the first cost of the structure. Precise data for computing taxes in the locality of the proposed structure may be obtained from the tax officers therein.

Premiums are paid for fire, casualty, fidelity, marine, title, employer's liability, and other **insurance.** In some states the employer is subject to special laws relating to employer's liability.

It is reasonable that a contractor be insured against risk of loss. If the work of construction is executed under the most common form of contract, the contractor risks his own labor and a part of his capital as well, for the cash cost of executing the work under his contract may turn out to be equal to or even greater than the sum received under the contract. Because of this risk and to cover it the contractor habitually increases his estimate under the head of either contingencies or profit by some percentage fixed by judgment; and such increase, if reasonable, is properly part of first cost.

Similarly, those who buy bonds which are issued to finance the construction of a structure, or those who contract to dispose of such bonds at a fixed price, run some risk of loss, and the issuers of the bonds pay insurance under the name of "bond discount." If such bonds are secured by a mortgage or other security which lacks nothing that the enterprise can reasonably give, and if further such bonds bear a rate of interest equal to that obtainable at the time of sale, such time being reasonably well chosen, on first-class commercial long-time securities, then the discount at such time of sale, if the sale is prudently made, is a proper charge for the insurance.

Good public policy requires that discount on bonds be treated in accounting as coordinate with, rather than subordinate to, first cost of structure, and accordingly that records of interest on, and amortization of, discount be kept coordinately with records of interest on and amortization of first cost. By this segregation foolish or fraudulent discounts are more likely to be recognized, and thus prevented; and in any case all interested parties are continually reminded of the price being paid for a deficiency in financial standing.

509. Any **interest** which at start of operation has accrued on capital tied up during the period of creation, is properly a part of the first cost of the structure. The interest on each payment on first cost of structure should be computed not for the interval between the date of payment and the earliest practicable date of operation, but for the interval between the date of first having the money on hand and the earliest practicable date of operation.

Reasons for this rule are as follows: The money to meet each payment on account of first cost must be on hand when the payment falls due. To have money on hand it must be withdrawn from the owner's other investments or borrowed. To withdraw or borrow a large sum of money requires time; and the owner must estimate the due date of each major payment and estimate the time it will take him to withdraw or borrow the money to meet the payment. Prudence requires that he adopt an early due date and allow a generous time for withdrawing or borrowing the money, for it is an act of common sense to provide against a conjunction of unfavorable circumstances; but since it is only in the exceptional case that all the circumstances turn out to be unfavorable, the exercise of prudence will, in the great majority of cases, result in having the money to meet the payment on hand a considerable time before it is paid out.

To make a separate withdrawal or borrowing to meet each minor payment is impracticable. Consequently to meet the minor payments expected to fall due within the next month or so, one withdrawal or borrowing is made and the money kept on hand to be paid out little by little.

Again, circumstances may virtually require that the entire capital for the cost of construction be on hand before the beginning of construction. If money is to be obtained from the sale of bonds, prudence may dictate that the bonds be offered for sale some time in advance of the need for the money, owing to a favorable condition of the bond market.

While the money is thus held on hand the owner is foregoing the interest on it if it has been obtained by withdrawal, or is paying interest for it if it has been borrowed or obtained from the sale of bonds; and in any case the interest is properly a part of the first cost of structure, provided that in case the money is obtained by withdrawal the interest rate should be no higher than that at which the money could be borrowed. For example, suppose that A withdraws $10,000 from other investments, paying him 10%, to meet a payment on first cost of a structure, and that he could have borrowed the money readily at 8%. The interest on this sum from the date of withdrawal to the first practicable date of operating the structure should be computed at 8%, not 10%, because the use of 10% money when 8% money is to be had is foolish, and the wasted 2% is rightly a loss to the owner and not a legitimate charge against the structure.

Sometimes the money held on hand can be placed in a safe depository which will pay a small rate of interest on such checking accounts; and when such is the case the interest received from the depository should be deducted from the interest foregone or paid by the owner for the money while on hand, and the remainder, only, entered as a part of the interest item of first cost.

For example, for the purpose of making a "betterment," a railroad sells at par one thousand $1000 bonds, bearing 5% interest. The betterment is immediately begun, and is completed in 1 yr., and the money, lying in a bank paying 2½% interest, is checked out from time to time during the year, a little at a time. If the average time during which all the money remains on deposit is ½ yr., the net interest, $1{,}000{,}000 \times \left(0.05 - \frac{0.025}{2}\right) =$ $37,500, is logically an item of the first cost of the betterment.

Shortening the building period of a structure tends to decrease the interest element of first cost not only of that particular structure but, more important still, also of a group of structures which with the particular structure constitute a structure of higher order.

For example, a railroad may be constructed in 1 yr., say, except for a single bridge which requires 2 yr. for completion by methods which give a minimum cost to the bridge itself. If all parts of the work are begun at the same time, the first cost of the other portions will be charged with the interest lost or paid during the extra year required for the building of the bridge. It may therefore be wise to hasten the construction of the bridge, even if its first cost is thereby increased, in order to save a part or all of the 1 yr.'s interest on the money tied up in the remainder of the road. Similarly, to shorten the time of construction, a long tunnel may be attacked through shafts as well as at both ends, the resulting increase in the cost of the tunnel itself being much more than offset by the saving in interest on capital tied up in it and in the other parts of the road.

It will occur to the reader that interest on tied-up capital can be reduced to a minimum by carrying on the work with utmost speed and so timing the beginning of work on each part of a structure as to bring all parts to completion simultaneously. For the construction of each component part of a complex structure, there are a beginning time, a method, and a speed of construction which together promise a minimum cost of construction exclusive of interest on tied-up capital. Beginning earlier, employing a quicker method, or working with greater speed will, it is true, decrease the item of interest on tied-up capital, but at the same time will increase some other item, or items, of cost by a sum which may be greater than the decrease in interest.

For example, the most favorable time for erecting the steelwork of a bridge on a railroad under construction is, as far as the cost of the steelwork itself is concerned, after the track has been laid to the site; for then the material and the erecting plant can be brought to the site by rail. It is a common practice to lay track to each bridge in turn, and use the track for bringing up material and plant. This method, of course, delays track-laying, because the track-laying beyond each bridge must await the completion of the bridge; but the elimination of these delays would, in many cases, increase the item of transportation by a sum greater than the corresponding saving thereby effected in the item of interest on tied-up capital.

It is for the engineer to study each case, and determine the time, method, and speed of construction, which, taken together, will give the minimum first cost including interest on tied-up capital. .

510. Experience shows that in listing the items of cost which together make up the estimate of cost of a structure or part of a structure, it not infrequently happens that one or more of the items are omitted from the list through oversight or ignorance. The more complex the structure or the shorter the time available for making the estimate, the greater the risk of overlooking some item. Even with ample time and all due care, the estimator is likely to overlook some items, particularly some of those which, though numerous, are individually relatively small. Such errors of omission are all negative, and the estimate of the whole will be too small by the arithmetical sum of the items omitted.

The estimator bases his quantities and unit prices upon a set of conditions which in his opinion will obtain during the preparation for and building of the structure. The actual conditions are likely to differ from those foretold, more or less—more if the forecast be long Unforeseen adverse conditions will introduce unforeseen items of cost, or increase the costs of some items, through an increase in either quantities or unit prices. If conditions prove to be better than anticipated the cost will, of course, be favorably affected.

There is, then, for one good reason or another, a likelihood that there will be some deficiency in any normal estimate. To cancel this deficiency, there is usually added to the normal estimate of cost an arbitrary item called **contingencies.**

The sum to be allotted to this item, contingencies, is usually found by taking some percentage of the normal estimate. The percentage taken depends on circumstances under which the normal estimate is made, the character of the item or class of items estimated, the general degree of stability of the conditions of weather, labor, and materials market, and the judgment of the estimator.

Sometimes a contingent sum is added to one or more of the subtotals of the normal estimate; sometimes a contingent sum is added only to the sum of the normal subtotals; and often the two methods are combined.

It is reasonable to apply the contingency item to the subtotals in all cases in which the percentage applicable is materially different for the different subtotals. For example, an estimate of cost of labor is usually much more uncertain than an estimate of cost of material, so that while the item, contingencies, is set in a given case at 5% of the normal estimate of cost of materials, it might reasonably be set as high as 15% of the normal estimate of the cost of labor.

Applying the corrective item to each subtotal does not preclude the application of a contingency item to the sum of subtotals, provided the contingency item for the whole has a basis not considered in the contingencies for the subtotal.

It is reasonable that the percentage for contingencies should be relatively great in an estimate of cost of work which may be materially affected by bad weather, underground water, flood, slides, or delays, or by the necessity of guarding against interruption of operating a neighboring structure.

From the foregoing, we see that contingencies is a sum added to the total of all other items of an estimate of a part or whole to supply any deficiency in that total.

511. The diagram, Fig. 511, gives a convenient **summary** of the preceding discussion:

- First cost
  - Labor
    - Wages, hire, salaries, fees, commissions brokerage, premiums, profits, percentage, lump sum.
  - (+)
  - Material things
    - Real estate, materials, manufactured parts, supplies, domestic animals.
  - (+)
  - Transportation
    - Steam railway, electric railway, highway, water, aerial, trail, pipe-line, telegraph, telephone.
  - (+)
  - Rights, etc.
    - Royalties, charters, franchises, permits, licenses, rights of way, rents, taxes, insurance.
  - (+)
  - Interest
  - (+)
  - Contingencies

Fig. 511.—Preliminary analysis of first cost.

512. Before leaving the subject of first cost, the following **two important principles** pertaining to it should be considered:

1. The first cost of a given structure includes such expendi-

tures on, and for, other structures (either imagined or real) as are necessitated by selecting and building (or buying and installing) the given structure.

This principle is illustrated by the following examples:

(*a*) A small waterway is required under a highway. Field observations are made and studied for the purpose of gathering data upon which the size of the required opening should be based, and the cross-sectional area of the opening is determined. Then data on pipes and culverts must be collected. Let us assume that 3 sq. ft. is the area of the required opening. For this a 24 in. pipe will do. There are different kinds of pipe: iron, steel (plain or corrugated), terra cotta, etc. To select an economical pipe requires a knowledge of the strength, durability, and availability, of the cost F.O.B., and of the cost of transportation of each variety of pipe. Also there are arch- and box-culverts of wood, brick, stone, concrete, etc. The monetary and other advantages and disadvantages of each type and of each kind of material must be considered. Finally, let us say, a stone box-culvert is selected as being on the whole the most economical. Detailed working plans must be made for the box-culvert. If the box be built all the expense incurred in studying all the rejected structures as well as the work on the stone box-culvert itself, must be included in the first cost of the box-culvert.

(*b*) Clearing the site of a proposed structure may involve the bodily or piecemeal removal of an existing structure. The cost of this removal is a part of the first cost of the new structure.

(*c*) The construction of the foundation of a proposed building may necessitate strengthening the foundation of an adjacent building. The cost of such strengthening is part of the first cost of the proposed building.

(*d*) When a highway bridge is worn out, the building of its successor may call for the construction of a temporary road around the site of the bridge. For this temporary road there may be required a right of way over private lands, some grading, and a temporary bridge over the stream; and the cost of all these is a part of the first cost of the new highway bridge.

2. Whatever is necessary to any step in the creation of a structure is an item of first cost of the structure, whether it be obtained from the owner or from an outside party; and any payment which would be properly a part of the first cost of the structure, if made to a party having no interest therein, is no less a part of said first cost if made to a party having interest therein.

A few examples will make this clear.

(*a*) If an architect designs his own house, the architect's commission, computed as for a client, is not improperly considered a part of the first cost of the house.

(*b*) Hardware is an item of the first cost of a house. If the owner of the house happens to be a hardware merchant and gets the hardware from his own store, the case will not be altered; and the cost should be computed on the basis of prices charged others.

(*c*) The cost of railroad transportation on material for a new bridge is a part of the first cost of the bridge, whether the bridge and the railroad are under one ownership or not.

OPERATION COST

513. As the foregoing diagram of the preliminary analysis of first cost will serve with slight modifications for operation cost, it is expedient to consider the latter in this place, by making such observations as appear to be justified, and pointing out the modifications.

It is by no accident that the diagram for first cost, which includes construction cost, serves fairly well for operation cost. Construction results from the operation of a construction plant (see §614).

514. The probable supply of labor, common or skilled, is often a factor of great weight in the choice of location for a proposed industrial plant.

Real estate and **domestic animals,** listed in the foregoing diagram under "material things," would not appear on a diagram for operation cost.

While the purchase price of real estate or domestic animals may be under some conceivable conditions appropriately chargeable to operation, it is almost always logically an element of first cost.

Under the head of **materials** is included all materials of whatever kind that are used in repairing and other upkeep (repainting, for example); but excluded are all raw materials for those articles which result from the operation.

Manufactured parts include machine parts, small tools, etc., that are bought ready made. In some cases the item "manufactured parts" is of considerable importance: relative convenience of obtaining repair parts sometimes determines which of two or more makes of machine shall be purchased.

Such material things as **supplies** are consumed in the furtherance of the operating processes, such as fuel, lubricating oil, electricity, stationery. Fuel and electricity frequently require relatively large outlays, and are properly given great weight in choosing the location for a proposed plant.

515. Where fuel is a large item of operation cost of a proposed structure, the cost of **transportation** may be the determining factor in the choice of kind of fuel or even in the selection of type and location of the structure.

Because of cost of transportation one community decides to burn coal, another oil or gas, another cord wood.

516. As a rule, **taxes** are higher within city limits than without, and sometimes this fact leads to locating a manufacturing enterprise just outside the city. Taxes and assessments are levied on a structure when it is idle as well as when it is operating, and so in a sense may be considered as not one of the costs of operation, but rather as a part of cost of capital, along with depreciation. It has seemed convenient, however, in this preliminary analysis of operation cost to include taxes.

517. One of the items often included in, but here *excluded* from, operation cost is the **interest on working capital** which consists of (1) cash on hand—either in the safe or in the bank, (2) the capital tied up in raw materials on hand, (3) the capital tied up in work in progress, and (4) the capital tied up in finished work not yet sold. (The money paid for raw material to be turned into finished work by the operation of the structure is not here considered a part of the cost of operation, nor is the interest on that money so considered.) A fund of cash, greater or less according to the characteristics of the business, must be kept on hand to meet all irregular and emergency bills and deficiencies in anticipated receipts caused by delayed collection of accounts.

518. In the formula for equivalent uniform annual operation cost, the actual annual cost is treated as a sum paid or payable at the *end* of each year. But payments are made for labor, materials, etc., of operation, from time to time during the year. Hence annual operation cost should include not only each of these payments, but also the interest accrued on the payments at the end of the year. In computing this interest, we should recognize the fact that the money to make each payment must usually be on hand some time before the date of payment. (In this connection see Interest, §509.)

CHAPTER VI

PRACTICAL ANALYSIS OF FIRST COST

FIRST COST

601. The practical analysis of first cost in any specified case will follow lines which are more or less plainly indicated by the nature and circumstances of the case. However, there are principles which may properly have a controlling influence in any such analysis; and some of these are exhibited in the following simple examples.

Example 1: Production

602. A master carpenter makes some bookcases for a customer. He keeps a little shop and employs a few assistant carpenters and a clerk. The shop is equipped with a variety of tools. Under the direction of the master two assistants build the bookcases in 3 da., making use of the equipment of the shop.

What is the cost of the bookcases to the master?

603. The total cost of the bookcases must be found by adding together the various elements of cost. Of these elements the most obvious are the materials of which the bookcases are made and the labor of the carpenters employed upon it. And after the work has been done, these two items are readily figured with exactness from memoranda of the quantities of materials consumed and labor expended, and of the unit prices paid for the materials and labor.

604. But the remaining elements are not as readily figured for this particular job, nor can they be determined with exactness, for they are elements shared with various other jobs turned out by the shop.

For example, the master pays cartage on incoming raw materials. If the materials for these bookcases were obtained on a special order and delivery, then the cost of cartage on these particular quantities of materials would be known exactly. But the master buys and places in stock considerable quantities of materials at one time, sufficient for several jobs. Two or more kinds of material may be brought in the same cart load. Under such conditions how is to be figured the element of cartage that enters into the cost of these bookcases? Before answering the question, let us run over the remaining elements of cost of the bookcases. The master pays rent for the shop; he pays for light, heat, and power; he pays the clerk who acts as keeper of time, books, and stores, and as secretary and general helper; pays for stationery, telephone, postage, lubricating oil for machinery. Maintaining the equipment in good working order takes labor and material for which he pays. He has also to carry fire and employers' liability insurance. Taxes are levied on his equipment. Then there is depreciation of equipment. And finally, some salary for the master's services as manager and supervisor is properly an element of cost of the output of the shop.

It is impossible or impracticable to determine directly or exactly the quantitative effect of any one of the foregoing elements on the cost of the bookcases. Those elements are often classified as **expense** (or **overhead,** or **burden**—all terms of more or less elastic definition).

605. The effect of expense on the cost of the bookcases is found by an indirect and approximate method called **pro-rating.** This method is based on the assumption that, from job to job during the year (or other chosen period of time), the element, expense, bears a constant ratio to some other cost element which can be readily determined with exactness on each job or unit of output. This constant ratio is found by dividing the year's total expense by the total for the year of the element chosen as a basis.

For example, suppose that in the master's shop the element expense is believed to be more nearly proportional to the element labor than to any other; that the past year's total of expense was $6000; the past year's total of labor, $12,000; and that the labor cost of the bookcases is $38.40. Then the expense element applicable to the bookcases is $({}^{6000}/_{12000})\ 38.40 = \$19.20$. The process shown in this example may be briefly described as "pro-rating expense on the **basis** of labor."

606. We have applied the method of pro-rating to the total of the indirect elements called expense. The method of pro-rating may be applied, if deemed desirable, to any component element of expense. Advantage is to be gained by pro-rating each element or group separately only when it is clear that substantial accuracy of results requires that one basis should be used in pro-rating one element or group, and another basis in pro-rating another element or group.

EXAMPLE 2: CONSTRUCTION

607. Let the problem be to find the cost, to the builder, of the master carpenter's shop of Example 1.

The shop, like the bookcases, is fashioned from materials. The cost of the materials incorporated in the building is found directly and exactly from quantities and unit prices recorded on invoices. So also is the cost of those used materials, which, while necessary to the work of construction, do not become a part of the shop; such as lumber for scaffolds and temporary office and storehouse. So also is the cost of labor wholly devoted to moving, fashioning, preparing, and placing the materials; and, too, the cost of the labor of superintendent of construction, timekeeper, etc., provided they devote their whole time to work on the shop.

608. All of the foregoing elements properly are collectively classified, it is evident, as **direct cost.** There may be other elements of direct cost; for example, the cost of such equipment as is bought new for, and lost or used up on, the job—shovels and hammers for instance—and the cost of various supplies used in considerable quantity, as fuel for hoist engine or concrete mixer.

609. As with the bookcases, so with the shop, there are several elements of cost which cannot be evaluated either directly or exactly.

For example: that part of the builder's home office costs that are properly a part of the cost of the shop (such as rent, office help; interest, repairs, and depreciation in office equipment; stationery, postage, telephone, etc.); advertising, interest and depreciation on construction equipment; taxes on property employed directly and indirectly in the work of construction; and so on.

610. These indirect costs of the shop, which, as stated above, cannot be evaluated directly or exactly and which we have called collectively **expense,** are, like the indirect costs on the bookcases, **pro-rated**—either the total expense pro-rated or its component elements pro-rated individually or in groups. As before said, the only object in pro-rating each element or each group by itself is to permit of the use of a variety of bases with the object of making a closer approximation to the truth.

SUMMARY OF EXAMPLES

611. It will be well to summarize from the two examples those common points with which we are chiefly concerned.

Entering into the cost of output (bookcases in Example 1, and shop in Example 2), there are two classes of elements: (*a*) elements which can be directly and exactly figured (such as direct material and labor), which are conveniently grouped under the head **direct cost,** and (*b*) elements which can be evaluated only indirectly and approximately by the method of pro-rating—**indirect costs** that we have called **expense.** The summary of Examples 1 and 2 is shown in brief by the diagram, Fig. 611.

- First cost
  - Direct cost
    - Direct materials
      - Quantities
      - Unit prices
    - (+)
    - Direct labor
      - Man-hours
      - Hour-rate
    - (+)
    - Other direct cost
  - (+)
  - Expense (indirect cost)
    - .... (Figured by the method of pro-rating.)

Fig. 611.—A practical analysis of first cost.

612. It must be remembered, as stated previously, that no one such analysis, when carried out to ultimate practical details, will hold for all cases; but the first stage of the analysis, which separates the elements of cost into two divisions—direct and indirect—is of general application, as is also the pro-rating method of evaluating the indirect elements, either collectively or individually.

613. Example 1 (finding cost of bookcases turned out by the master carpenter's shop) is typical of the work of all shops, factories, farms even, and all other plants which, like the master's shop, are engaged in what is commonly called **production.** Example 2 (finding cost of the shop itself) is typical of the complementary field of endeavor, universally known as **construction:** the building of shops, factories, and all other industrial works; of houses, barns, stores, grain elevators, garages, office and commercial buildings; dams, canals, railroads, bridges; pavements, sidewalks, sewers, town halls, jetties, harbors, and all other public works; and so on. Therefore the foregoing analysis of first cost of output is applicable equally to the fields of production and of construction.

614. This conclusion might have been reached more directly by comparing somewhat closely the operations of production with the operations of construction. On reflection we find that their essential difference is this: in production the operating plant is fixed in position, the output is portable and is moved away for use elsewhere; in construction, on the contrary, the operating plant is portable and its output stands fast. It is not apparent that this essential difference can have any material effect on the fundamentals of the problem of finding first cost of output.

615. It may be well, before leaving this chapter, to state that some materials that enter directly into the output enter in such small quantities that, to save clerical work, they are arbitrarily placed under the head of expense. For example, the glue and nails used on the aforesaid bookcases would be more cheaply charged indirectly against the bookcases through expense than treated as direct materials, and the resulting figured cost of bookcases would not be sensibly different. The reason for including so insignificant an item as glue in the cost of the bookcases is that the year's cost of the glue is not insignificant.

616. The fact that the greater the output of a given plant the smaller will be the expense chargeable to each unit of output or job turned out, is an important point in the economics of operation.

MATERIALS COST

617. A more detailed consideration of cost of materials is now in order. The diagram, Fig. 617, of this analysis needs no introduction.

$$\text{Cost of materials}\left\{\begin{array}{l}\begin{array}{c}\text{Cost of material}\\\text{(Kind A)}\end{array}\left\{\begin{array}{l}\text{No. of units}\\\text{(quantity)}\\(\times)\\\text{Unit cost}\end{array}\right.\\(+)\\\left.\begin{array}{c}\text{Cost of material}\\\text{(Kind B)}\end{array}\right\}\text{(Same as above)}\\(+)\\\dots\dots\dots\dots\dots\end{array}\right.$$

Fig. 617.—An analysis of materials cost.

The diagram briefly says that if we know the quantity and unit cost of each of the materials under consideration, the cost of that material can be immediately found by multiplication; the cost of materials can then be found by adding the products.

618. Quantity of material in a structure is obtained directly or indirectly from the plans (*i.e.*, drawings) of the structure.

The quality or grade of each kind of material is given either on the plans or in the accompanying specifications. The work of thus determining and recording the quantities is commonly called **taking off** the quantities.

Each quantity of material is of course expressed in units, and the particular unit used for any kind of material is largely a matter of commercial custom. For example: (1) bricks go by count, the customary unit being M (*i.e.*, 1000) bricks. So with shingles: so many M's of shingles. So also with many other materials, and with fixtures. (2) Moulding is given in linear feet; (3) hardwood flooring in square feet; (4) roofing in square feet, or in squares (1 square = 100 sq. ft.); (5) concrete in cubic yards; (6) paint, in pounds or gallons; (7) nails, in pounds; and (8) lumber in board-feet (1 board-foot = $1/12$ cu. ft.) or, if the quantity is considerable, in the unit MBF (1000 board-feet). Thus to express quantity of material we use units of count, length, area, volume, and weight.

619. It is stated above that quantities are taken directly or indirectly from the plans. Let us illustrate from the plan of a house. Linear feet of pipe, moulding, etc., are given directly on the drawing by scale or by written figures. The number of fixtures, doors, etc. is directly taken off by count. But area and volume must be calculated from dimensions given by the drawings, or from dimensions and count. Thus square yards of plastering are derived from dimensions; board-feet of floor joists, from dimensions and count. Again, the commercial unit of building paper is the roll; and the quantity required must be calculated from the area of the surface to be covered by the paper (computed from dimensions taken from the drawing) and the area in the roll. The number of bricks, though bricks are bought by the count, is calculated from the volume of brickwork and the volume of 1 brick, some allowance being made for the volume of mortar. The number of cubic yards of mortar is found from the number of brick and the volume of mortar required to lay 1000 brick, this volume being known from experience. Shingle nails are bought by the pound. Handbooks give the number of nails in a pound, but it is impracticable to count the nails required; hence the custom of obtaining the quantity of nails by multiplying the M's of shingles required by the pounds (learned from experience) of nails required to lay 1 M shingles.

620. Finally, it must be noted, not all of the material of every kind required for the house is accounted for in the plans. The difference between the quantities "taken off" and the quantities required to be bought is **waste.** Breakage and loss account for part of this waste, but the greater part is the necessary result of cutting and fashioning the raw materials for placement, and from rejection of imperfect pieces of material.

621. The examples of units given above, and of indirect methods required to turn counts and dimensions, taken from plans, into commercial units, and of ever-present waste, are drawn from house building; but similar examples are furnished by every other field of construction, and by the fields also of production.

LABOR COST

622. Labor is usually paid on the basis of time or on the basis of achievement or output. When time is the basis, payment is in proportion to the number of man-days or **man-hours.**

The man-hour is for many purposes a better unit than the man-day, because a work day may mean anything from 7 to 12 hr. of work, according to the custom of the place, craft, and time. The man-hour has the advantage, too, when the workman changes from one kind, or piece, of work to another during the day, because of fewer fractions required in keeping the record.

623. The **unit of achievement** depends on custom and the character of the work done.

In house building, for example, we find the following used as units of achievement: (1) hanging 1 door; (2) laying 1 M shingles, laying 1 M brick; (3) placing 1 lin. ft. of moulding; (4) plastering 1 sq. yd. of surface, 1 coat; (5) painting 1 square (100 sq. ft.) of surface, 1 coat; (6) completing 1 cu. yd. of excavation or, more in detail: loosening 1 cu. yd. of earth, loading 1 cu. yd. of earth, wheeling in a barrow 1 cu. yd. of earth a unit distance, dumping 1 cu. yd. of earth; (7) framing 1 MBF of lumber; and so on.

The number of units of achievement of any kind required for a structure is obtained directly or indirectly from the plans and specifications, just as the number of units of each kind of material is obtained.

624. The analysis of cost of labor that is paid by "piece-work," that is, on the basis of output (units of achievement), follows the lines of the diagram, Fig. 624a.

$$\text{(I) Labor cost} \left\{ \begin{array}{l} \text{Units of achievement} \\ \quad (\times) \\ \text{Unit price} \end{array} \right.$$

FIG. 624a.—An analysis of labor cost.

The analysis is made on the lines of Fig. 624b if the pay of labor is on the hour basis:

$$\text{(II) Labor cost}\begin{cases}\text{Man-hours} & \begin{cases}\text{Units of achievement}\\ \quad(\times)\\ \text{Man-hours per unit}\end{cases}\\ \quad(\times) & \\ \text{Hourly wage} & \end{cases}$$

Fig. 624b.—An analysis of labor cost.

The manner of determining the number of units of achievement has been stated above. The hourly wage on future work must of course be predicted.

625. The number of man-hours required for a unit of achievement is a variable quantity whose limits and usual values are to be learned only from experience and the records of experience. This point is further considered in chap. XIV (Cost Keeping).

626. Each of the foregoing diagrams shows the analysis of cost of one kind of labor. The total cost of labor expended on a structure is indicated by the diagram, Fig. 626.

$$\text{Cost of labor}\begin{cases}\text{Cost of labor (Kind: A)} & \begin{cases}\text{(Either I or II above)}\end{cases}\\ \quad(+) & \\ \text{Cost of labor (Kind: B)} & \begin{cases}\text{(Either I or II above)}\end{cases}\\ \quad(+) & \\ \ldots\ldots\ldots\ldots & \end{cases}$$

Fig. 626.—An analysis of labor cost.

627. In some production plants there are in use plans of payment for skilled labor other than the hour (or day) rate, and piece rate. Among these are the Halsey premium plan, the Rowan premium plan, the Taylor differential wage rate plan, the Gantt task-and-bonus plan, and the Emerson efficiency bonus plan; all of which will be found fully explained in various books on cost keeping and on industrial management.

UNIT COST

628. It often proves convenient to express cost in terms of **unit of product.**

In house building, for example, the units of product may be: (1) 1 door hung; (2) 1 M shingles laid, 1 M brick laid; (3) 1 lin. ft. of moulding placed; (4) 1 sq. yd. of surface plastered, 1 coat (or more); (5) 1 square (100 sq. ft.) of paint applied, 1 coat (or more); (6) 1 cu. yd. of excavation completed, 1 cu. yd. of concrete completed; (7) 1 MBF of lumber placed; and so on.

This example, it may be noticed, parallels the example used to illustrate units of achievement, in §623.

The number of units of product of any kind, required for a structure, is obtained directly or indirectly from the plans and specifications.

629. The analysis of the so-called unit cost of product will depend on the nature of the product. One example will suffice for illustration.

The diagram, Fig. 629, indicates in general terms a typical analysis for certain classes of product.

Unit cost of product
- Cost of materials (per unit of product)
  - Cost of material A (per unit of product)
    - Units of material A (per unit of product)
    - (×)
    - Unit cost of material A
      - Unit price of material A
      - (+)
      - Other costs per unit of A
  - (+)
  - Cost of material B (per unit of product)
    - (Same subdivisions as for A)
  - (+)
  - . . . .
- (+)
- Cost of labor (per unit of product)
  - Cost of operation 1 (per unit of product)
    - Man-hours (per unit of product)
    - (×)
    - Wage rate
  - (+)
  - Cost of operation 2 (per unit of product)
    - (Same subdivisions as above)
  - (+)
  - . . . .
- (+)
- Cost of plant (per unit of product)
  - Assembling, setting up, dismantling, dispersing (per unit of product)
  - (+)
  - Depreciation (per unit of product)
  - (+)
  - Supplies (per unit of product)
- (+)
- General expenses (per unit of product)

Fig. 629.—An analysis of unit cost of product.

The number of units of each kind of material entering into a unit of product is learned from experience or is fixed by the specifications or the plans. "Other costs per unit of" material include such items as freight, unloading from cars, hauling, storing, etc., and are learned from experience. Likewise the number of man-hours required for each "operation" and the various items of cost of plant are learned from experience.

OTHER ANALYSES OF FIRST COST

630. In §§203–205 it is pointed out (1) that although the ultimate elements of a problem are the same, the compound elements will vary according to the lines along which the analysis proceeds; (2) that an analysis is carried no further in any direction than is necessary to reach a known quantity; and (3) that the analysis is often carried along lines determined by the purpose and circumstances. That this is true of the problem of estimating cost will be further exemplified in the next chapter.

CHAPTER VII

PRACTICAL ANALYSIS OF FIRST COST (CONTINUED)

Procedure in Estimating

IN GENERAL

701. In the preceding chapter certain fundamental principles pertaining to cost estimating were developed, and the conditions which give rise to the use of different methods of estimating were explained. The purpose of this chapter is to describe some of the more common methods of estimating and give the student a closer view of the estimator at work.

702. The estimator makes continual use of his faculty of **visualization.** As he looks over the plans and specifications for a structure, he visualizes the structure as a whole, and each of its parts in turn as a whole; and then inspects most carefully a mental moving picture showing the various materials and operations which day by day or hour by hour carry the structure from beginning to completion. The **analysis** that he makes by the help of visualization is carried along lines indicated by the fact that the problem he is analyzing is a cost problem, and so influenced also by the character of the cost data available to him.

703. In making an estimate the **estimator is controlled** by: (1) the purpose of the estimate; (2) the available data; and (3) the time available for making the estimate.

The **purpose** of the estimate decides as to whether the estimate should be (*a*) closely approximate, (*b*) fairly approximate, or (*c*) roughly approximate. Usually the closer the desired approximation, the more time will be required to make the estimate. The purpose fixes the desirable degree of closeness of approximation. But on account of the two other controlling factors, **available time** and **available data,** the desirable closeness of approximation is frequently not obtainable. Making an estimate of desired closeness may be prevented, either by lack of sufficiently accurate data, or by lack of time necessary to make the estimate on the basis of such data. In either case the estimator and his client must be content with an estimate which, though the closest possible from the available data in the available time, is not as close as desired.

Evidently the estimator has here a need for continual exercise of judgment.

The application of the foregoing discussion to procedure in cost analysis may be illustrated by the following very simple example:

Let it be required to estimate the cost of a steel bar of rectangular cross-section. General principles justify the analysis of Fig. 703a.

$$\text{(I) Cost of bar } (f) \left\{ \begin{array}{l} (a) \text{ Length (ft.) of bar} \\ \qquad (\times) \\ (d) \text{ Cost per foot} \qquad \left\{ \begin{array}{l} (b) \text{ Weight (lb.) per foot} \\ \qquad (\times) \\ (c) \text{ Cost per pound} \end{array} \right. \end{array} \right.$$

FIG. 703a.—Analysis of cost of bar.

But general principles also justify the analysis of Fig. 703b.

$$\text{(II) Cost of bar } (f) \left\{ \begin{array}{l} (e) \text{ Weight (lb.) of bar} \left\{ \begin{array}{l} (b) \text{ Weight (lb.) per foot} \\ \qquad (\times) \\ (a) \text{ Length (ft.) of bar} \end{array} \right. \\ \qquad (\times) \\ (c) \text{ Cost per pound} \end{array} \right.$$

FIG. 703b.—Analysis of cost of bar.

If the data, *a*, *b*, *c*, *d*, *e*, are all on hand and of equal quality, the estimator will carry his analysis only one stage; and he will calculate either $f = a \times d$ or $f = e \times c$ (the one product is as dependable and as easy to find as the other). He will not bother to calculate either *d* or *e* from its elements, because, under the assumption of uniform data, to do so would be to throw time away. The foregoing remarks obviously hold for any desired degree of closeness of estimate.

If the data, *a*, *b*, *c*, *d*, *e*, are all on hand, but *d* and *e* are less reliable than *a*, *b*, and *c*; and if also the closest possible estimate, *f*, is desired; then if sufficient time is available the estimator will calculate $f = a(b \times c)$ from I, or $f = (b \times a)c$ from II (and of course the two ways are practically identical).

If time will not permit this, he must necessarily be content to calculate either $f = a \times d$ (using the *d* on hand) or $f = e \times c$ (using the *e* on hand), though in either case the resulting *f* is less accurate because the *d* is not as reliable as $b \times c$, and *e* not as reliable as $b \times a$.

Again, suppose *a* and *d* are the only available data, and of inferior quality at that. Then even if the closest estimate, *f*, is desired, the estimator is under the necessity of calculating $f = a \times d$—unless his client will authorize him to spend the requisite time and money to obtain by special search or research more reliable data than a and d.

704. The estimator **diagrams** each step of his analysis as he makes the step, in such a way as to indicate not only the various disclosed parts, sub-parts, sub-sub-parts, and so on, but also the relation between the cost of each segregated part and the cost of the whole structure.

Experienced estimators do not as a rule make actual diagrams (like Fig. 707, for example) of analysis of cost on paper, but build up and carry in their heads the equivalent of such diagrams. The inexperienced estimator should test the carrying capacity of his head before giving up the practice of paper-diagramming his analysis as he makes it, step by step. See §§723, 724.

TYPICAL PROCEDURE FOR ESTIMATING IN DETAIL

705. A typical **procedure** for making a detailed estimate of cost of a proposed structure fully described by plans and specifications at hand, will now be given.

706. The estimator **visualizes** the structure. If it is made up of separate component structures, he recognizes and notes each structure.

For example, the estimator, about to figure the cost of a section of proposed highway, might recognize as separate component structures: a cut, a bridge, a culvert, the pavement, etc. He notes these component structures and the relation between the individual costs of these and the compound structure, either mentally or on paper: preferably on paper and in diagrammatic form, as in Fig. 706, for example.

$$\text{Cost of highway}\left\{\begin{array}{l}\text{Cost of cut}\\ \quad(+)\\ \text{Cost of bridge}\\ \quad(+)\\ \text{Cost of culvert}\\ \quad(+)\\ \text{Cost of pavement}\end{array}\right.$$

FIG. 706.—Analysis of cost of highway.

707. In estimating the cost of a part the estimator visualizes it as a distinct structure, and recognizes and notes its major component parts, each part being distinctive by reason of the material used or of the trade of the men who make it, or some other peculiarity.

In the case of the culvert, for instance, he might, for estimating purposes, recognize: excavation, cast-iron pipe, concrete headwalls—the latter, say, corresponding to "Cost of part A" in the diagram, Fig. 707.

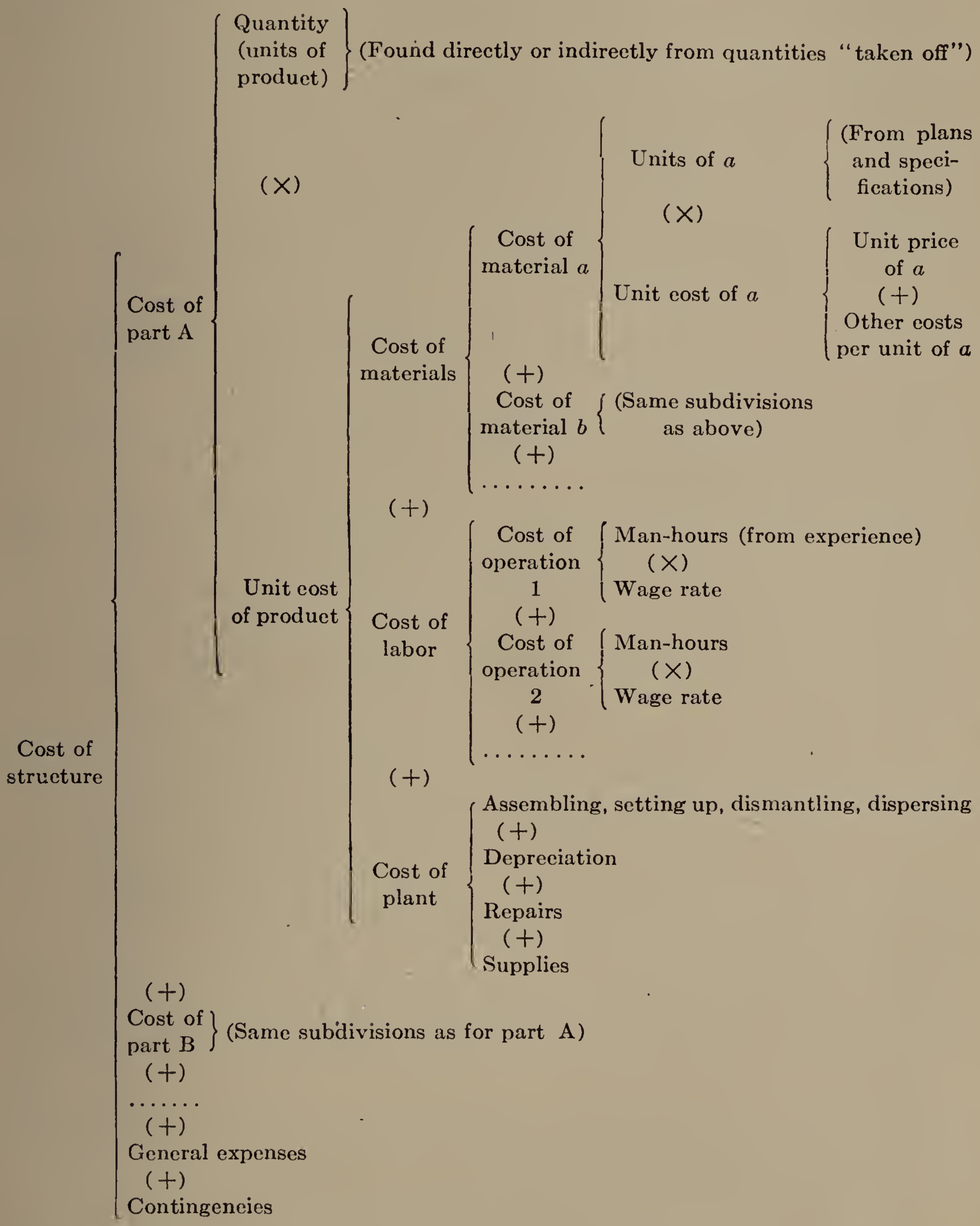


Fig. 707.—Analysis of cost of structure.

708. The estimator then visualizes each distinctive part in turn, following in imagination the work of fashioning that part, and recognizing the component materials and the operations by which they are incorporated to form the part; and thus he builds up his **diagram** of analysis of cost, until all the necessary data are recognized as available in numerical form.

709. Guided by the diagram, he takes off **quantities,** and lists all other cost data.

Take the concrete headwalls for illustration: The estimator would picture to himself the operations of building the forms (out of lumber, nails, wire, etc.); mixing the cement, sand, crushed stone, and water, and placing the resulting concrete in the forms; and later stripping the forms from the concrete. And further, he would call to mind the means by which the materials and plant are likely to be assembled, and the plant dispersed. His analysis diagram, actual or mental, would be similar to Fig. 707 except that it would show specific instead of general terms.

In the work of taking off quantities a saving of time and a decrease of risk of error are effected by recording the quantities on paper ruled into columns suitably spaced for entering the following: location or identification of item; description of item; quality; dimensions; quantities; unit prices; and extensions.

Also to facilitate the taking off and to reduce the risk of error through omission or duplication of items, the estimator follows some systematic sequence in taking off. For example, he might begin with the foundation of a building and finish with the roof; or begin at the northeast corner of a basement wall and follow around the building in clockwise direction; and so on.

It is a common custom, in offices in which a few lines of construction or production are regularly carried on, to have printed or manifolded for each line, copies of an **estimate sheet** on which appear the elements of cost usually found in that line of work. After repeated use, and revision from time to time, such a sheet approaches completeness and is a great aid to the estimator, and insures against omission of any item of the average job.

Such an estimate sheet is that issued by the Associated General Contractors of America, and prepared in detail for highway work.

710. Having taken off the quantities, the estimator determines and notes the **unit price** applicable to each kind of material, and the **wage rate** applicable to each labor craft.

711. The next step is to make and total the **extensions,** that is, make the multiplications, additions, etc., indicated in the diagram, Fig. 707.

712. Next, an estimate is made of **other direct costs** than those estimated above.

As examples of such costs may be cited transportation charges, watchman's wages, and so on.

713. The elements of **indirect cost** (expense or overhead) are next compiled and totaled.

Such elements may embrace depreciation and repairs on construction plant, general office expense, liability insurance, etc. These are usually found by pro-rating.

714. A grand total is taken; and to this is added for **contingencies** a percentage corresponding to the circumstances of the case in the judgment of the estimator; thus obtaining the **estimated cost of the structure.** To this estimated cost may or may not be added a percentage for **profit,** according to the circumstances of the case.

ABBREVIATED PROCEDURES

715. The foregoing procedure is typical of cases in which the estimator must carry the analysis to the degree of subdivision indicated by the diagram, Fig. 707, in order to disclose elements whose values are known to him. It is often the case, however, that through foresight or good luck the estimator is in possession of values of some of the compound elements, or their ratios, disclosed during the earlier stages of the analysis—values derived from cost records of precisely similar existing structures—and in such case he gains nothing by carrying the analysis beyond the points where he discloses these known compound elements; and on the contrary gains time both in analyzing and in computing, by stopping the analysis at these points.

Or, the purpose of the estimate may permit a sufficiently close estimate to be made from values of compound elements taken from cost records of other structures more or less similar to the one under consideration. In this case, also, the estimator would direct the analysis toward and carry it only far enough to reach those values.

Or again, as not infrequently happens, the time available for making an estimate will not permit calculating the estimated cost from ultimate elements, no matter how superior their values may be to the compound-element values on hand; and in cases like this the estimator is compelled to use the compound-element values on hand even if they are from records of structures more or less unlike that for which the estimate is being made, either in size, shape, material, or conditions controlling construction; and he directs and abbreviates his analysis accordingly. Of course he modifies the recorded data before entering them into his calculations; or, if not, modifies the result of his calculations, to the best of his judgment, to neutralize the effect of the deficiencies in the data.

Five abbreviated procedures will now be given.

First Abbreviated Procedure

716. If the estimator has on hand the unit cost of product for each of the various products that go to form a structure, his analysis of the cost of such structure need not be carried beyond the point where (Fig. 707) "Unit cost of product" appears. That diagram, under these conditions, reduces to Fig. 716.

$$\text{Cost of structure}\left\{\begin{array}{l} \text{Cost of part A}\left\{\begin{array}{l}\text{Quantity (units of product)}\\ \qquad\qquad(\times)\\ \text{Unit cost of product}\end{array}\right.\\ \qquad(+)\\ \text{Cost of part B}\left\{\text{(Same subdivision as above)}\right.\\ \qquad(+)\\ \dots\dots\dots\dots\\ \qquad(+)\\ \text{General expense}\\ \qquad(+)\\ \text{Contingencies}\end{array}\right.$$

Fig. 716.—Analysis of cost of structure.

The estimator ascertains from plans and specifications the number of units of product in each part. The other factors are known from cost data on hand or available. See §§809–817 (Bringing Old Cost Data up to Date).

Second Abbreviated Procedure

717. If the cost data on hand include "Cost of part A," "Cost of part B," and so on, of the structure, the estimator's work in finding the cost of the structure is of course reduced to the steps indicated in Fig. 717.

$$\text{Cost of structure}\left\{\begin{array}{l} \text{Cost of part A}\\ \qquad(+)\\ \text{Cost of part B}\\ \qquad(+)\\ \dots\dots\dots\dots\\ \qquad(+)\\ \text{General expense}\\ \qquad(+)\\ \text{Contingencies}\end{array}\right.$$

Fig. 717.—Analysis of cost of structure.

Third Abbreviated Procedure

718. Sometimes the estimator has on hand the ratio which the cost of each part of an existing structure bears to the cost of the structure. In estimating the cost of a proposed structure of the same general type, he estimates the cost of one part by the easiest method permitted by his store of cost data, and divides this cost by the previously ascertained ratio of cost of structure to cost of this part.

A builder analyzed his cost records pertaining to several completed frame dwellings and found that the per cent ratios of items to total ran about as in Table 718.

TABLE 718.—PER CENT RATIOS OF PARTS TO WHOLE

General item	Per cent of whole cost
Masonry	5
Lumber	23
Carpenter labor	21
Plastering	9
Millwork	17
Plumbing	8
Shingling	6
Hardware	4
Painting	7
Total cost of dwelling	100

The builder estimated the lumber for a proposed frame dwelling at $960; his past experience, as represented by the schedule above, showed that the cost of lumber was about 23% of the total cost; consequently he computed the probable cost of the proposed dwelling at $(^{100}/_{23})960$ which, in round numbers, is $4200.

The foregoing is to be taken merely as an example illustrating a method of estimating; the percentages given are not intended for information.

Of course the degree of confidence to be placed in an estimate made by this method will depend, in part, upon the number of structures upon which the schedule is based, and the variations which the schedules of individual completed structures show in comparison with the adopted average schedule. In general, it is to be expected that the larger the percentage of the item chosen as a basis, the more reliable will be the result; for instance, the lumber item (23%) in the schedule above should be more reliable as a basis than the plumbing item (8%). The number of items into which the whole cost should be divided for the purpose of compiling a percentage schedule will depend upon circumstances and the character of the original data.

It is to be supposed that the mason would incline to use the masonry cost, the plumber the plumbing cost, the millman the millwork cost, and so on, as a basis for computing the total cost of a dwelling from the schedule.

Fourth Abbreviated Procedure

719. The estimator sometimes finds that the structure of which he is about to find the cost is very like one or more existing structures of which he knows the cost. In such cases he may assume the probable cost of the proposed structure to be the same as the known cost; or he may add to or subtract from the known cost a sum which in his judgment corresponds to changes in prices and wages, and to such difference as he may recognize between the proposed and existing structures.

To illustrate: An experienced builder inspects the plans and specifications for a proposed dwelling with a view to making a rapid estimate of its cost; and mentally passing in review all the dwellings that he has built, notes that the dwelling which he built for Henry Hillhouse at a cost of $5500 is on the whole very like the proposed dwelling as to quantity and quality of materials and labor required; recognizes the fact that prices are a little higher now than when the Hillhouse dwelling was built, and accordingly estimates the cost of the proposed dwelling at $6000. Or, perhaps, he classifies the proposed dwelling as lying (as to cost) somewhere between two dwellings already built by him—one for Brown at a cost of $5500, the other for White at a cost of $7200; judges that the proposed dwelling will cost nearer $7200 than $5500, and estimates its cost at $6500.

The method here exemplified is used more than all other methods of estimating. It is as quick as, if not quicker than, any other method. The estimator applies the method with equal facility, if not with the same sureness, to the structure of which he has only a dim mental picture and to that of which he holds complete working drawings. This method is, or should be, always used as a check on cost estimated by other methods; and is universally employed by executives to test the work of subordinates for blunders. Furthermore, this method is more reliable in some cases than the more elaborate methods—in cases where the plans and specifications and conditions which have controlled the cost of several actual structures will control the cost of the proposed structure.

Fifth Abbreviated Procedure

720. Finally there is the procedure based on the assumption, often more or less closely approximating the truth, that the cost of a structure varies directly with the number of units of some chosen kind which the structure contains—either a unit of material, or a unit of product.

The estimator finds the number of units in the structure, multiplies the number by the cost per unit as ascertained from experience.

For instance, to find the cost of a proposed bridge by this quick method, the estimator calculates the approximate weight of the bridge in pounds and multiplies the weight by a pound price that has been learned from experience; or to find the cost of a building of a certain type he takes off the number of square feet of floor area, and multiplies this number by the square-foot cost supplied by experience; or he may estimate the cost of a building by multiplying the number of cubic feet of space that it encloses by the cubic-foot cost ascertained from experience; and so on.

Abbreviated Procedures (Concluded)

721. It should be understood that the estimator **varies his methods** from structure to structure, indeed from part to part of structure, guiding himself in choice of method by cost data available, time at command, and purpose of estimate.

He may for example completely analyze the cost of one part of a proposed structure and adopt a lump-sum figure on record for the estimated cost of another part.

722. Since the estimator's object is to turn out an estimate which shall have some predetermined degree of closeness, with a minimum of effort and time, he takes every available short cut the use of which will not lower the predetermined closeness of approximation. He bears in mind that the uncertainty in the total estimated cost is no more affected by a large percentage of error in a small item than by a small percentage of error in a large item; and proportions his time on small items and large items accordingly.

If the actual cost of, say, nails in a structure is to be $18.33 and the actual cost of labor is to be $3736.00, it is plain that forgetting the nails altogether would have no greater effect on the total estimate of cost than estimating the labor too low by 0.5%.

USE OF ANALYSIS DIAGRAM

723. Making the analysis of a problem is of course an indispensable preliminary to the solution. Making a diagram of the analysis on paper is not an indispensable preliminary. Nevertheless the diagram should be actually constructed. The student will find this statement justified by the following considerations.

1. By analysis the estimator discovers the elements of the problem and the relations which they bear to one another. In solving the problem none of the elements must be overlooked, and none of the relations confused.

The mental effort required to prevent omission and avoid confusion is far less when the diagram is on paper than when it exists only in the mind. The effort saved by the use of the written diagram is the greater as the problem is more complex. It is safe to say that it has been far easier for the reader to follow the analysis of investment choice in this volume than it would have been if no diagrams had been inserted.

2. Having made the diagram on paper, the estimator can attack each sub-problem with a mind entirely free from thought of other sub-problems and of the problem as a whole.

3. When he has solved a sub-problem, the result is a known factor, ready to use in a subsequent step in the solution of the original problem. He should record the result where it may be conveniently found when wanted. The diagram suggests to him a convenient form and a convenient place for the record of his result.

4. When the estimator has completed the solution of a sub-problem he checks it off on the diagram. By this practice he makes the diagram a clear progress chart from which he can tell at any time what has been done and what remains to be done.

5. By a glance at the diagram at any time he gets a bird's-eye view of the elements of the problem and their relations. Many times during the progress of the solution he will need this view, particularly at the beginning and end of the work on each sub-problem.

6. The logical arrangement of the parts and sub-parts of the report on the solution of a problem is indicated by the analysis. The only way of testing the logic of the arrangement of the parts of the report is to compare the arrangement with the analysis of the problem. A written diagram of the analysis is therefore a

powerful aid in drafting an outline of a report, and in judging a drafted outline.

7. If an assistant makes a reasonable analysis of a problem assigned to him by his chief, and a clear diagram of that analysis, the chief can know from a brief inspection of the diagram that the problem has been assigned to the right man.

8. Finally, the benefits arising from the use of a diagram of analysis on paper, enumerated above, become greater when many men are engaged on the solution, some on one part, some on another, under the direction of a chief.

The diagram is a material aid to the individual workers, but especially helpful to the chief. The chief analyzes the problem, gives each major sub-problem to the subordinate best qualified to undertake it, and finally receives from the subordinates their results, and properly combines them. For large problems each subordinate, acting as a sub-chief, may subdivide his sub-problem, and allot the different parts to *his* subordinates for solution. The diagram in such cases is indispensable.

724. From the foregoing discussion we may derive the following useful **working rules**:

1. Analyze the problem, making an outline or diagram as the analysis progresses.

2. Carry the analysis forward until the values of all factors on the right of the diagram are known to be on hand or ascertainable by search or inquiry.

3. Indicate by signs or equations the relations which exist between the factors disclosed by each step in the analysis. (When the relations between elements in a group are complex it is well to write on a convenient margin of the diagram the formula which expresses the relation.)

4. Draft a list of the major divisions of the solution, naming them in the order in which it is planned to take them up.

CHAPTER VIII

PRACTICAL ANALYSIS OF FIRST COST (CONCLUDED)

Procedure in Estimating (*Concluded*)

801. In the preceding chapter the general frame work of the analysis of the problem of estimating cost has been erected. The solution of the problem involves the substitution of numerical values for the elements ultimately disclosed by analysis and the combination of the elements in accordance with the indications of the analysis diagram.

The student needs no instructions in regard to the calculations required to effect the combination of numerical values of elements (that is, to figure the total cost from the elements), once those elements are evaluated; but for the work of evaluating the elements he will need some instruction. It is the main purpose of this chapter to supply this instruction.

PRICES, WAGE RATES, AND MAN-HOURS

802. Customarily the estimator uses current **price** and **wage rate,** or his best approximations to them, in making an estimate, even when he anticipates that, at time of construction, price and wage rate will be higher or lower.

Any anticipated change in price or wage rate between the date of making the estimate and the proposed later date of building, he usually takes into account by adding to or subtracting from the cost, figured on the basis of prices of date of estimating, such a percentage of that cost as he judges will make the necessary adjustment.

803. The estimator draws on several **sources** for price and wage data. An estimating office is likely to accumulate records of unit prices along the lines of interest to the office; but often the estimator will require a current price which does not appear in the office records. Sometimes he can conveniently obtain the needed prices from the purchasing department of the organization of which he is a member. Often he finds it best to obtain a current price directly from a dealer. Prices current or recent can be obtained also from catalogs and discount sheets, and from market quotations published in technical and trade papers and, in some instances, in newspapers.

804. Occasionally the estimator does not know the current price of a particular material and does not wish to take the time and trouble to obtain it. He then falls back on a price of some earlier date—some price that he remembers or finds on record—bringing the price up to date by means of his knowledge of general price changes subsequent to the date of the chosen price.

805. When the desired unit price of a material is not available, the estimator may deduce an approximation to it from the known unit price of some similar material.

For instance, the price of reinforcing bars being unknown, it may be approximated by making such modification as judgment directs, in the known price of angle bars or even of pig iron.

806. The only basis for predicting the number of **man-hours** of labor likely to be required for one unit of achievement of any kind, on proposed work, is past experience. Where a good cost-keeping system is in force, records of man-hours per unit of achievement are built up almost automatically. But in many cases the estimator has no such record to draw on; and must do the best he can in the time available to extract from the time keeper's records the desired rate of achievement; and must appeal to published records of such rates when the office records fail him.

807. The number of man-hours of labor required for one unit of achievement of any kind is the resultant of a great many factors. Hence it is that records give widely differing figures for man-hours per unit of achievement; and that the confidence with which any one figure from the records can be used in estimating is dependent on a knowledge of the **conditions** that affected the work from which the figure was derived.

Among the conditions which affect the number of man-hours required, say, to load 1 cu. yd. of sand are the following: the mental and physical make-up and condition of the man, the condition of the sand, and the general conditions under which the work is done. With conditions all favorable there might be required 0.4 man-hour to load 1 cu. yd.; while with conditions all unfavorable there might be required 0.7 man-hour.

808. Occasionally the number of man-hours per unit of achievement is unknown to the estimator, and unobtainable from his records or associates. In such event his only course is to form a **mental moving picture** of the work involved, from start to finish, and act as timekeeper.

To find how many man-hours will be required, for example, to unload by hand a car of 2 ft. cast iron pipe in 12 ft. lengths, the estimator will visualize the job—the number of men required, the manner in which they will carry on the work—time the operations as well as he can, and so form an estimate of the total man-hours required to unload the car. Then, from the known number of pipe lengths or total weight of pipe in the load, he can obtain the number of man-hours required to unload 1 length, or 1 ton, of pipe.

BRINGING OLD COST DATA UP TO DATE

809. In following any one of the abbreviated procedures (§§715–722), the estimator frequently must use an old cost—after, of course, having brought it up to date.

810. The current **methods** of bringing costs up to date are merely abbreviations of the following described precise method, which is seldom employed.

This precise method is based on the assumption that the hourly output of labor remains constant in spite of changing wage rate—an assumption that will be called in question after the method is described.

811. The **precise method** of bringing the cost of an earlier date up to the present is as follows: (1) Divide the given lump cost into parts, each of which is the total cost of one kind of labor or of one kind of material. (2) Multiply each resulting labor cost by the wage rate ratio (that is, the ratio between the present wage rate and the wage rate of the earlier date). (3) Multiply each resulting material cost by the price ratio (that is, the ratio between the present price and the price of the earlier date). (4) Add these products, obtaining the most reasonable figure for the present lump cost corresponding to the given lump cost; *provided* that the same total number of man-hours of each kind of labor will be required for the proposed structure as were required for the earlier structure.

812. There are two reasons why the precise method given above is not used in practice: (1) It is seldom the case that there are available sufficient data regarding the earlier structure to permit a literal following of the method; and (2) it would not be desirable to apply the method literally even if all needed data were at hand, because a result of substantially the same reliability can be obtained with much less effort by greatly abbreviating the process.

813. Often a former unit cost or lump-sum cost for a given structure may be satisfactorily brought up to date by multiplying the former cost by the general cost index ratio for that class of structure (that is, the ratio of the general cost index number of the present to that of the date of the former cost).

For example, the cost of building a certain structure in 1913 was $20,000. The building cost index number of November 1921 was 168 as compared to 100 in 1913. What would have been the cost of a like structure in November, 1921?

The building index ratio is here $^{168}/_{100} = 1.68$, which multiplied by 20,000, the cost in 1913, gives $20{,}000 \times 1.68 = \$33{,}600$, the probable cost in November, 1921.

814. Sometimes, instead of the building cost index number ratio, the price ratio of that material (or the wage rate ratio of that kind of labor) most largely used in the structure is used as the factor for bringing the cost of a former date up to the present.

It is obvious that closer approximations could be secured by multiplying the given lump cost by the average of two or more ratios pertaining to the more largely used materials and kinds of labor. At times the estimator is justified in weighting the ratios and using the weighted average.

815. In converting an old cost into new, the estimator bears in mind the fact that when wage rate rises (or falls) because supply of labor lags behind (or outruns) demand, the hourly output of labor tends to fall (or rise); and that hence cost of labor has two sources of variation: (1) The direct variation due to change in wage rate; and (2) an indirect variation. in the same direction, due to change in rate of labor output.

He draws the conclusion, when he multiplies an old cost by a price ratio or wage rate ratio to bring it up to date, that he should make some further adjustment to correspond to the indirect effect of change in wage rate on unit cost of labor achievement; but when he rejuvenates the old cost by multiplying by a cost ratio, no such adjustment is required on his part, for the simple reason that the cost ratio itself contains the corrective element.

816. From personal observation, and the related experience of others, I judge that many workmen and many groups of workmen accomplished in 1 hr. during the World War at double the 1913 wage, not more than one-half the hourly output of 1913. This judgment suggests the idea that the cost of a specified labor output varies approximately as the square of the wage rate.

I have tested this idea by the only set of data I have been able to find for this purpose, and found that for this particular case the cost of labor varied over a 20 yr. period more nearly with the square of the wage rate than with the wage rate.

817. Finally, a recorded cost must be modified by judgment for use in estimating the cost of a proposed structure, according as the **conditions** expected to obtain on the proposed work differ from the conditions that surrounded the work which supplied the recorded cost.

This brings out the fact that, in order to use records of costs of past work with confidence, the records must include a full description of the conditions that had a material influence on those costs. In short, the circumstances of the work from which costs have been derived are **indispensable** cost data.

That the statement just above is not exaggerated is shown by the following: The cost of excavation per cubic yard, for example, depends on the number of cubic yards in the excavation; on the nature and condition of the material excavated; the kind and condition of the excavating plant used; the intelligence and vigor with which the work is carried on; the shape of the excavation; the length of the haul; wages; and other conditions and circumstances. For example, rock costs more than earth, mainly because it requires a greater amount of labor or other power to loosen it; wet earth requires more effort than dry, because it sticks to the shovels and vehicles; and in some cases, as in tunnels and foundation pits, the cost of excavating soft material greatly exceeds that of solid rock because of the water that has to be pumped to keep the excavation in workable condition; in an excavation of considerable extent machine work is usually cheaper than hand work; very shallow excavations, deep trenches, shafts, wells, and tunnels, all cost more per cubic yard than excavations in similar material, which, due to more favorable shape, present fewer hindrances to the free movement of the forces; work is usually done less rapidly when laborers are scarce than when they are plentiful; work done under contract is as a rule pushed more vigorously than work done by day's labor; higher wages and higher cost of explosives and plant make a higher cost per cubic yard of material moved; and so on. The significance of all these conditions to the estimator will be appreciated by even the novice when he reads that the cost of excavation per cubic yard runs from below 10 cts. to upward of $5, according to the circumstances of the work.

AIDS IN ESTIMATING

818. The foregoing pages of this chapter give some indication of the great variety of data required by the estimator. It is entirely impracticable to give in this book anything more than this indication, for there are a great number of distinct lines of construction and of production, and each line calls for a great variety of cost data, differing in many ways from the cost data required by each of the other lines.

Something worth attention, however, can be said in a reasonable space regarding facilitating the work of estimating.

819. We have seen that time is one of the limiting factors in making an estimate. Obviously, then, the estimator's data, such as he has, should be in the most readily available form. It is not sufficient to have on file the pay-rolls and receipted bills for one or more past pieces of work of character similar to that of the one for which the estimate is being made. It is seldom practicable, at the time of preparing a preliminary estimate, to take the time to extract the useful data from a great mass of original records of construction, and as a result many an estimate is made in an office stored with original records which, though filled with potentially usable data, lend little or no aid to the estimator.

The original records of construction costs should be studied, sorted, and sifted as soon as made, or as soon thereafter as practicable; selected data should be tabulated; and ratios, relations, and laws, which may be of direct use in making preliminary estimates, should be derived from the data and recorded in the shape of formulas, or tables, or charts, all of which should be accompanied by descriptions of all pertinent circumstances and conditions of the work from which the data are drawn.

In short, the data of experience should be reduced to their lowest or most convenient terms at the earliest opportunity.

The point of the foregoing paragraph may be emphasized by quoting Dr. J. A. L. Waddell:

"In concluding this chapter on 'Office Practice' the author desires to call again the reader's attention to the necessity for adopting the most systematic methods possible for doing all kinds of work, keeping all kinds of records, and filing all kinds of accumulated material. As soon as a large piece of work is finished a thorough systemization should be made of the knowledge obtained in making both the design and the various calculations, so that the office force shall be able to use the same to the best possible advantage when starting on another piece of work. And whenever there is any spare time in the office for any of the employees it should be devoted to accumulating, digesting, and putting in convenient form for use the results of previous investigations, and to doing such work as tabulating and recording on diagrams the weights of metal per lineal foot of span for bridges of all kinds.

"Finally, in bringing this little treatise to a close the author feels that he cannot do better than repeat from chap. II the following principle: 'The systemization of all that one does in connection with his professional work is one of the most important steps that can be taken toward the attainment of success.' "[1]

820. Attention has been called previously to the use of specially ruled paper as a practical advantage to the estimator in taking off quantities; and to the desirability of making lists of cost elements entering into total costs of structures, and perfecting those lists as experience dictates. Such lists not only save time, which may be a point of great moment in some cases, but also reduce greatly the risk of errors of omission.

821. The estimator preserves a copy of every estimate that he makes, not only for the purpose of meeting any question that may arise in regard to the estimate, but also as an addition to his stock of estimating data.

According to present custom, a constructor or producer, in preparation for bidding on a job, makes an estimate of cost of the job. If, say, ten bids are made for one job, then there will be nine estimates made in vain, because only one of the bidders will obtain the job. The cost of making estimates for unsuccessful bids is a part of the bidder's general or overhead expense, and is all charged to and helps swell the costs of the jobs that he does get.

Thus in every constructor's and producer's office, the number of estimates made is many times the number of production or construction jobs secured. The estimator in such an office therefore automatically increases his stock of estimating data quite rapidly.

822. In former years it was not the practice of business units to permit any of their cost records to be given to outside parties, but more recently business units are releasing some of their cost records. Thus the estimator has now the resource of a great variety of published cost data.

---

[1] *De Pontibus, a Pocket Book for Bridge Engineers* (New York: John Wiley & Sons), p. 346.

823. If the estimator is working in the office of a constructor or producer, his best source of cost data should be the cost records of work done by the constructor or producer, or of work in progress. But some cost records of construction and of production give little more than pay-rolls and invoices of materials and plant—the bookkeeper's records. In order that construction or production may yield the various kinds of cost data required by the estimator, there must be kept on the work a specially devised set of records, supplementing the bookkeeper's records. Devising and keeping such supplementary records is called costing, or cost accounting, or **cost keeping,** which is described in chap. XIV.

CHAPTER IX

BUSINESS UNITS

Use of Borrowed Capital—The Individual Business —The Partnership

901. Engineering Economics deals with those business problems which require for their solution more or less knowledge of engineering. Business problems differ one from another in many ways, but they all have this in common: for each one of them some or all of the characteristics of the business unit concerned are pertinent data.

It follows therefore that the characteristics of the business unit are among the elements of Engineering Economics. The chief characteristics of a business unit are represented in its organization, control, management, and methods of raising capital.

It is the purpose of this chapter and the two following to give the student some picture of the structure and working of each of four kinds of business units.

It is hoped that the statements of legal doctrine made in these chapters, although purely incidental to the primary purpose of the chapters, are accurate. Perhaps however it is necessary to warn the reader that no such brief and incidental statement can safely be made the basis of any extension by inference. In any concrete business situation involving a *legal problem* the engineer must call to his aid competent *legal advice.*

Before entering upon a description of the business units it will be well to consider the custom, universal among private business units, of using more or less borrowed capital, and to learn the cause of the custom and some of its effects.

USE OF BORROWED CAPITAL

902. Carrying on business with capital of which one part is borrowed and secured by the other part owned, is sometimes called **trading on the equity.**

The owner's equity is his right to that part of his own property which may be over and above the contingent right given by him as security to his creditor.

The cause and effects of trading on the equity are most readily discovered by considering a numerical example:

Example

Smith carries on a business for a year with his own capital, $10,000; with a resulting yield of $2000, or 20% on his capital.

He then borrows $5000 at 10%, pledging his own capital as security, and puts it into, and thereby expands, the business. During this second year the business yields $3000, out of which he must pay $500 as interest on the borrowed $5000, leaving him $2500 net from the business.

Now this net result, $2500, has been made possible by Smith's ownership of the $10,000 to serve (1) as part of the business capital, and (2) as security to obtain the other (borrowed) part of the business capital. Hence $2500 net (the total return, $3000, from the business, less the $500 paid for use of the borrowed $5000) is properly the yield on Smith's $10,000 capital. Thus the second year, through the use of $5000 borrowed capital, he has increased the yield on his own capital from 20 to 25%—a strong incentive to doing business on borrowed money.

But the whole of Smith's story has not been told. Suppose that during the third year, the business, capitalized as during the second year, yields only $500. The whole $500 is absorbed by interest so that the yield on his own capital as a result of the whole business is zero. Had he used only his own capital in the business his yield would have been $333.33, or 3⅓% on his investment, $10,000.

903. The foregoing example makes it clear that if part of the capital used in a business is borrowed, the yield derived from the owned capital is augmented or diminished (as the case may be) by the difference between the operating income derived from the borrowed capital and the interest paid for use of that capital.

Thus, if we consider the owner's annual yield on his capital, borrowing holds out the advantage of making good business better; but at the same time threatens the disadvantage of making bad business worse.

904. Trading on the equity may, at a time of bad business, result not only in reducing the owner's yield to zero as we have seen, but in reducing his equity to zero, that is, in his losing all of his capital. The greater the ratio of borrowed to owned capital, the greater of course is the **risk** of losing the owned capital at a time of declining business.

The worst of Smith's story is still ahead:

> In the fourth year, the business, capitalized with $10,000 of owned capital and $5000 borrowed, failed to yield anything over operating cost; the interest due on the borrowed $5000 was defaulted; the lender of the $5000 foreclosed; the assets of the business were put up at public sale by the sheriff; and, because of the general depression, brought in all together only $5500—just enough to pay Smith's debt with interest. Thus Smith's story and his equity in the business terminate simultaneously.
>
> If during the fourth year Smith had used only his own capital, he would have cleared nothing on the business, to be sure, but he could have continued safely in business by disposing of a small part of his assets (at a sacrifice if necessary) or by borrowing the needful amount (and this he could have readily done, because his assets would have been clear of encumbrance). He would have lost income and some small part of his capital perhaps, but he would have held a safe grip on the bulk of the capital and so been ready to take advantage of coming improvement in business conditions.

905. Risk to the lender (as well as to the borrower) is in proportion to ratio of borrowed to owned capital; and the lender aims to proportion the interest rate to the risk. Hence, for each succeeding increment of borrowed capital the higher is likely to be the interest charge; and hence the smaller may be the net return (*i.e.*, the income derived from the borrowed increment less the cost, including interest, of deriving it) on the increment borrowed, which net return is the incentive for borrowing the increment.

From this it follows that the owner's incentive grows, up to a certain total of borrowed money; then diminishes as the total is increased; and at some point becomes zero.

906. The owner's incentive to borrow is partially counteracted by the fact that each additional increment of borrowing further jeopardizes his own capital, as we have seen. The sagacious owner will not make an additional borrowing unless the promised net return thereon is several per cent of the sum borrowed—3 to 10% or upward, according to the risk as judged by the borrower.

907. Summing up, the incentive to trading on the equity is the owner's expectation of increasing the annual increment of his ownership in spite of the fact that his own capital is limited. The drawbacks to such trading are (1) that borrowing makes bad business worse (although it may make good business better), and furthermore (2) that borrowing introduces the risk of the owner's losing all of his capital when business is depressed.

908. Considering the foregoing facts together, we should expect the prudent owner of a business to diminish his borrowed capital in anticipation of declining business, and increase it again in anticipation of a business revival.

THE INDIVIDUAL BUSINESS

909. We are well acquainted with the **sole proprietor,** the owner of the one-man business: drayman, butcher, baker, blacksmith, shopkeeper, farmer, and so on.

Any person may start and carry on almost any kind of business (there are exceptions) without let or hindrance from the government.

Among the exceptions: Lotteries, for example, are prohibited in the United States by law; drug stores are licensed by the state, tobacco shops by the federal government, and laundries by the municipal.

910. The individual business is owned solely by an individual —hence the name "sole proprietor." All credits and profits belonging to the business are the individual property of the sole proprietor.

911. All debts incurred in the business are by law the personal debts of the sole proprietor. He is said to have unlimited personal **liability** for the debts of the business.

For example, A owns a business and, besides, $10,000's worth of property, partly in other ventures and partly in his home. The business fails with liabilities exceeding assets by $10,000. The creditors of the business can lay hold on the outside $10,000's worth of property. (Exception: some state laws exempt certain household goods and trade tools from claims of creditors.)

912. The owner of the individual business has absolute and undivided **control** of the business so long as he meets his obligations.

One who lends capital to the sole proprietor acquires thereby a contingent control, that is, a potential control whose change tc active participation in control is contingent upon failure of the sole proprietor to meet his obligations to the lender. The lender can merely sit by so long as interest and principal are paid according to terms of agreement. There is nothing however to prevent the lender's taking advantage of his position when the business is declining, to give advice to the sole proprietor, which advice the proprietor is free to accept or reject; but the lender has no legal control until and unless the proprietor fails in his obligations to the lender.

913. The sole proprietor may manage the business personally, or through one or more agents; but these agents are merely his employees, responsible solely to him, who hires and dismisses them at pleasure.

914. We have seen that to the owner of a business there is the temptation, if the business be good, to employ more or less borrowed capital. The sole proprietor as a rule employs borrowed capital with his own.

He borrows of his bank (*i.e.*, of the bank which he patronizes) or of capitalists.

He gives his note for the money borrowed, and he may be required to give some kind of security for the payment of the note, and is the more likely to be required to give security the larger the sum borrowed.

He may give as security a mortgage on some of his property, or deposit with his creditor as collateral any negotiable instruments (*e.g.*, promissory notes, certificates of deposit, bonds, bills of exchange) that he may own.

If the sum needed is very large so that there is little likelihood of obtaining the sum from a few capitalists, the sole proprietor can issue and sell a series of bonds, which are promises to pay equal sums of money under certain conditions, and are usually secured by one mortgage held by a trustee. However, long term bonds, 10 yr. or more, issued by a sole proprietor are not likely to sell to advantage, because of the uncertainties as to what 10 yr. of time may do to a sole proprietor.

915. If the sole proprietor sets about expanding his business beyond the limit of his own capital and credit, he finds it necessary to obtain the additional capital needed by taking in one or more partners (thereby changing the business unit from an individual business to a partnership), or by incorporating the business (thereby converting the individual business into a private corporation).

916. In any individual business the yield is likely to fluctuate from year to year, even if there is first class management. The **risk** that the yield may be meager or nothing, or even negative (*i.e.*, a loss), depends upon the kind of business, the location, the total capital employed; the ratio of borrowed capital to the total, and the general policies pursued (factors controlled by the owner); and upon the routine of operation, under the direction of the manager, who may be either the owner of the business or his employee.

The risk of losing capital is, as we have seen, proportionate to the ratio of borrowed capital to the total capital used in the business; and fixing this ratio is a function of control. And it may be well to repeat here that since under the law, all his capital —that outside as well as that inside the business—is subject to claims against the business, this risk pertains to all his capital.

THE PARTNERSHIP

917. A **partnership,** also called a "firm," is a business unit or association formed by a mutual agreement of two or more persons.

918. The ordinary partnership enters into business with as little restriction from government as does the sole proprietor.

919. The **articles of agreement** creating the partnership usually include: (1) the amount of capital to be provided by each partner, (2) the services to be contributed by each, (3) the share that each shall take in the management (the firm may delegate the management to one or more employees), (4) the proportions in which the profits are to be divided among the partners, (5) the proportion of losses to be borne by each, and (6) the proportion of the firm's property each shall receive on dissolution of the partnership.

This partnership agreement may be changed by mutual consent of the partners at any time.

The business is owned by the partners jointly, and the partners therefore have joint interest in the credits and profits of the business.

The peculiar nature of the individual partner's interest in the firm property is indicated in the succeeding paragraphs.

920. The withdrawal of a partner, the death or insolvency of a partner, or the sale of his interest by a partner, **dissolves** the partnership. Partners A and B cannot take C into their existing partnership; but A and B may dissolve the existing partnership and with C form a new partnership by a new mutual agreement.

If a partnership is formed for a specified period of time, the withdrawal of a partner before the end of the period may give ground for damage, but nevertheless dissolves the partnership.

If a partnership is dissolved by the sale of his interest by one of the partners, the remaining partner or partners may form a new partnership with the purchaser under a new mutual agreement.

When a partnership is dissolved for any reason, any assets remaining after all debts of the partnership have been paid are distributed to the partners in proportions prescribed by the partnership agreement.

921. Partners may or may not contribute equal sums of capital, but in every case they have equal powers of **control** of the business. Each partner of the firm may transact the business of the firm to the extent authorized by the partnership agreement (§919, (3)).

As to persons who do not know or have not been put on notice of the provisions of the terms of partnership agreement, the firm is bound by the act of a partner whether or not the act exceeds the authority given by the agreement. But if one did know, in contracting with a particular partner, that this partner was exceeding his authorized powers of management the contract or other act would not bind the firm. A firm may be fairly well protected by giving notice to regular customers or creditors.

A managing partner usually may buy and sell the goods in which the firm deals; borrow money for use of the firm; pay the firm's debts out of its money; receive and receipt for money owed to the firm; and hire and discharge employees of the firm: taking each individual action, it may be, without the other partners' specific consent for it or knowledge of it. Thus it is plain that the business welfare of each partner is largely in the hands of each of the managing partners.

922. Each partner is personally **liable** for all debts of the firm. Unsatisfied creditors bring action, not against the firm, but against the partners jointly and severally.

For example, A and B form a partnership, A putting in all his capital, and B all of his except $100,000 which he has invested elsewhere. The firm fails with assets $100,000 short of liabilities. The creditors may legally lay hold of all of B's outside capital to satisfy their claims.

Exception: Some states authorize "limited partnerships" in which all except one partner may be limited in liability for firm debts to the extent of the capital which they have contributed; and in such cases the limited partnership agreement is required to be made of public record for the protection of outside persons.

923. The partnership business unit enjoys the same opportunities to **borrow capital** as are enjoyed by the sole proprietor, and has the same incentive as he for using borrowed capital. A partnership may increase its capital by taking in a new partner who will contribute capital (this means dissolving the existing partnership and forming a new one including the new partner). Practically there is a limit to numbers in a partnership, and consequently there is a limit to the capital which a partnership can control. This limitation of the partnership has given rise, at the coming of large enterprises, like railroads, to another form of business unit, the corporation, to which we presently turn our attention.

924. The discussion on **risks** run by the sole proprietor applies to the risks of the partnership. Each partner, however, runs one risk that is not run by a sole proprietor, namely, that of loss through unwise or unfriendly or dishonest acts of one or more partners. And, too, it should not be forgotten that all of a partner's capital placed outside the partnership business, as well as that placed in the business, is exposed to the risk of being taken to satisfy the debts of the firm.

CHAPTER X

BUSINESS UNITS (CONTINUED)

Private Corporation

1001. With the coming of enterprises like the canal and railroad, requiring a much greater capital than could be marshalled by any practicable partnership, another kind of business unit was needed, capable of raising the larger quantities of capital. The **private corporation** was introduced to fill this need.

1002. The private corporation is, like the partnership, a business unit, but it is sharply differentiated from the partnership by the possession of the following **characteristics**: (1) it is created by law or under specific authority of law; (2) its powers and liabilities are given and prescribed by law, and are separate and distinct from those of its members; and (3) its life is fixed by law and unaffected by all changes that may occur in its membership. A corporation can sue, or be sued by, one of its own members. In fact, in the law books the corporation is often called an **artificial person.**

1003. The **powers** of a private corporation are limited to such as are specificially given by or under the law and to such others as may be needful for effecting the authorized acts and purposes of the corporation. All corporations are not given the same powers, but all have the following powers: (1) to make contracts in furtherance of the authorized purposes of the corporation (to borrow money, to give security, to issue negotiable paper, to sell its property, to buy property in furtherance of its authorized purposes), and (2) to sue and be sued.

CREATION AND ORGANIZATION

1004. By far the greater number of our corporations have been created under general corporation laws enacted by state legislatures under authority of state constitutions. The general corporation laws differ materially from state to state.

1005. Some corporations have been created by and under Acts of Congress. These include: national, federal reserve, farm loan, and foreign-business banks; certain transcontinental railroads; and certain private corporations in the District of Columbia.

1006. The legal **procedure** for the incorporation of an ordinary business unit is, in very general terms, as follows: The interested parties, called **incorporators,** file with a designated state officer (usually the secretary of state) a paper prepared as prescribed by law and called **articles of incorporation,** and receive from him a paper called a **certificate of incorporation.** (See Appendix A for specimens of these papers.)

While the procedure in every state is in general terms virtually that described above, there is variation in detail from state to state.

1007. In California the prescribed procedure is in essence as follows: Three or more persons, two thirds of whom must be residents of the state, associate as incorporators and draw up articles of incorporation, which articles they then acknowledge (*i.e.*, subscribe to before a notary public), and file in the office of the county clerk of the county in which the principal business of the company is to be transacted. The county clerk issues to the incorporators a certified copy of the articles as filed. The incorporators file the certified copy with the Secretary of State for California, who then issues to the incorporators a certificate of incorporation declaring that a copy of the articles in due form has been filed in his office. The issuance of this certificate is the final step in the procedure of incorporation, and from the instant of issuance, the incorporators and their associates and successors become a "body politic and corporate," the life of the corporation begins.

1008. The articles of incorporation in California, a specimen of which is given in Appendix A, must set forth the following: (1) the name of the corporation; (2) the purpose for which it is formed; (3) the place where its principal business is to be transacted; (4) the term of its life, not to exceed fifty years; (5) the number of its directors, three or more; (6) the names and residences of the persons designated as directors for the first year; (7) the amount of its capital stock and the number of shares into which the capital stock is divided; and (8) the amount of capital stock actually subscribed, together with the names of the subscribers.

1009. The articles of incorporation for a railroad or telegraph organization must set forth in addition to the foregoing the following: (9) the kind of road or telegraph intended; (10) the proposed termini and branches; (11) the estimated length of the line; and (12) the fact that not less than 8% of the subscribed capital stock has been paid in to the treasurer of the proposed corporation.

1010. A specimen articles of incorporation and a specimen certificate of incorporation, relating to a California corporation, are presented in Appendix A.

1011. The articles of incorporation and the corporation laws under which the articles are issued may be considered as together constituting the **charter** of the corporation.

The earlier corporations of this country were created by the state not under general corporation laws but each by special legislative act called the corporation "charter."

1012. The charter prescribes only a few points for the organization of the corporation and the management of its affairs. Such further rules and regulations as may be necessary or desirable for effective organization and management are formulated and adopted as **by-laws** by the corporation itself by vote of its stockholders.

The by-laws must not be inconsistent with the articles of incorporation, or with the constitution or laws of the state or of the United States.

The by-laws usually contain, besides the provisions supplementary to the charter, a simple restatement of the articles of incorporation and of the laws under which the articles are issued, not as a legal requirement but merely for the convenience of the officers and agents of the corporation.

The by-laws usually contain all necessary provisions as to stock, stockholders, directors, standing committees, officers, dividends, finance, amendments, etc.

1013. If in the charter no other provision has been made, a California corporation is authorized to prescribe in its by-laws: (1) all procedure in respect to meetings—calling, place, conducting—of stockholders and of directors; (2) the number constituting a quorum in meetings of stockholders and of directors; (3) the manner of voting; (4) the qualifications and duties of directors; and (5) the manner of appointing officers other than directors, together with their compensations and duties.

1014. In California the stockholders must adopt by-laws within one month after the certificate of incorporation is issued.

If the by-laws are to be adopted at a meeting of stockholders, two weeks' notice of the meeting must be given by the acting president by advertisement in some newspaper published in the county in which the principal place of business of the corporation is located, or, such paper being non-existant, in a paper published in an adjoining county; and at such meeting the assent of the stockholders representing a majority of all the subscribed capital stock is necessary to the adoption of the by-laws. By-laws may be adopted without a meeting of stockholders, by the written assent of the stockholders representing two thirds of the subscribed capital stock.

As each corporation makes its own by-laws we should not expect to find those of any two corporations just alike. In general the length and comprehensiveness of a set of by-laws will correspond to the number of purposes of the corporation and the magnitude of its business.

1015. A specimen of recent by-laws of a California corporation is given in Appendix A.

OWNERSHIP

1016. The **capital stock** of a private corporation is the total number of equal shares, or the total par value of the equal shares, into which the right of ownership in the corporation is divided, as set forth in the articles of incorporation. Each share is a **share of stock.** The owner of one or more shares is a **member** or **stockholder** of the corporation.

1017. Very commonly each share of stock has a **par** value or face value of $100. Some corporations, such for example as those engaged in mining, fix the par value of their stock at $1 or even less. And a few corporations assign no par value to a share. Whatever the par value may be, the actual value of a share of stock is what it will sell for.

1018. The corporation, through its officers, issues to each stockholder a **stock certificate** in testimony of the fact that he is by name registered in the books of the corporation as owner of a certain number of shares of stock of a stated par value. Usually a stock certificate is issued to a subscriber for certain shares of stock only after he has paid in the par value of the stock in cash or in property declared by the board of directors to be of the par value.

1019. A stockholder may sell any number of his shares at the best price he can get. The seller writes on the back of his certificate an authorization for the buyer to have the buyer's name substituted for the seller's on the corporation books.

If this authorization is written with the buyer's name left blank the certificate may pass from buyer to buyer in succession until some buyer finds it desirable to insert his name and have his name substituted on the books for the name of the seller to whom the certificate was originally issued.

When the buyer's name is substituted for the seller's on the books of the corporation, the corporation issues a new stock certificate to the buyer, and cancels the old.

1020. Such of the **income** of the corporation as may be over and above that required to pay all running expenses and fixed charges and to maintain the capital, is **net income.** In this net income each shareholder has an interest proportional to the number of shares he holds, but he cannot legally receive any part of it except in the form of **dividends.**

The total of dividends received by the shareholders for any given period of time may be greater or very much less than the net income for that period, according to circumstances. See §1025.

1021. In case of **dissolution,** each stockholder receives his share (proportionate to the number of shares of stock he owns) of the property remaining to the corporation after all debts of the corporation have been paid.

If all of the corporate property is taken for debt at dissolution, each share of stock will be worthless.

1022. The corporation is **liable** for the debts which the directors or officers have with due authority contracted. The corporation may be liable, under the laws of many of the states, also for certain other debts—for example, debts contracted outside of the due authority of the directors or officers but for which innocent creditors have given value to the corporation. The corporation may be liable also for damages on account of wrongs done by the directors or officers acting on behalf of the corporation. (Any such wrong is of course without due authority, for no corporation is authorized by law ever to do wrong.) The creditors of the corporation may lay hold of the corporate property to satisfy over-due claims; but they may not lay hold of property belonging to individual stockholders, except (1) property of original stockholders who have paid in less than the par value of their stock, and then only to the extent of the shortage; and (2) when the charter prescribes some personal liability, and then only to the extent of the prescribed liability.

✓ The constitution of the state of California makes a stockholder in any corporation personally liable for such proportion of corporate debt as the number of his shares at the time the debt was incurred bear to the total of subscribed shares.

✓ In some states—Ohio for example—the stockholder is not personally liable for the debts of the corporation except to the extent of the difference between the par value of such shares as he may have subscribed for and the sum that he has paid in on those shares. California and Ohio represent the extremes of personal liability imposed on stockholders of the ordinary private corporation by the various states. The personal liability of the subscriber or buyer of a stock is evidently an important point for investigation when considering the purchase of the stock.

CONTROL AND MANAGEMENT

1023. The stockholders, being the owners, **control** the corporation. They perform their acts of control only when assembled in legal meetings and by vote. They elect the directors; adopt and revise by-laws; determine increase or decrease of capital stock; determine on the issue of securities, such as bonds; and fix the general policy of the corporation. They decide such important questions as they have in the by-laws reserved for their own decision; but in other matters they delegate their control to their board of directors.

The stockholders' control may be, as a matter of fact, rather slight. Once a board of directors is elected, unless the articles give to the stockholders a power of recall or reserve specificially the power to instruct the directors by stockholders' resolution, most jurisdictions permit the directors to disregard any such resolution or other attempted control by the stockholders, except (1) in matters specifically reserved to the stockholders by statute and (2) matters of (*a*) fundamental policy as distinguished from matters of (*b*) mere business operations. (See §§ 1024, 1026, 1027.) Examples of (*a*) are, even in the absence of statute: enlarging the capital stock, changing the term of existence of the corporation; and of (*b*) are: purchasing property for or selling property of the corporation, dismissing an officer, etc.

California specifies in its constitution that the stockholder may cast in an election for directors a number of votes equal to the number of his shares multiplied by the number of directors to be elected, and that he may distribute his votes equally among the candidates, or cast them all for one candidate, or follow any compromise of these two plans.

This system, which is called **cumulative voting,** is included in the corporation laws of many states to enable the holders of a minority of stock to elect, in spite of the majority, a director to represent their interests.

It is important to note an exception to the control exercised by stockholders: though they are the owners of the net income, they cannot individually receive any part of it except such as the board of directors votes to declare as dividends.

1024. The **board of directors,** in exercising the control delegated to it by the stockholders, is governed by the charter, the rules and regulations in the by-laws, and resolutions passed in stockholders' meetings (qualification: §1023, fine print).

Each director has one vote on every question that comes before the board, regardless of the number of shares he may own. A majority of the directors suffices for a quorum, and, at a meeting of a quorum, a majority of those present, rules, except where otherwise ordered by the charter or by-laws.

1025. The board of directors has control of the *net income* which the corporation derives during a given period, as a year, from the enterprise that it conducts. The board may make appropriations from net income for reserves (for depreciation of capital, for improvements, for investments, for future contingencies, etc.), and the remainder only is then available for dividends (for distribution to the stockholders in proportion to their share holdings). But all of this remainder may or may not be distributed as dividends at once: the board may hold a part of it as a dividend reserve from fat years to augment the earnings of lean years, with the object of stabilizing the dividend rate.

If the board fails to declare a dividend on a customary dividend date, the dividend is said to be **passed.**

1026. The board of directors takes the necessary action to carry out the resolutions passed by the stockholders—as in issuing securities, bringing or defending suits, and purchasing property.

The board may purchase land or other property for use in the business within the limits laid down by the charter and by-laws.

1027. The board of directors may, if it judge best to do so, enter into and determine any point or detail of management of the business carried on by the corporation, but with infrequent exceptions the board delegates this management to officers and agents whom it elects or appoints.

Immediately after an election of directors the board of directors must organize by the election of officers: a president (elected from the directors), a secretary, and a treasurer. The board may also elect one or more vice-presidents, and may appoint a general manager, a general superintendent, a general counsel (legal adviser) a chief engineer, an auditor (in short, any subordinate officers and employees deemed by the board necessary for the proper conduct of the corporation's business); and may prescribe their duties, determine, raise, or lower their salaries, or remove them at any time.

1028. Usually the **president** is the supreme head of the management under the board of directors. He alone regularly reports directly to the board. (Any other officer or employee reports to the board only at the specific request of the board.) All other officers report to him. Under the board he directs and is responsible for all internal and external activities of the corporation.

The president directs and is responsible for the following: purchase and sales; finances; the institution, defence, and settlement of suits; the accounting; the reports required of the corporation by the state or the United States; the employment and discipline of employees; the care of the property; relations with the public; and so on. The president or a vice-president signs deeds, contracts, etc., under the name of the corporation.

1029. The **vice-president** or vice-presidents have such powers as are given them by their superiors.

They may sign deeds, contracts, etc., only in the absence of the president or to the extent that they are authorized by the president to sign. In large corporations it is quite customary to have one vice-president in charge of finances, another in charge of accounting, another in charge of operation (in which case he might have the title of Vice-President and General Manager), another in charge of sales, and so on.

1030. The **general manager,** subject to the orders given from time to time by the president, has full authority over all matters stated above to be under the direction of the president (except the execution of papers); but his powers may be limited.

For example, he may take care of everything except finances; or everything except accounting; or everything except suits; and so on; or he may have charge only of operation. Usually the general manager does not have authority over the general counsel, who in that case reports to the president.

1031. The general manager may delegate certain of his duties to one or more assistant managers or managers of departments of the business, who are responsible to him and act under his general and specific instructions.

1032. Under a manager or assistant manager there may be one or more superintendents. A **superintendent** is the head of a main department or of a sub-department. There may be a general superintendent of a department with superintendents directing sub-departments under him.

1033. Next come the **foremen,** each of whom, under the guidance of a superintendent or assistant superintendent, directs the activities of a group of workmen.

1034. The offices of secretary, treasurer, auditor, general counsel, chief engineer, etc., although necessary or desirable adjuncts to management, do not participate in the management of the corporation.

1035. The **secretary** keeps the seal of the corporation; attends the meetings of the board of directors, and records the minutes; and affixes the seal to proper papers and attests it by signing his name and title.

1036. The **treasurer** has the care of the money and securities. He sees that they are safely kept; that idle money is drawing interest; that provision is made for future needs for pay-rolls, taxes, and other anticipated payments. He looks after accounts receivable and accounts payable; signs checks; pays the pay-roll. He takes in all the money and pays out all the money.

1037. The **auditor** keeps the accounts; makes up the checks; keeps accounts of receipts and disbursements; audits the accounts of the treasurer; and furnishes to the management any desired statistics about the business, including statements of earnings and expenses, either in general or in any desired detail.

1038. The **general counsel** is the chief lawyer of the corporation and has direction and supervision of the legal phases of all concerns of the corporation.

1039. The **chief engineer** is advisor to the management in all engineering matters, and directs and supervises such engineering work as the corporation undertakes in furtherance of the enterprise on which it is embarked.

1040. The foregoing paragraphs on control and management are not to be taken as strictly applicable to every private corporation. They hold in the main for many such corporations, however, and give some picture of the relations between the internal units of the organization, to give which picture is the chief object of the paragraphs.

In order that all concerned in the control and management—all officers and employees—may have a clear-cut idea of the organic relations between offices, between officers, and between employees, some corporations post or circulate a chart called an **organization chart,** setting forth in pictorial or diagrammatic form these organic relations.

HOW THE PRIVATE CORPORATION RAISES CAPITAL

1041. It is possible for a corporation to start business with cash in its treasury, equal to the full par value of its total capital stock, and very likely there have been actual instances of corporations starting off thus well supplied with cash. However, as a rule, only a part of the necessary capital is supplied by the owners (stockholders), and in some cases a very small part of the total capital employed in the enterprise is supplied by them; the rest being borrowed.

A corporation has, for obtaining capital over and above that originally paid in for stock, four commonly used **methods:** (1) by giving a promissory note, as to a bank, or by selling, for what they will bring, an issue of promissory notes; (2) by issuing and selling bonds; (3) by issuing and selling preferred stock; and (4) by selling additional common stock. These methods will now be briefly described.

1042. The corporation may borrow money at a bank on its promissory **note,** just as does the sole proprietor. But if the needed sum is large enough, the corporation usually prints and executes a number of notes—hundreds or thousands—all alike, each for $100 or $500, or perhaps $1000, and bearing a certain rate of interest; and offers these for sale. In many cases the issuing corporation places with a trustee sufficient negotiable instruments to secure the note buyers against loss.

The corporation sells the notes, directly or indirectly, for what it can get for them, through the agency of investment bankers and bond houses. The advantage of the series of small notes over one or more large notes obviously lies in the fact that small investors are numerous while large investors are scarce. Further, a capitalist does not choose to loan a very large sum without specific security. On the other hand it costs just as much to sell a $100 note as to sell a $1000 note; and this fact acts as a damper on popularizing small-note issues among corporations.

The notes thus issued in series are written often to mature in two years, sometimes in three or four, or even five.

Notes are issued usually to meet some temporary shortage of capital needed in the business, but sometimes they are issued for capital desired for a very long period, with the thought that by the time the notes mature the corporation will pay them off with capital obtained therefor at a rate of interest lower than that borne by the notes.

1043. A **bond,** like a promissory note, is a promise to pay a stated sum, usually either $500 or $1000, to bearer or to a specified person or to his order, on a specified date, and to pay interest, usually semi-annually or quarterly, at a specified rate per annum.

Bonds are issued by corporations usually in series; and in each bond is stated the particulars pertaining to the issue, including a specification of the mortgage security, if any, upon which the whole series rests. Such mortgage as may be back of a bond issue is made to a trustee for the benefit of those who buy the bonds.

Bonds now on the market are of great variety, and appear under a multitude of names that are partially descriptive. For example, a **registered** bond is made payable to a specified person whose name is registered in the books of the issuing corporation. The transfer of such a bond from the registered holder to another person cannot be effected except by registering in the corporation's books the name of the transferee. The interest when due is paid to the registered owner of the bond.

On the other hand, a **coupon** bond is one made payable to a specified person or to his order (a coupon bond may be registered as to principal), or to bearer, and bearing a series of interest coupons (printed on the same sheet with the bond), one for each interest date. Each coupon is a promise to pay the interest due on a specified date, and when severed from the sheet may pass from hand to hand like a promissory note. Each coupon on or after the date named therein is surrendered to the bond issuer in exchange for the interest due.

These and various other kinds of bonds are named in the following classification, which includes bonds issued by corporations public as well as private. The numbered classes are not mutually exclusive.

Classification of Bonds[1]

1. Bond names derived from the character of the issuer:
    - (*a*) Railroad bonds
    - (*b*) Public utility bonds (such as street railway, water-supply, electric light and power, and gas company bonds)
    - (*c*) Municipal bonds
    - (*d*) Industrial bonds (issued by manufacturing companies)
    - (*e*) U. S. and other national government bonds
    - (*f*) State bonds
    - (*g*) Miscellaneous (such as timber, irrigation, drainage, and real-estate bonds)
2. Bond names derived from the purpose of the issue:
    - (*a*) Purchasing or consolidating properties or companies
    - (*b*) Construction and extension
    - (*c*) Improvement
    - (*d*) Unifying, refunding or consolidating other issues

---

[1] This classification with slight modifications is taken, by permission, from *Corporation Finance* (Vol. 11 of Modern Business Texts of the Alexander Hamilton Institute, N. Y.) by William H. Walker, LL.D.

3. Bond names derived from the privileges and conditions of payment of principal:
    - (*a*) Convertible bonds
    - (*b*) Redeemable bonds
    - (*c*) Serial bonds
    - (*d*) Registered bonds
    - (*e*) Negotiable bonds
    - (*f*) Sinking fund bonds
4. Bond names derived from the privileges and conditions of payment of interest:
    - (*a*) Coupon bonds
    - (*b*) Registered bonds
    - (*c*) Income bonds
    - (*d*) Participating bonds
    - (*e*) Profit sharing bonds
5. Bond names derived from the character of the security:
    - (*a*) Simple obligations:
        - Plain bonds
        - Debenture bonds
        - Long term notes
        - Certificates of indebtedness
    - (*b*) Reinforced obligations:
        - Bonds secured by guaranty
        - Assumed bonds
        - Guaranteed bonds
        - Indorsed bonds
    - (*c*) Bonds secured by lien on personal property:
        - Sinking fund
        - Equipment
        - Paper collateral
        - Stocks
        - Bonds
        - Mortgages
        - Leases
        - Certificates of interest
    - (*d*) Bonds secured by lien on realty, according to:
        - Character of property mortgaged
        - Priority of lien

1044. Some corporations issue, besides the ordinary common stock, what is called **preferred stock,** which carries with it (*a*) all of the rights of the common stock unless specifically restricted by the charter, and (*b*) certain specified preferences as compared to the common stock. These preferences may lie (1) in participation in dividends, (2) in distribution of assets on liquidation, or (3) in voting rights.

For example, a preferred stock may carry (1) the right to secure a fixed dividend (quite commonly 7%) before any dividend is declared to holder of common stock; or (2) the right, on liquidation, to receive assets to the amount of its par value before any assets may be distributed to the holders of common stock; or (3) the right, in effect, to veto, by vote, certain policies voted by the common stock.

In any case the charter, by-laws, and certificate of preferred stock, taken together, specify all of the rights carried by that stock.

A holder of preferred stock is a participant in the risks of the business though not to the same extent as the holder of common stock. The holder of preferred stock is not a creditor who comes in either with the secured bondholders or the general creditors in claims against the corporation. Only after the creditors are paid in full has he any rights to a distributive share of the corporate assets.

One of the purposes of issuing preferred stock is to raise capital under conditions more advantageous to the corporation than those which would obtain through sale of bonds or common stock.

1045. If at any time the corporation has not issued all the authorized stock, it may issue part or all of the remainder; but if all the authorized stock has been issued, the corporation can issue no more until it has secured an amendment to its charter, increasing the capitalization and thus permitting a **new issue of stock.**

The usual legal requirement is that the new stock may not be issued below par. If the market value of the old stock is below par at the time, of course none will subscribe for the new stock, except under special circumstances. For example, $100,000, par value, of new stock might be accepted, as "Hobson's choice," by a creditor in full payment of his $100,000 claim against the corporation.

FINANCIAL CONDITION

1046. The private corporation enjoys certain advantages and runs certain **risks** in trading on the equity (employing outside, that is, borrowed, capital), just as does the individual business or the partnership.

The outside capital of the corporation is represented by its issued notes and bonds. The greater the total of bonds issued by the corporation, the greater the danger that net income will not suffice to pay the bond interest, and the first failure to pay interest due is likely to result in transferring the control of the corporation from the stockholders to the bondholders, with disastrous results to the stockholders. The corporation therefore may safely issue bonds only up to a point such that the yearly interest on the bonds will be well below the most conservative estimate of net earnings of the leanest years.

As to preferred stock, if it has voting rights with common stock, the greater the issue of preferred, the greater the risk that the common may be outvoted and lose control of the corporation. If the preferred has voting rights only when the corporation fails to pay dividends on it, the greater the issue of preferred the greater the risk of the common's losing control.

Bonds and preferred stocks are so diverse in the rights which they give to their holders that no attempt can be made here to cover all of the cases, but the foregoing examples suffice to illustrate the vital importance to the corporation of planning its maximum issue of securities (bonds, etc.) with a keen eye to the probable net income of the leanest years.

1047. The financial condition of the corporation, present and prospective, is one of the most important of irreducible data to be taken into account by the **engineer** when making decisions of major importance.

A. M. Wellington wrote more than forty years ago: "Railways are not undertaken unless they are expected to be profitable, not to the general public nor to other parties in the near or distant future, nor to those who lend money on them, *but to those who at first control the enterprise.* If the means in hand be not sufficient for the projectors to complete the road for operation and to control its operation afterwards, the result to them is usually complete loss. Remembrance of this fact becomes the more important because the available means (the great bulk of which is borrowed money) are almost always over-rated, and the demand upon them under-rated."[1]

---

[1] *Economic Theory of Railway Location* (New York, John Wiley & Sons), 1886.

CHAPTER XI

BUSINESS UNITS (CONCLUDED)

Public Service Corporation—Public Corporation—Promotion

PUBLIC SERVICE CORPORATION

1101. Some private corporations which supply service deemed necessary or of special interest to the public welfare are called **public service corporations** or quasi-public corporations.

Many, but not all, public service corporations are so circumstanced as to have the exclusive privilege of rendering the service.

This exclusive privilege of rendering service which is generally recognized as public is a **monopoly,** legal or virtual.

The street railway company that has been given an exclusive franchise to lay its tracks and run its cars on certain streets has a legal monopoly of the street car business on those streets.

Still other private corporations, by virtue of conditions other than legal, control the supply of a service that is deemed a public necessity. A company supplying the water, for example, to the inhabitants of a town or city and having control, by ownership or lease, of the only practically available source of water supply, has a virtual monopoly of the local water supply service.

A corporation may have a virtual monopoly of supplying a recognized public service from the mere magnitude of the sum of money that would be required to erect a competing plant and the improbability that sufficient business could be diverted from the original corporation to make the investment profitable.

Among the public service corporations that are readily recognized as often enjoying a monopoly are the following: railroad, electric railway, express, gas, electric light and power, water, telephone, and telegraph.

As examples of public service corporations which do not usually have a monopoly, may be cited hotel, bus-line, and grain elevator.

1102. We have to note two **opposite effects** of the public demand for and the monopolistic character of the service rendered by the public service corporation: (1) The demand for the service is steady and tends to increase as fast as, or faster than, population, with the result that the income tends to increase with time in spite of hard times or poor management; and (2) to prevent or correct abuse of its monopoly by the public service corporation, the public takes a hand in the affairs of the corporation through governmental regulation. Many regulatory laws have been passed or executed without due consideration of all elements entering into the desired result, namely, satisfactory service; and in many cases the effect has been to depress the corporation's net income to zero or below in spite of most skilful management.

1. That **demand** for public service remains fairly steady in the face of a prevalent decline in business and despite poor management is a fact generally recognized. When a family begins to feel the pinch of lessened income (or of income virtually lessened because of rising prices), it will immediately cut down on renewal and maintenance of wardrobe and household furnishings, and even forego the luxuries of the table; but habit will moderate or even prevent altogether the cutting down of use of street car, water, light, etc. And the effect of poor management is rather to irritate the consumer than to cause him to cut down his use of service.

2. Governmental **regulation** of the public service corporation has been brought into existence by the people of the state in self-defense against exorbitant charges for and indifferent rendering of service considered necessary to the public welfare. The history of public service corporations indicates that their regulation was necessary and is likely to continue to be necessary.

This is apparently recognized by all. But unfortunately the early regulatory acts were designed by legislators with the idea of pleasing their constituents who clamored for revenge for corporate offenses, rather than with the far-seeing purpose of securing to the public the most advantageous service possible in the long run. Thus the community followed the example of the man who, finding his lazy work-horse gorging itself in the granary, thereupon and thereafter hobbled and short-rationed the offending animal.

Reasonable regulation is obviously that which secures to the public a satisfactory service, quality and price considered. Such service requires plant continually up to date. Such plant is possible only at the cost of continually increasing investment. Capital for investment in an enterprise is to be had under practicable terms only when the enterprise promises a return that is satisfactory compared with the returns promised by other types of enterprise. Therefore reasonable regulation is that which permits a return on investment just sufficient to attract to the enterprise such additional capital as is required from time to time for expanding and improving service, to keep pace with demand.

The effect of progress in invention, and change in public standards, is to scrap a plant before it is worn out to make way for a more modern plant. This results in a sacrifice of old capital; and usually the new plant has a greater first cost than the old. For example, consider the change in street railway service from horse car to cable car, from cable car to overhead-electric car, and from overhead-electric car to underground-electric car.

The earlier custom of regulating blindly has given such unsatisfactory results as to lead governments to set up commissions to study the circumstances in each case with a view to reasonableness in regulation.

For example the federal government, through the Interstate Commerce Commission, has made a detailed physical appraisal of the steam railroads of the country, to obtain pertinent facts to aid in the formulation of reasonable regulations.

Under the "Transportation Act of 1920"—the so-called Esch-Cummins Bill—the Interstate Commerce Commission is empowered to prescribe just and reasonable railroad rates so that under reasonable management the railroads can "earn an aggregate annual net railway operating income equal . . . to a fair return upon the aggregate value of the railway property . . . ." The Act fixes 5½% as the fair return till March 1, 1921.

1103. Towns and cities have from time to time become owners and operators of existing public service enterprises. To acquire ownership of the property and business of an existing public service corporation, as a water-supply company, the municipality may employ one of the following methods: (1) the municipality may buy the plant and business of the company at a price agreed upon by the company and the municipality; or, failing to agree on price, (2) the municipality may acquire the property by condemnation proceedings—in which case the price to be paid is finally fixed by the courts; or (3) the municipality may gain control of the enterprise by purchasing in open market the majority of the stock of the company. To what extent any particular municipality may acquire public service enterprises and the manner of acquirement which may be employed are both determined by the charter of the municipality.

PUBLIC CORPORATION

1104. Without unduly stretching terms it may be said that *a government is a business unit.*

It is not difficult to imagine all of the officers and employees of any government as being the officers and employees of a single individual or private corporation which, under contract with the people, is rendering them the very services now rendered by the government.

Indeed, in times past, the situation supposed above has actually existed: the lord of the castle, an individual, made a business of supplying, at a price, the primary objects of government: he settled disputes, preserved the peace, and defended against foreign foes. He cared for the helpless, and even dispensed religion. He employed such help as he needed for these purposes; and no doubt he followed the common custom of private business and charged what the traffic would bear. The lord of the castle ran the business for profit. The business was finally taken over by the people, for their own profit; their motive being to dispense benefits of government to themselves, and thus keep the profits in their own pockets. In our country the people have from time to time taken over a service enterprise previously conducted only by private corporations, and in each case a contributory motive has been their belief that by running the business themselves the people would reap the profits formerly taken by the private corporation.

1105. A government is here called a **public corporation.**

For the present purpose the following may be called public corporations: the federal government; a state government; a county government; a county-subdivision government (*e.g.*, a township government); a city government; a "district" government (*e.g.*, the government of a school, health, or irrigation district).

Each public corporation is identified with and carries on its corporate activities within a particular territorial area having boundaries precisely defined; and in this respect is unlike the private corporation.

1106. A public corporation, like a private corporation, is created by a **charter** or a virtual charter.

The **federal** government's charter is the Constitution of the United States as granted by the original thirteen states, amended from time to time by the combined action of the component states of the Union, and interpreted by the United States Supreme Court.

The charters of the colonies, which became, in 1789, the original thirteen states, of course had been granted by European sovereigns. The charter of each of the newer **states** consists of its home-made constitution and the act of Congress which created the state out of a "territory" (itself a public corporation created by Congress).

The **county** charter may be said to consist of those parts of the federal constitution, of the state constitution, and of legislative enactments, which pertain to the county.

The charter of a county subdivision (*e.g.*, township) may be similarly described.

A **city** charter is drafted and adopted by the voters of the territory of the proposed city under specific authority of state law, and may or may not (depending on the provisions of the Home Rule statutes of the state) have to be subsequently approved by the legislature.

A **"district"** charter is a particular law, generally passed by the legislature of the state, but in some states Home Rule charters may be adopted by counties.

1107. The **purpose** of a public corporation is specified in its charter.

The primary purpose of government is to protect its subjects, collectively and individually, against aggression. Thus government undertakes to oppose invasion, preserve order, and administer justice.

As a secondary purpose government engages in various other enterprises undertaken for the public welfare. Of such enterprises the most widely familiar, perhaps, are highways, public schools, and the postal service.

1108. The public corporation, like the private corporation, has only such **powers** as are specified in its charter. But the charter of the former, unlike that of the latter, gives the power of levying **taxes,** and also power of **eminent domain,** by which the corporation may, by legal process for fair compensation, take and use private property.

The right of eminent domain is granted to railroads, and some other public service corporations; thus furnishing some exceptions to the statement above.

1109. The **organization** of a public corporation is determined in a more or less general way by its charter. Some of the details of organization are left to be filled in by the corporation itself by the enactment of laws (called "ordinances" in the case of a city). Under and to the limit of the authority given by its charter the public corporation enacts such laws or ordinances, and adopts such resolutions, rules, and regulations, as may be necessary or expedient to carry out the undertakings and enterprises for which the corporation was chartered.

The enactments of the public corporation bear the same relation to its charter that the by-laws of the private corporation bear to its charter.

1110. In a general way the voters of a district established as a public corporation bear the same relation to the **control and management** of the public corporation as the stockholders of a private corporation bear to the control of the private corporation. Here the parallel ends.

While there is great diversity among charters of private corporations, yet all such charters are uniform in this, that they prescribe that the stockholders shall elect the board of directors and that the board shall elect the officers. Charters of public corporations are not uniform in corresponding points. However, it may be said that usually the laws passed under authority of charter are enacted by a legislative body composed of individuals elected thereto by the voters.

The will of the voters, in carrying out the purpose of the public corporation, is effected by a great variety of types of legislative and administrative machinery. The type of machinery employed in cities varies from state to state, and from class to class of cities in the same state.

How Public Corporations Raise Money

1111. A large sum of money is annually required by the public corporation for the purpose of carrying on its enterprises. This sum is derived, in any particular case, from some or all of the following **sources**: (1) taxes (on property, manufactures, income), (2) poll tax, (3) licenses, (4) special assessments, (5) sale of privileges, (6) fees, (7) fines, (8) forfeits, (9) gifts, (10) revenues from productive property owned by the public corporation, (11) grants from the federal or state government, and (12) loans.

Of these sources the more important are taxes, special assessments, loans, and revenues from productive property of the public corporation. Loans are sometimes obtained to raise the money necessary to pay off maturing loans; in other words, loans are sometimes refunded.

The federal government derives the greater part of its regular income for meeting routine expenses by taxing imports (tariff), certain domestic manufactures (tobacco, for example), and incomes; and, to meet extraordinary demands, as the cost of war, obtains additional money by issuing bonds, or other negotiable instruments (treasury notes, war savings stamps, etc.). After bonds have been issued, the interest on them, and provision for retiring them at maturity, become items of routine expense.

The state government relies chiefly on taxation to meet its ordinary expenses; and, so far as its charter permits, like the federal government, issues bonds to raise money for new and large enterprises which it undertakes from time to time. Counties and townships, and incorporated towns, cities, and districts likewise levy taxes and issue bonds, in so far as their charters permit.

Most incorporated cities, towns, and districts are empowered by their charters to levy special taxes or assessments, to pay for local public improvements (paving, for example), on the real property directly benefited by such improvements. For example, the owners of some greater part of the frontage of property along a street may compel the improvement of the street and the assessment of all that property in proportion to frontage, at a time when the popular vote would be overwhelmingly against improving the street at general expense. The special assessment power therefore enables parts of municipalities to advance without waiting for a majority vote of all the corporate voters.

Some incorporated towns and cities obtain considerable income from productive property owned by them—for example, from the municipal water-supply.

It is seen that the public corporation, like the private corporation, issues bonds to obtain large sums of money, not for ordinary running expenses, but for the purpose of building new structures or the improvement or expansion of old. The federal government issues bonds, not only for particular projects, but whenever it is deemed necessary by Congress to supplement the regular income.

The public corporation enjoys one advantage over the private corporation, namely, the authority to levy taxes; and thus has the power to meet its obligations, so long as there remains sufficient value in the private property within its territory. But in times past many a state, or lesser public corporation, has willed to repudiate some of its obligations represented by bonds.

1112. The **engineer** as a member of a community has the same interest as the lawyer, or merchant, or workman, or any other citizen, in the public corporation as an instrument for promoting the well-being of the members of the community; but as an engineer his interest is confined to such of the corporation's undertakings as require for their successful initiation or prosecution the services of an engineer. Thus the engineer's professional interest centers on the promotion, creation, operation, and maintenance of **public works**—all those structures employed by the public corporation in furthering its purpose, public welfare.

The public works of the **federal** government include fixed and portable means of national defense (including naval vessels), highways, canals, reclamation works (for drainage and irrigation), railroads (in Alaska), harbor works, river improvements, custom houses, post offices, arsenals, barracks, warehouses, mints, and other public buildings.

The **states** build highways, bridges, canals, harbor works, river improvements, and various kinds of public buildings.

Counties, also, build highways, bridges, drainage ditches, and public buildings.

Among the public works created by **cities** are streets, bridges, sewers, water-works, tunnels, docks and wharves, street railways, subways, electric light and power plants, gas works, parks and play grounds, refuse disposal plants, and various public buildings.

The incorporated **district** builds such structures as are necessary to the purpose for which the district is incorporated. For example: a school district constructs school buildings; an irrigation district constructs dams, canals, and irrigation ditches.

1113. The fact, that the financial situation of an enterprise often has an important bearing on a question of economic choice of engineering structure, gives the engineer an interest in the public corporation's means of raising money; and, if he is serving a public service corporation, also gives him an interest in the power held by the public corporation to regulate rates and service of his company.

1114. Since the engineering department of any business unit must act in harmony with other departments, the engineer must necessarily be interested, also, in the organic relations by means of which the various departments coordinate their efforts.

PROMOTION OF BUSINESS UNITS

1115. The creation of a new business unit requires the belief on the part of the prospective investor, that rendering for sale some particular kind of service offers a desirable investment opportunity. The conditions must be such as justify the belief not only that the rendering of such service is feasible, but also that it can be rendered at such cost, and sold at such price and in such quantity, as to bring a return (on the necessary investment) which compares favorably with the return promised by other opportunities.

1116. But however favorable the conditions may be, a business unit does not, of course, come spontaneously into existence: it must be created by conscious human effort. As stated in §107, there must first be a recognition of the promise of the conditions ("conception of the enterprise"); next, an "investigation to determine the volume of service which probably will be required; the feasibility of rendering the service; the cost of rendering it; and the probable return;" and finally, a business unit must be organized, and the necessary capital enlisted. The conscious effort required to create a business unit is called **promotion.** The person or business unit that puts forth this effort is a **promoter.**

Any number of persons may have conceived the idea that it would be profitable to produce and sell service of a certain kind, before this idea comes to the promoter. The promoter differs from the others in this, that he is willing to venture his work, money, credit, and reputation in promoting the enterprise.

There are numbers of professional promoters of private corporations. The way of the professional is to settle, and part company, with each corporation of his creating as soon as it is a going concern. But in many a case the individual who promotes a corporation is a promoter only incidentally, as when a sole proprietor, in order to obtain more capital or to reduce his personal liability, incorporates his business.

1117. Let us consider, first (for reasons which will become apparent), the promotion of a private corporation, in order to get some glimpse of the promoter at work.

Promotion of Private Corporation

1118. The promoter, having conceived or taken over the idea that it would be desirable to establish a particular enterprise, quietly **investigates** all conditions which are likely to have a material influence upon the success of the enterprise, such as: (1) the feasibility of obtaining by lease or purchase a suitable location, and by purchase or construction a suitable structure for producing the proposed service; (2) the future supplies and prices of raw materials, and of fuel and other supplies, and of labor; (3) legal and social conditions which are likely to affect favorably or unfavorably the erection or operation of the structure; (4) the degree of competition which the enterprise is likely to encounter; (5) the relation between price of service and the corresponding volume of sales; (6) the probable price and sales. He finally calculates the annual yield on the capital.

The promoter's first step is to make the investigation in its entirety. Having in view the enterprise as a whole, he may subdivide this problem, and set one assistant at one part, another at another, and see that each assistant remains in the dark as to the existence and work of the other assistants, and, of course, as to the idea of the enterprise as a whole.

The promoter takes the results of the assistants, and combines the results to form the grand resultant of the investigation. (Here is likely to be absence of team work, and it is not unreasonable to expect the promoter's pronouncement as to the promise of his enterprise occasionally to be not thoroughly well supported.)

In this investigation various qualifications are needed, and the promoter is likely to call to his aid the engineer and the lawyer, and often one or more other specialists, such as accountant, architect, builder, chemist, real estate appraiser, and industrial expert.

1119. Next, the promoter plans the business organization and the methods of obtaining the capital needful to launch the enterprise and keep it going. Having formulated the plans and prospects of the proposed enterprise and compiled the data to support his belief in the success of the enterprise, the promoter lays all these before one or more capitalists, with the object of getting them to join with him in forming a corporation.

1120. Each **capitalist** privately investigates, as far as he deems prudent, the claims of the promoter; and in this work he may employ as many different kinds of experts as the promoter has employed.

The promoter has a lively imagination, and on conceiving the enterprise sees it a flourishing reality, and a good thing for all concerned. It is natural that of the conditions disclosed by the investigation the favorable should be welcomed and magnified by the promoter unconsciously, and the adverse shunned or minimized. The promoter's judgment in the course of promotion is likely to become warped in weighing the *pros* and *cons* because every *con* points to his throwing up the work and losing all the time and effort he has put on it and a part or all of the money expended on it. Therefore the capitalist, sought to be enlisted in the enterprise, must check the promoter's data by an independent investigation if he would intelligently decide whether or not to take an interest in the enterprise.

1121. Having satisfied each of the capitalists needful for his purpose, the promoter by an agreement associates them with him as incorporators. The **incorporation** is finally effected and the corporate organization completed.

1122. The corporation cannot be held for contracts which it has not made; and since it could not make contracts before it existed, it follows that it is not bound by any agreements made by the promoter or his associate incorporators either among themselves or with others. The promoter, therefore, makes such terms with the new-born corporation as he can in regard to the reward for his labors in promoting the enterprise.

The promoter may take options on properties that he thinks will be advantageous or necessary to the enterprise, and in some cases, being unable to secure an option, he may buy the properties outright, and thus by the control of these properties make terms, satisfactory to himself, with the corporation when it comes into existence. The promoter, in buying each option or property, acts with as much secrecy as he deems necessary to avoid exposing his plans as a whole.

1123. If the promoter has acted in good faith and has done his work well, and the enterprise is meritorious both in fact and in popular belief, he may obtain a handsome return for his work. And under such conditions he deserves no less, for he has put his time, efforts, property, and credit, at his own risk, into the creation of an enterprise to supply needed service. The fact that the promoter's chief motive is the hope of a liberal return from his risk and effort should in no way cloud the fact that he has performed an indispensable and difficult work in creating a business mechanism which will convert capital into service desired by the community or by the people at large.

Of course, if a man promotes a corporation ostensibly for the purpose of carrying on a mis-called meritorious enterprise, but really for the sole purpose of selling at a hold-up price to the corporation some property which he controls through ownership or option, he is carrying on a swindle rather than a true promotion; and is legally liable to the corporation if he has defrauded it of value and to individuals who have been injured by his misrepresentation.

1124. The foregoing description of the work of the promoter of a private corporation perhaps does not accurately represent any one case, for the circumstances of no two cases are precisely alike, and of some cases are as widely different as can be imagined. For example, the promoter may find it necessary to plan his financial scheme with an investment banker, because the latter will consider advancing the needed capital only on condition that he have a hand in the planning. And sometimes the control and direction of the promotion is taken over at this stage by the banker, who thereupon becomes for the time being himself a promoter. Sometimes, where large capital is needed, two or more bankers may join together in taking over from the original promoter the control and direction of the final steps in the promotion of the enterprise. The financial plan in such a case is likely to be not the choice of any individual but a mutually acceptable compromise.

Again, the prospects of the enterprise may be so uncertain—as in the case of a new mine, for instance—that investment bankers, and others in control of large sums, will decline to furnish the necessary capital. The promoters must then turn to the multitude of small capitalists, hoping to find a number who, in the hope of great return, are willing to risk small sums sufficient to produce the required total.

Promotion of Other Business Units

1125. Investigation, financing, and organization are as necessary to the intelligent setting up of one kind of business unit as another; hence, though it is not usual to do so, it is as logical to speak of promotion and promoter in connection with the individual business, the partnership, and the public corporation, as with the private corporation.

1126. The **sole proprietor** is his own promoter, of course. His aim in promotion may be either (1) to establish the business, and thereafter to control and manage it, or merely to control it; or (2) to establish the business in order to sell it soon at a profit.

1127. The idea of forming a **partnership** ordinarily occurs first to one of the partners-to-be; and that one, in prevailing on one or more individuals to join him in a partnership, acts as a promoter. (Of course, the one or more who are invited to enter the partnership should make a thorough investigation not only of the promoter's plan and data, but also of the promoter himself.)

1128. The promotion of a **public corporation** is practical politics, that is, "the conduct of the immediate public affairs of the state." Therefore, in promoting a public corporation, an individual, whatever his regular occupation, becomes for the time a politician.

As a rule the promotion of a public corporation is the work of a group of individuals, which pushes a plan pleasing in all points to no individual but agreed upon as the most promising compromise.

Even the taking over of a private enterprise by a municipality is the result of promotion. The most active promoter in some cases is the original owner of the enterprise who believes that he will gain more from selling than from continuing the enterprise. Over-regulation is likely to create such belief.

The promoter of a private corporation works in the hope of making money for himself. He pretends no other aim, and none is deceived in this regard. But in a case of a man promoting the idea of a municipality's entering upon a new line of enterprise the motive is not so clear. There may be one unselfish or disinterested motive, or one or several selfish ones. Hence, the voters of the municipality should make as thorough an investigation of the promoter's plan before deciding the question of accepting it as the banker makes of the plans of the promoter of the private corporation; and the investigation by the voters cannot be considered as complete so long as the promoter's real motive remains undiscovered.

CHAPTER XII

IRREDUCIBLE DATA OF PROBLEM OF INVESTMENT

1201. The diagram (Fig. 1201) of the first stage of the analysis of the problem of investment may be recognized as the initial part of Fig. 414.

Choice (based on comparison of:)
- Yield (Opportunity I)
- Yield (Opportunity II)
- Irreducible data

Fig. 1201.—Analysis of problem of investment.

The analysis of yield has been carried through several stages in chap. II, IV, V, VI, VII, and VIII. To the analysis of irreducible data we have given some consideration in connection with a particular example in chap. II. To give final consideration to the subject is the present purpose.

1202. The **ideal** of the capitalist is to put his funds in service in such place and manner and under such conditions as to: (1*a*) give the maximum possible yield; (1*b*) give the highest assurance of maximum appreciation (increase in value); (1*c*) incur minimum taxes; (2) leave no doubt of the return of the principal; (3) assure the periodic return in uniform payments; (4) subject him to a minimum of liability; (5) permit his selling his rights in his investment quickly and advantageously at any time; (6) permit borrowing money advantageously at any time on his rights in his investment, as security; (7) suit his wishes as to duration of investment; (8) permit him to control his investment to just the extent that he desires and no more; (9) permit him to manage the service to just the extent of his wishes; and (10) have the units of investment of most convenient size or denomination.

1203. Though opportunities for placing capital are endless in variety, there is none that reaches the ideal as described above.

For example, U. S. Government bonds, which are practically ideal in point of safety, are far from ideal in the point of size of return. (This is because there are so many capitalists who prize safety above other considerations that, in bidding against one another for the bonds, they run the price of the bonds up to such a point that the return on the investment is comparatively low.)

Qualities 1*a*, 1*b*, 1*c*, as we have seen, are "reducible," and readily combined into the quality "yield" which serves as the numerical basis of comparison. All of the other qualities, 2 to 10, come under the general head "irreducible data," and collectively serve as a non-numerical basis of comparison.

The foregoing diagram (Fig. 1201), when developed to show the first stage in the analysis of the element, irreducible data, becomes the diagram of Fig. 1203.

- Choice (based on comparison of:)
  - Yield (Opportunity I)
  - Yield (Opportunity II)
  - Irreducible data
    - (Data pertaining to:)
    - Safety of principal
    - Safety and uniformity of yield
    - Liability
    - Marketability
    - Collateral value
    - Duration
    - Control
    - Management
    - Units
    - Probability of appreciation

Fig. 1203.—Analysis of problem of investment.

It is evident that the prudent capitalist must consider each one of these qualities which together comprise irreducible data, and so far as he is able determine the degree of each—for example, the degree of safety of the principal.

1204. The majority of these qualities are more or less dependent on one or more of the following **conditions:**

1. The investor's financial relation to the business unit;
2. The type of business unit;
3. The individuality of the unit;
4. The general type of opportunity;
5. The trend of conditions.

1205. From the existence of the two kinds of **financial relations** of investor to business unit, namely, that of ownership and that of creditorship; and of the four common **types of business unit,** with which we are already acquainted, results the following classification of opportunities:

1*a*. Lending to a sole proprietor;
1*b*. Becoming a sole proprietor;
2*a*. Lending to a partnership;
2*b*. Entering partnership;
3*a*. Lending to a private corporation:
    On a note;
    By buying its notes;
    By buying its bonds;
3*b*. Becoming a stockholder in a private corporation:
    By buying its common stock;
    By buying its preferred stock;
4. Buying bonds (or other obligations) of a public corporation.

1*a*. In **lending to a sole proprietor** the capitalist assumes no part in control or management of the business, and his risk is tempered by the fact that all of the sole owner's property, wherever found, is subject to the claims of his creditors. The lender assumes none of the risk of the business directly, but he is insured against that risk only to the extent of the assets of the sole proprietor.

1*b*. In **setting up as a sole proprietor,** the capitalist assumes the entire burden of control and responsibility for the management of the business unit, and of course assumes all such risk as may be inherent in the business. Any profits from the enterprise belong to him.

2*a*. In **lending to a partnership,** as to a sole proprietor, the capitalist assumes neither control nor management, but his risk is likely to be less than when lending to a sole proprietor, by reason of the fact that all of the personal assets of all of the partners is subject to the claims of creditors. In lending to a partnership, the capitalist assumes only such indirect risk as may result from excess of liabilities over total assets of the partners in case of the failure of the business.

2*b*. In **entering a partnership** the capitalist assumes joint control, and so much of the management as may be agreed upon, but he may assume a greater risk than does the sole proprietor, inasmuch as creditors may fall upon all of the property of any one partner in seeking satisfaction of their claims. Any profits made by the business unit are shared by the partners according to agreement.

3*a*. The capitalist takes over no part in control or management by **lending to a corporation.** The risk, however, may be greater than when lending to a sole proprietor or partnership because creditors of a corporation can go only to a limited extent, or not at all, beyond the corporation property to the private property of the corporation members, in seeking to satisfy their claims.

In buying **bonds** issued by a private corporation the capitalist acquires only a contingent ownership and therefore only a contingent control in the corporation, and of course no part in the management. Bonds are usually secured by mortgage on the property of the corporation, while preferred stock is not. The rate of return on the bond is fixed, whereas that on the preferred stock is fixed only as to the upper limit.

Notes issued by private corporations differ from bonds chiefly in maturing in from 2 to 5 yr. instead of in from 5 to 100 yr. and upward, and generally in not being secured by mortgage.

3*b*. While the **stockholders** of a corporation have entire control of the business unit, exerting this control through the ballot—one vote for each share of stock—the real control is exercised by one or more stockholders who, by ownership or influence, control the votes of a majority of the shares. The minority stockholders are likely to have little influence in the control of the business unit. Thus the degree of control acquired by the capitalist who buys stock of a corporation will depend upon what proportion of the total issue of stock he buys and on his influence among other stockholders. The stockholder, as such, takes no part in the management of the business. As to risk, whether he be a majority or a minority holder, he shares this in proportion to his holdings; but the risk extends no further than to the sum paid for the stock; unless personal liability of stockholder is expressly fixed by statute. In this, his position is decidedly more favorable than that of the sole proprietor or partner. The stockholder is entitled to share in the profits according to the number of shares he owns, but the board of directors decides what part of the profits each year shall be distributed to the stockholders and what part retained in the business—which decision is limited by the fact that the directors cannot pass (*i.e.*, fail to declare) dividends with the purpose of freezing out the small or needy stockholder—the directors must act in good faith for the interest of the corporation.

The degree of ownership and control acquired by the capitalist in buying **preferred stock** depends on the characteristics of that stock as set forth in the charter and by-laws of the corporation. The holder of preferred stock devoid of voting rights has no part in the management of the business unit. While the capitalist assumes less risk of loss in buying preferred than in buying common stock, his income therefrom may be limited to a stated dividend regardless of the profits gained by the enterprise.

4. Bonds issued by the United States and by states rest only on the ability and willingness of the issuing bodies to meet their obligations. Bonds issued by counties, cities, districts, etc., rest upon the ability of

the bodies to pay, and the willingness of the courts to compel (as they can do if the corporation is in existence and has officers reachable) the corporation to levy and collect taxes to pay the bondholder.

1206. The **individuality of a business unit** is the resultant of those characteristics and combinations of characteristics which distinguish that particular unit from all others. For example, the business reputations of the controlling men of a corporation are so many elements comprised in the individuality of the unit.

As further examples of elements of individuality may be cited the following:

A certain drayman has long-term contracts with a number of well-established and substantial mercantile houses for transporting all of their incoming and outgoing freight.

A certain mercantile partnership is sole agent for a celebrated and popular brand of staple goods.

A certain new manufacturing corporation engages as manager of works a man who has previously won an enviable reputation in a like position with an old successful company.

A certain county is known to have been delinquent over a certain period covering a number of years, in the payment of interest on its outstanding bonds.

1207. We have heretofore used the expression "investment opportunity" in a general way to signify any opportunity for placing capital in remunerative use, without regard to the characteristics of the opportunity (and, by the way, we shall so continue to do beyond this section). We have now, under the head of **general type of opportunity** to consider "investment" in a restricted sense, namely, in contradistinction to "speculation." **Investment** has been defined as an outlay of money, or its equivalent, with a view to a later specified return; and **speculation,** as an outlay with a view to an unspecified return.

The opportunities for investment—using investment in the restricted sense—are the following: loaning to (1) a sole proprietor; (2) a partnership; (3) a private corporation; (4) a public corporation. (Buying a bond is of course one way of making a loan.)

All other opportunities for using capital remuneratively are opportunities for speculation, and comprise: acquiring ownership as (1) sole proprietor; (2) partner; (3) holder of common stock; (4) holder of preferred stock.

While the foregoing clear-cut classification makes it easy to pronounce a given opportunity for employment of capital a speculation or an investment, it is open to the possible objection that some of the resulting "investments" involve much more risk of loss than some of the "speculations"—for example, buying the bond (investment) issued by one company may be far more risky than buying stock (speculation) in another.

The popular distinction between investment and speculation is rather a distinction between opportunities involving smaller risk of loss and those involving greater risk; and no sharp line can be drawn on this basis between the two resulting classes: what one man considers an investment another will call a speculation.

1208. We come now to inquire into the last of the list of data (§1204) which underlie the qualities of choice of opportunity for remunerative use of capital; namely, the **trend of conditions.**

In the first place we may for convenience classify conditions which tend to influence investment choice as (1) political, (2) social, (3) financial, (4) economic, and (5)—for want of a better term—miscellaneous.

1209. The question as to the probable course of **political events** has often an important bearing on choice of investment. The passage of any law designed to neutralize the effect of free economic forces is of moment to a multitude of business units.

The passage of a new tariff law has a direct and favorable effect on many enterprises, and a direct and adverse effect on many others; while it affects indirectly and in different degrees a multitude of still other enterprises.

The prospect of greater liberality, or the opposite, in government regulation of public service corporations, directly influences the attractiveness of investment in those corporations, and, to only a less extent, in the business units which sell to those corporations.

Again, taxes must be considered in connection with certain classes of investment opportunity. Each state and each municipality (to the extent permitted by its charter) makes its own tax laws, and a very great diversity of laws is the result. Some states levy income tax (each state according to its own schedule), while other states do not. Some states levy inheritance taxes (no two on precisely the same schedule), and others do not. Some states have profession, trade, or occupation taxes; others do not. And so on. Obviously the prudent investor in long-term offerings will not only inform himself as to the present tax laws affecting a proposed investment, but will also forecast to the best of his ability the probable direction and amount of change that will be made in those laws during the life of the proposed investment.

1210. Changes in fashions, habits, tastes, customs, standards of living, public opinion—in fact all **social changes**—have direct and indirect effects upon the financial success of countless enterprises.

The following examples may be cited in illustration of the foregoing statement.

The passing of the fashion of riding bicycle had of course a momentous effect on all makers of bicycles.

The risk of arousing adverse public opinion in regard to appearance, or safety, or sanitation, or public health, may be greater in choosing one type of structure than in choosing another type. Again, the prospect for continuous and constant or increasing demand for service may be better with one type of structure than with another.

1211. The future trend of **financial conditions** will have its effect on choice of investment.

To illustrate: A business unit decided last year to build a cheap temporary structure and put off building a proposed alternate permanent expensive structure. While anticipation of lower prices and wages had considerable influence on the decision, a factor of moment was the anticipation of obtaining funds 2 or 3 yr. later under terms much more favorable to the borrower than the terms then offering.

1212. Choice of investment may be materially influenced by prospective **economic conditions.**

For the illumination of this statement the following examples may be helpful.

The probability of rise in prices of materials and labor may be much greater for one proposed structure than another.

The outlook for prompt delivery of ample materials, and for sufficient supply of dependable labor may be brighter for one type of structure than for another.

1213. A multitude of **miscellaneous** prospective conditions, pertaining particularly to the construction and operation, is necessarily included as pertinent irreducible data when choosing between two or more structures proposed for a specified service.

A typical example of such data is the following: Of two types of highway bridge proposed for a given site, let us say that No. 1 can be built in 6 mo. and No. 2 in 12 mo., and assume that the traveling public will be materially inconvenienced throughout the period of construction. What is the present justifiable expenditure to save the traveling public from 6 mo. material inconvenience? This question is in no case easy to answer. Ordinarily the only question that need be answered is the following: Is it justifiable to pay \$ ... (a stated sum) to save the public from 6 mo. material inconvenience? And this question is answered with comparative ease. It is always far easier to decide whether or not you will take a thing at a named price than to decide on the maximum you would pay for it.

Additional examples of irreducible data of the same general nature are:

(*a*) It may be more difficult to obtain capital to build one kind of structure than to build another.

(*b*) Promise of early date of operation may be greater in the case of one proposed structure than in that of another.

(*c*) Where the proposed structure is to be an addition to or alteration of an existing operating structure, or is to be in close proximity to a neighboring independent structure, one of the proposed structures may appear to have the advantage over another in one or more of the following points: (i) Extra cost of construction to prevent interruption to operation of existing structure; (ii) extra cost of operation during construction; (iii) net loss by reason of interruption to operation from and during construction.

(*d*) One proposed structure may have the advantage of another in the point of probability of loss from accidents to property or persons, inside and outside, whether of owner, employees, or other.

(*e*) One structure may offer less risk than another, of trouble from uncertainty as to the legal status of patented methods or devices, or as to titles to property.

(*f*) There may be more risk of failure of guarantee of performance with one structure than with another.

(*g*) Facility of future extension or expansion of structure may be more marked in one structure than in another.

(*h*) Two structures proposed for a stated service may be quite unequal in adaptability to possible or probable future change in kind, and even in quality, of output.

Conclusion

1214. All the irreducible data which we have considered in this chapter are of concern of course to the investor, and many of them are directly pertinent to questions which the engineer must answer.

CHAPTER XIII

BUSINESS STATISTICS

Financial Statements

1301. It goes without saying that the **owner** and the **creditor** (prospective as well as actual) are vitally interested in the financial status of a business unit, and hence also in the internal and external conditions of the business upon which the financial status depends.

1302. Since the **manager,** whose work it is to make the business render profit as well as service, spends the greater part of his time in directing and controlling the internal conditions to that end, his interest in the conditions must be vital.

1303. It has been pointed out previously that the financial status of the business unit for which an **engineer** is deciding a major question, is one of the most important of the irreducible data which he has to consider in making such choice. Moreover, investigation with a view to the improvement of certain important conditions pertaining to a business is an ever recurring engineering problem.

1304. Thus the engineer, as well as the owner, the creditor, and the manager, must be vitally interested in the conditions whose resultant is the financial status.

1305. Financial status of a business can be determined only from the numerical facts of the internal and external conditions of which the status is the resultant. These numerical facts, when systematically collected, are **business statistics.**

The easiest way of getting acquainted with the types of facts, and their relations to financial status, is to put oneself in the place of him who seeks to know the financial status of a particular business unit from a particular standpoint. And this is the way which we shall follow.

BALANCE SHEET

1306. Let us be dealers in building materials, and let our interest now center in the affairs of **Mr. John Doe, contractor,** of whom we know nothing except that he is a builder in concrete and has just written us asking for cement on credit—say $500's worth.

We ask to be informed of his financial status; and he replies:

"Net assets, $5000."

1307. Probably the fact of first importance among the data revealing the financial condition of a business unit is **net assets,** which is the value of that portion of its property which is over and above the total of claims outstanding against the business unit.

1308. Knowing now the meaning of net assets and the figure, $5000 of Mr. Doe's, ought we to be satisfied? Shall we be able from this information to judge intelligently of the advisability of trusting him for $500's worth of goods?

1309. The discussion of this question can be materially shortened by using the following two business terms: assets and liabilities. The **assets** of a business unit may be described, with sufficient precision for the present purpose, as being the value, expressed in terms of money, of the total property possessed by the unit. The **liabilities** of a business unit are the debts or pecuniary obligations of the unit. From definitions just given, it is evident that

$$\text{net assets} = \text{assets} - \text{liabilities}.$$

Now, returning to the question—

1310. Let us consider: Net assets change from day to day—from moment to moment of the business day, as business is transacted. This requested credit of $500 is only $\frac{1}{10}$ of the net assets, $5000. Is it likely that Doe's net assets may materially decrease before the proposed date of payment of the $500? This likelihood depends not so much on the ratio of the requested credit of $500 to the net assets of $5000 as on the ratio of net assets to total assets. If Doe's net assets of $5000 come from, say, $500,000 assets and $495,000 liabilities, it is evident that a small relative decrease—1% in fact—in assets or a small relative increase in liabilities would be sufficient to wipe out entirely the net assets; and such a change is quite possible in the ordinary course of business. On the other hand, if the net assets, $5000, arise from $6000 assets and $1000 liabilities, it would require a relative decrease of 80% in assets, or a 400% increase in liabilities to reduce the $5000 net assets to zero; and changes of these magnitudes are not likely to occur in short periods of ordinary business. It is quite plain then that we must know the assets and liabilities, before passing judgment.

1311. On request, Mr. Doe sends this statement of his business:

Assets	$8500
Liabilities	$3500
Net Assets	$5000

Now should we, or should we not, on the showing in this statement give him credit for $500's worth of cement?

1312. We cannot reach an informed decision without some knowledge of the character of the assets and of the liabilities. The reason for this answer will appear.

Many business men classify an asset on the basis of time required to liquidate it (convert it into cash) in the ordinary course of business, as either a **current asset** or a **fixed asset.** Similarly they classify a liability, on the basis of time which will elapse before its maturity, as either a **current liability** or a **fixed liability.**

The distinction in accounting practice between "current" and "fixed" liabilities is not uniform. For example, some accountants fix the character of a liability at the time it is incurred, while others revise their classification of certain kinds of debts at each balance-sheet date.

1313. On further request, Mr. Doe sends us a statement of his financial condition, as in Fig. 1313.

Assets:		
Current assets	$5500	
Fixed assets	3000	
Total assets		$8500
Liabilities:		
Current liabilities	$2500	
Fixed liabilities	1000	
Total liabilities		3500
Net assets		$5000

Fig. 1313.—Statement of Doe's financial condition.

That this statement sheds additional light on our question is plain. But after all it leaves us in twilight, as it were, because current assets shade almost insensibly into fixed assets, and current liabilities shade into fixed liabilities. An asset may, as to degree of liquidity, lie anywhere between cash (which is 100% "liquid") and "a drug in the market" (which is not "liquid" at all). And a liability may, as to time of maturity, lie anywhere between an account that is now due or over-due and a bond that has a century to run.

1314. Carrying the analysis of net assets one step further yields the diagram of Fig. 1314.

- Net Assets
  - Assets
    - Current assets
      - Cash in hand
      - Cash in bank
      - (+)
      - Cash claims
        - Drafts, checks, money-orders
        - Accounts receivable
        - Notes receivable
        - Bonds
      - (+)
      - Stock certificates (value of, on hand and in safety deposit box at bank)
      - (+)
      - Materials (value of, on hand)
    - (+)
    - Fixed assets
      - Land
      - (+)
      - Improvements (buildings, etc.)
      - (+)
      - Equipment
    - (+)
    - Prepaid expenses
      - Values not received, but paid for in advance (*e.g.*, unexpired insurance)
  - (−)
  - Liabilities (*i.e.*, debts)
    - Current debts
      - Accounts payable
      - (+)
      - Notes payable (short-term notes)
      - (+)
      - Accrued debts
        - Accrued interest
        - Accrued wages
        - Accrued taxes
        - etc.
    - (+)
    - Fixed debts
      - Long-term notes
      - (+)
      - Bonds
      - (with or without mortgage)

Fig. 1314.—An analsyis of net assets.

It should be understood that the diagram, Fig. 1314, is merely a specimen, and that any actual diagram constructed from the data of a particular business unit for a particular date would probably differ from the specimen by the omission of some of its items or by the inclusion of items which it does not show, or by both omission and inclusion.

1315. Let us critically examine some of the items shown as elements of current assets, fixed assets, etc., in the diagram.

Cash on hand is 100% liquid, for it will be taken on the spot at its face value in settlement of debt. Cash in the bank is the same as cash on hand; and the same is practically true of **drafts, checks,** and **money-orders,** because cash, to the amount specified in these papers, is forthcoming when they are presented for payment.

Accounts receivable call for specific sums of cash, but there is more or less uncertainty as to when cash will be delivered, and even in regard to whether all of it will ever be delivered.

Notes receivable are payable on specified dates and call for specific sums of cash, but there is always the question in regard to any note as to how good it is. If it is good enough it may be sold before its due date nearly or quite at its face value plus accrued interest. **Bonds** are like notes, and must be valued by the market price, regardless of their face value, interest rate, or due date; and this market price may fluctuate more or less according to the reputation of the company issuing the bonds, and other conditions.

Stock certificates also must be valued by market price, and this may be widely different at the two ends of a week, or even of a day.

The value of **materials** may be given as their cost, or the present market price, but in any case the net cash that can be obtained for them at forced sale is likely to be much less than cost.

The value of **land, improvements,** and **equipment** cannot be appraised with any degree of certainty; and to sell at approximately their appraised values may require many months. So little commercial mobility have these assets, that they are commonly designated as **fixed assets.**

The **prepaid expenses** are items which cannot be liquidated during the continuation of the business.

The above brief discussion is sufficient to show the variety of uncertainties existing in any cash evaluation of the assets of different kinds.

On the other hand liabilities at any date are exactly knowable.

However, there is a great difference between debts that are now due or that fall due in the near future, and debts that will not become payable for a year or several years, because, so long as a business unit remains solvent, the short-time creditors are not rivals of the long-time creditors.

Returning to the question of our extending credit to the contractor, it should be clear now that the answer cannot be given intelligently until he has supplied us with the actual or appraised values of the various elements of both his assets and liabilities.

1316. On request for more particulars Mr. Doe furnishes a formal statement, Fig. 1316, known as a **balance sheet.**

JOHN DOE

Balance Sheet, Dec. 31, 1921

Assets		
Current Assets		
Cash	$ 37.16	
Accounts receivable	491.07	
Notes receivable	100.00	
Stock certificates	2000.00	
Materials	2871.77	
Total current assets		$5500.00
Fixed Assets		
Land and improvements	$2600.00	
Office equipment	400.00	
Total fixed assets		3000.00
Total assets		$8500.00
Liabilities		
Current Liabilities		
Accounts payable	$ 800.00	
Notes payable	1700.00	
Total current liabilities		$2500.00
Fixed Liabilities		
Mortgage note	$1000.00	
Total fixed liabilities		1000.00
Total liabilities		$3500.00
Net assets		5000.00
		$8500.00

Fig. 1316.—Doe's balance sheet.

Could we make a show of reason for asking Doe to supply us with the sub-items making up the items listed as assets? We could reasonably ask for particulars concerning some of the items. For example, pertinent to the matter of extending credit are such questions as the following: What proportion of the Notes Receivable is long overdue? Of what corporations are the stock certificates, and do the certificates represent common stock or preferred? Where is the land and what is the quantity? When does the mortgage note fall due? It is plain that the answer to any one of these questions is a factor of some influence in reaching an intelligent decision on Doe's request for credit.

1317. Assuming, now, for the sake of getting on in the discussion, that Doe has supplied us with the ultimate elements of net assets; have we all requisite financial data for making our decision?

The answer to this question is, No; because, when we come to think of it, the complete balance sheet after all gives us only an instantaneous view of Doe's business: shows the conditions at a certain moment—the end of business hours of Dec. 31, 1921. The balance sheet gives no indication whatever of the past course of the business—whether it has been improving or declining—and so leaves us without this valuable aid to forecasting the probable future course of the business; which probable future course has a real bearing on the question of extending credit to Doe.

1318. Some indication of the past course of business could be obtained by comparing the balance sheet above with an earlier balance sheet—say that made a year earlier, of date Dec. 31, 1920. If the net assets of Dec. 31, 1921 exceed the net assets of Dec. 31, 1920, the excess is the **net income** (see qualification of this statement, made below) or profit resulting from the year's business; if the net assets of the later date are less than those of the earlier date the year's business has resulted in a loss; and if the net assets of the later date equal those of the earlier date, the result of the year's business has been zero—neither profit nor loss.

This statement above holds true only when there has been, between the two balance-sheet dates, no withdrawal of profits or of capital, and no addition of capital from sources outside the business. If there have been withdrawals or additions, a statement of them must accompany the balance sheets in order to permit calculating the net result of the business during the interval between the balance-sheet dates.

1319. Suppose that Doe's balance sheet, of date Dec. 31, 1921, shows net assets greater than the earlier net assets of Dec. 31, 1920; in short, suppose that his business during the year has resulted in a net income, say $2000. Knowing this we must still however be curious, because this net income may have been due to either good management or good luck. Good management promises continuing success; good luck promises nothing.

1320. To judge of the management, we must know something of the transactions which constituted the current business of the year and resulted in the net income aforesaid.

INCOME SHEET

1321. Doe takes a contract for a piece of concrete work, buys materials—cement, sand, and crushed rock—and employs labor and rents equipment to convert them into concrete, for which he is paid the contract price. His business consists of repetitions of this series of operations. Besides outlays for materials, labor, and plant rental, there are taxes, interest, office help and expenses, etc., which must be paid.

1322. A partial analysis of Doe's net income, conducted on the lines indicated above, would yield a diagram something like Fig. 1322.

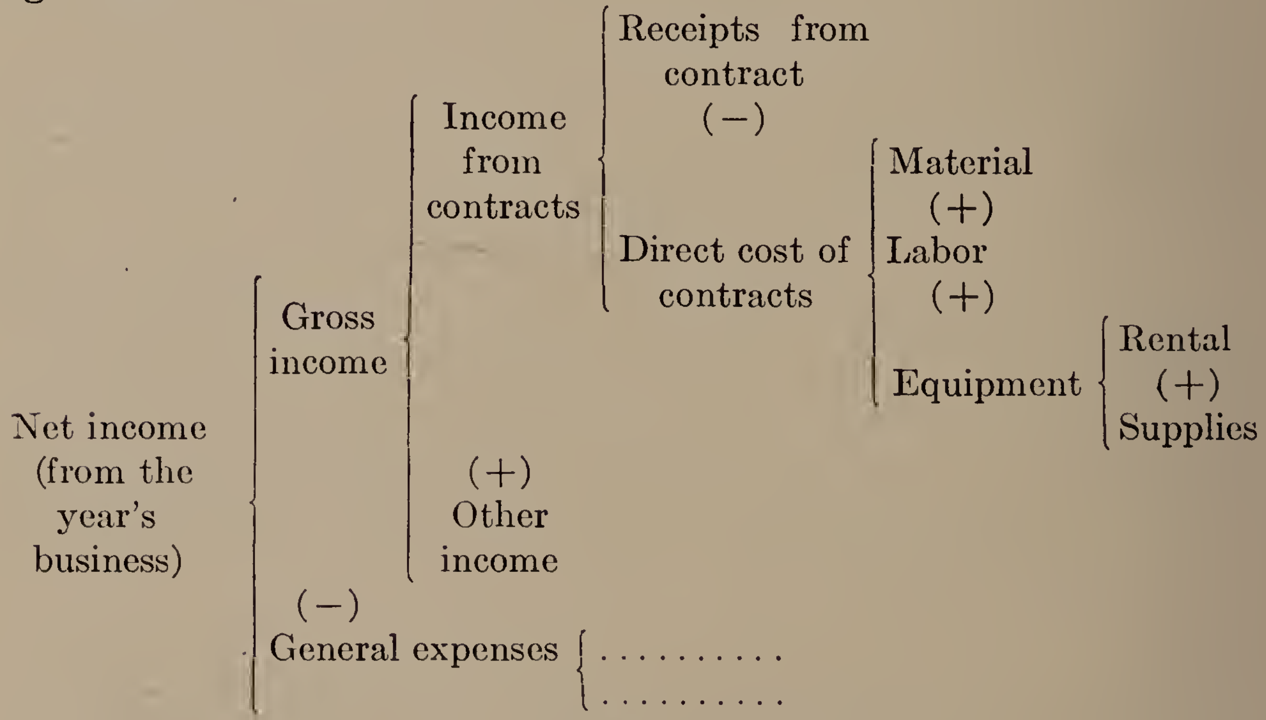


Fig. 1322.—An analysis of net income.

The arrangement of the elements in Fig. 1322 is in no way fixed: many other arrangements are possible. And the terms "Receipts from contracts," "General expenses," etc., are merely descriptive, and might be varied indefinitely.

1323. The customary financial statement corresponding to the foregoing diagram of analysis of net income is called an **income sheet,** and is designated also by various other names, such as "profit and loss statement," and "loss and gain statement."

1324. Suppose that we ask Mr. Doe for an income sheet of his business for the year ending Dec. 31, 1921. and he responds with that shown in Fig. 1324a.

JOHN DOE

Income Sheet for Year Ending Dec. 31, 1921

Receipts from contracts			$15,000:00
Cost of materials used		$5000.00	
Cost of labor		8000.00	
Equipment:			
Rental	$700.00		
Supplies	400.00		
		1100.00	
Direct cost of contracts			14,100.00
Income from contracts			$ 900.00
Other income			2600.00
Gross income			$ 3500.00
General expenses			1500.00
Net income			$ 2000.00

Fig. 1324a.—Doe's income sheet.

From this income sheet we gather that while Doe apparently made $900 on his contracts during the year he in reality lost $600 on them, because the general expenses, $1500, are chargeable to the contracts.

The fact, that it requires some figuring to determine that the net result of the year's contracting was a loss, indicates that the arrangement of the items of the income sheet is not as convenient as it might be, and that the analysis of the diagram might have been carried along more satisfactory lines.

Doe's income sheet would have better served its purpose had it followed the lines of such an analysis as that indicated in Fig. 1324b.

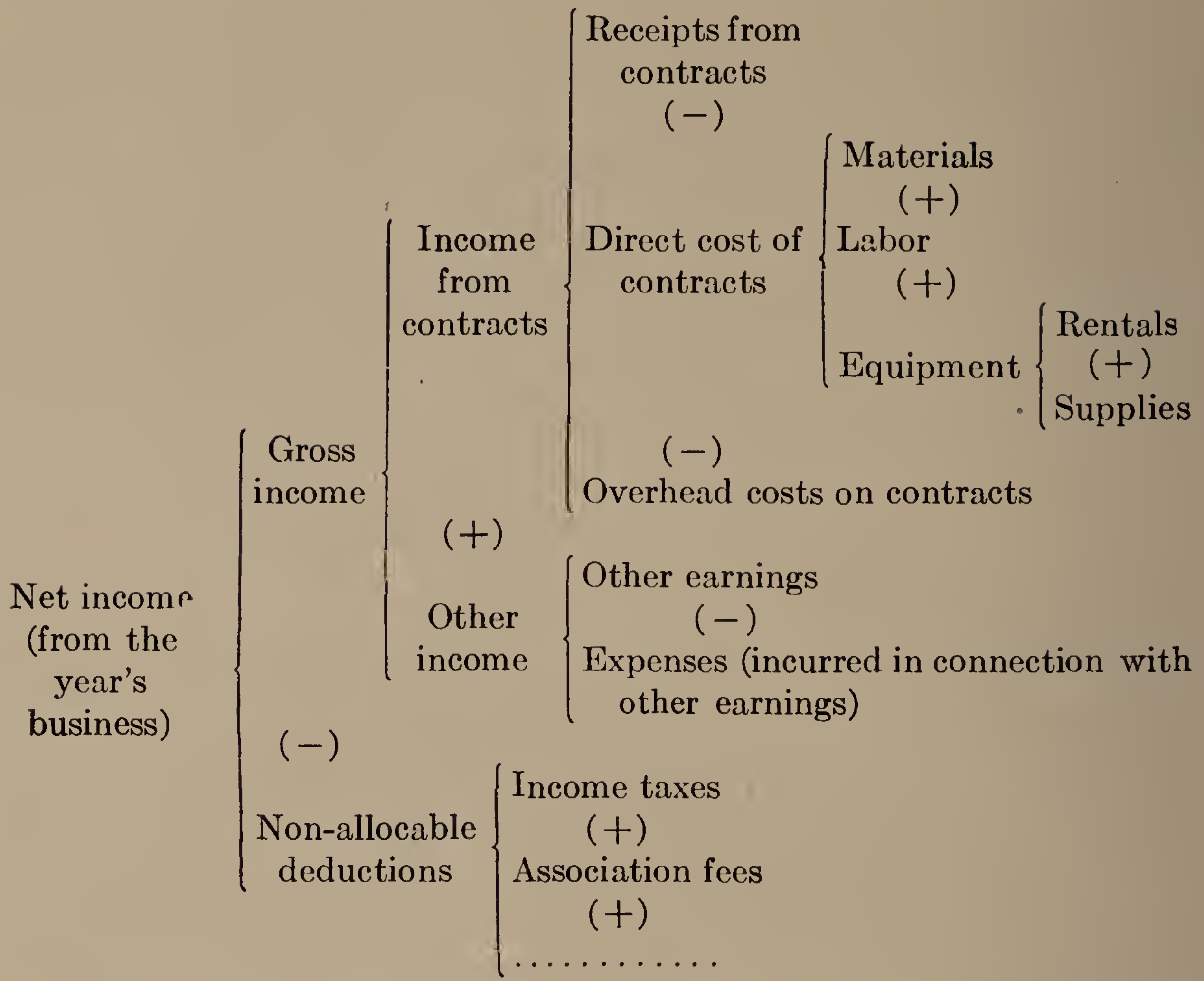


FIG. 1324b.—A better analysis of net income.

An inspection of this diagram will make it apparent that had Doe's income sheet been based upon it, the sheet would then have shown at a glance that there had been a loss from contracts.

The year's net income would have been *minus* $600 had it not been for "other income" of $2600.

The loss of $600 on the year's contracting does not on its face promise well for the future. As prospective creditors of Doe we are in need of particulars sufficient to form an opinion as to whether the loss was due to poor management or to circumstances over which he had no control.

It is not to be assumed that a contractor is a poor manager because he does not foresee and provide against the coming of a war (which suddenly jumps prices and wages), or the flood or earthquake which comes once in a life-time (and destroys partially completed work); and so on.

And "other income:" we should know some particulars concerning this item. If, for example, it consists mostly of profit from stock speculation during the year, it will count against, rather than for, our extending credit.

However, it will be more profitable and less monotonous to pursue the investigation under another aspect.

1325. In concluding the discussion of the question as to whether or not we should let Mr. Doe have $500's worth of materials on credit, it should be stated that his **business reputation** is an indispensable factor to be considered.

This point of business reputation (as to both integrity and success in past undertakings) is, by many experienced lenders of credit, considered as properly having more weight than any other single factor in deciding a question of extending credit; and this late notice of a point so vital in practice is due to the fact merely that the point has no part in the purpose of the preceding discussion, which purpose is to build up intelligible pictures of balance sheet and income sheet.

1326. The reader, it is hoped, has in following the foregoing discussion gained an appreciation of the purpose of the balance sheet and of the income sheet, and a fundamental knowledge of the general method of constructing them.

SUPPLEMENTARY STATEMENTS

1327. Of course the analysis of net assets might be carried so far as to disclose the ultimate elements of value, as, for example, the number of sacks of cement on hand and the cost per sack; and it would be possible to draw up a balance sheet to correspond with the full analysis. But experience has shown that a balance sheet best serves its purpose when it is brief, and such exhibit of detail as may be needful to explain or justify an item is supplied not in the balance sheet itself but in a **supplementary statement.**

Experience has shown the same desirability for brevity in an income sheet, and that one or more supporting statements, rather than the sheet itself, should carry all needful explanatory data.

Just the degree of brevity to be attained in a balance sheet or an income sheet in a particular case is determined by judgment based on a knowledge of the particular and general questions that the sheet is designed to answer.

1328. Statements supplementary to balance sheet and income sheet range from (1) those which exhibit purely financial data, through (2) those which set forth unit prices or unit costs, or other ratios between financial data on one hand and physical data on the other, to (3) those which contain nothing beyond physical data.

As an example under (1) may be mentioned a statement of details making up the item, Notes Receivable. The cost schedule for a unit of product exemplifies (2). An illustration of (3) is found in a statement giving details of labor turnover.

ACCOUNT

1329. Having determined the characteristic items which must appear in a balance sheet and in an income sheet, in order that these sheets may serve their purposes, we now face the question: How are these items obtained?

1330. Looking at Fig. 1314 we find the first item to be "cash in hand." Evidently the cash in hand can be ascertained by actual **count.** For the next item, "cash in bank," however, there can be no count, because there is in the bank no number of pieces of money identifiable as the property of any one depositor; and the cash in bank can be determined only indirectly by subtracting the total of withdrawals from the total of deposits, and these two totals can be found only from the record of withdrawals and the record of deposits made by the depositor. These two records together constitute what is called an **account.**

1331. So going down through the list of items on the right side of the diagram we find that the values of some of the items are ascertainable, like cash in hand, by count; of other items, like cash in bank, from accounts; and of still others, like stock certificates, by **appraisal** or count and appraisal.

1332. Although the values of some items can be determined by count and some by appraisal, it is nevertheless the universal custom to keep an account for each of these items (for cash in hand, for example, as well as for cash in the bank), to learn the value of the item from the account, and to check this value by count or appraisal. So, too, it is the custom to keep an account for each of the items which involve appraisal, and to enter the appraised values in the accounts.

1333. We have seen that an account is a two-way record: it shows the inflow and the outflow of value pertaining to an item of ownership.

For example (Fig. 1333), if each receipt of cash is entered, along with date of receipt, under "Cash received" on the *left* half of the account, and similarly each disbursement of cash is entered on the right half of the account (this use of right and left halves is universal in practice), then, at any instant the value of cash on hand can be found by subtracting the total of the Cash Disbursed column from the total of the Cash Received column.

Cash

Cash received				Cash disbursed			
1922				1922			
Jan. 1			1000.00	Jan. 2			200.00
5			20.00	3			400.00
					(Balance)....		(420.00)
			1020.00				1020.00
Jan. 6	On hand.....		420.00				

Fig. 1333.—Cash account.

In practice it is the custom to enter the larger total beneath its column (*e.g.*, the total 1020.00 of Cash Received, as shown in Fig. 1333); to enter in the opposite half of the account a value just sufficient to make that half balance the other half (*e.g.*, 420.00 in the diagram); and to enter the total of column thus padded. Then double lines are ruled across the two halves of the account as shown; and the balance (420.00 in the diagram) is entered, together with the date of entering, as cash "on hand" in the left half of the account.

It may be well to remark, in passing, that sub-heads, such as "Cash received" and "Cash disbursed," have been inserted in Figs. 1333, 1336*a*, 1336*b* for the purpose of explanation. In practice such sub-heads are often omitted (see Fig. 1339) as being unnecessary, and on the same grounds it is not unusual to omit the Dr.'s and Cr.'s of Fig. 1339.

1334. In describing an account, above, we have used the terms "left side" and "right side;" but in practice the left side is regularly called the **debit** side and the right side is regularly called the **credit** side. And in practice "to debit an account" means to make an entry on the debit (or left) side of the account; while "to credit an account" means to make an entry on the credit (or right) side of the account.

Thus if a sale is made for cash—involving, say $100—the Cash account is debited, and the Sales account is credited, with that sum.

1335. Such is the structure of an account pertaining to an item.

One Transaction: Two (or More) Accounts

1336. When, in the course of business, cash is received, it is in exchange for the same value (nominally, at least) in some other form; and when cash is paid out it is in exchange for the same value in some other form. And so with the receipt and disbursement of values in forms other than cash. Every transaction thus involves the receipt of value in one form and the disbursement of the same value in other form.

A transaction therefore affects two different accounts by the same sum but in opposite directions (see §1337 for extension of this statement).

Dr.			Cash				Cr.
Cash received				Cash disbursed			
				1922 Apr. 1			1000.00

Dr.			Purchases				Cr.
Receipts of goods				Disbursements of goods			
1922 Apr. 1			1000.00				

Fig. 1336*a*.—Two accounts affected by one transaction.

Dr.			Cash				Cr.
Cash received				Cash disbursed			
1922				1922			
				Apr. 5			1000.00
May 16			1000.00				

Dr.			Notes Receivable				Cr.
Notes receivable accepted				Notes receivable paid and returned			
1922				1922			
Apr. 5			1000.00				
				May 16			1000.00

Fig. 1336*b*.—Two accounts affected by one transaction.

For example, suppose that a merchant buys goods to the extent of $1000, paying cash therefor. The cash fund is by the transaction depleted by $1000; also the value (nominal) of goods on hand is augmented by $1000. Suppose the account, in which are made entries of bought goods, is called Purchases account. If we follow the same conventions as before—debiting receipts and crediting disbursements (*i.e.*, putting disbursements, or subtractive values, on the right side of an account, and putting receipts or additive values on the left side of an account)—the entries in the Cash account and Purchases account, representing the transaction above, will appear as in Fig. 1336*a*.

Similarly for the payment of $1000 for a $1000 promisory note: in representing the transaction the 1000 would be entered on the credit (*i.e.*, right) side of the Cash account and on the debit (*i.e.*, left) side of the Notes Receivable account, as shown in Fig. 1336*b*.

Suppose the foregoing transaction was on Apr. 5, 1922, and that on May 16 the note is paid in cash by the drawer. Then the corresponding entries in the two accounts would be as shown, in Fig. 1336b, opposite the two entries of May 16.

1337. It is stated above that each transaction pertains to two accounts. But often that which is considered as one transaction pertains to three or more accounts.

For instance, if I make a sale at $100 and accept in exchange $50 in cash and a note for $50, I credit the Sales account with $100 and at the same time debit both the Cash account and the Notes Receivable account each with $50.

Classification as to Debit and Credit

1338. When one of the two representatives of equal value, which are exchanged in a transaction, is cash, there is no difficulty in determining on which side of each account the recording entry should be made. There can be no question as to whether, in the transaction, cash has been received or disbursed: if received the entry in the Cash account will be on the debit (left) side according to custom; if disbursed, on the credit (right) side; and whichever side is proper for the entry in the Cash account, the opposite side of the other account must be the proper side for the entry in that account.

It is not so immediately clear on which side the entry should be made in each account in cases in which the value in neither side of a transaction is represented by cash. For example, when a sale is made—say to the extent of $200—and a note is accepted therefor, we know immediately that the $200 should be entered on one side in the Sales account and on the opposite side in the Notes Receivable account, but how shall we know on which side in either account?

The answer becomes apparent on considering an imaginary parallel case in which the sale of $200's worth is made for cash. The $200 would be entered on the debit (left) side of the Cash account, and hence on the credit (right) side of the Sales account. Now, as it makes no difference to the Sales account whether the return is in cash or in the form of a note, it follows that in the original example the $200 should be entered on the credit (right) side of the Sales account, and consequently the $200 should be entered on the debit (left) side of the Notes Receivable account.

All similar questions of classification can be resolved by similar recourse to an imaginary parallel case in which cash is substituted for one of the representatives of value in the actual transaction.

LEDGER

1339. The book—bound or loose-leaf—which contains all of the accounts (as defined above) pertaining to a business is called a **ledger** (Fig. 1339). A separate ledger may be kept for the accounts of each department of a business; but if such ledgers are kept they are called *auxiliary* ledgers, and their summaries are carried periodically to *the* ledger which is then called a *controlling* ledger.

1340. It has been noticed no doubt by the reader that each entry in each of the accounts shown by way of illustration has been very brief indeed—showing in fact merely the date and value concerned in the transaction.

In practice some entries in the ledger are nearly or quite as barren as those illustrated above, but usually, between the date and the value (expressed in dollars and cents), there is inserted the name of the other account in which the opposing entry of the same transaction is recorded, as shown in Fig. 1339.

Ledger

Dr.				Cash			Cr.
1922 Jan. 6	Sales.........		75.00	1922 Jan. 7	Notes payable...		500.00

Sales

				1922 Jan. 6	Cash............		75.00

Notes payable

1922 Jan. 7	Cash.........		500.00				

FIG. 1339.—Part of a page of a ledger.

It is possible so to construct the ledger spaces that all details pertaining to each transaction may be entered along with the date and value in each of the two accounts involved, but this would so multiply details as to obscure the more important.

Experience has shown that the ledger should not undertake to exhibit more than the value (in terms of money) that is exchanged in a transaction, together with the date, the name of the account in which the opposing entry is made, and the book and page in which the full details of the transaction can be found.

Sources of Data for Ledger

1341. The accounts in the ledger then are merely abstracts of **original records** of transactions. The original records themselves may all be kept in one book in chronological order or in various books of different forms, or they may exist as invoices, vouchers, check stubs, etc. The present purpose does not require a description of any of these books or of the manner of keeping them. It suffices to say that in every case the system of such books, and methods of collecting and recording the original data of transactions in them, are designed with a view to completeness, definiteness, and convenience, and of producing more or less automatically the data required for the ledger accounts.

Ledger Balance

1342. Since every transaction affects two accounts, and affects the two accounts by the same sum but in opposing manners—

Ledger

Dr. (Left side)		(Right side) Cr.
	Account No. 1	
a d		b c
	Account No. 2	
b		
	Account No. 3	
c		a d
Totals: $a + b + c + d$		$a + b + c + d$

FIG. 1342.—Ledger balance.

the one on one side, the other on the other side—the one positively, the other negatively—it follows that the sum of the totals of all left sides in a ledger must equal the sum of totals of all right sides.

For example, when transactions involving respectively the sums a, b, c, . . . are entered each in its two proper accounts, the record as a whole will stand as in Fig. 1342.

Obviously the total of the left side of the ledger is equal in size (though opposite in sign) to the total of the right side.

1343. It follows, further, that if the balance is struck in each account in the ledger, then there must result an equality between the total of left side balances and the total of right side balances —the algebraic sum of the balances must be zero.

Ledger

Dr. (Left side)		(Right side) Cr.	(Balance)
	Account No. 1		
a d		b c	
$a + d$		$b + c$	$(a + d) - (b + c)$
	Account No. 2		
b			
b			b
	Account No. 3		
c		a d	
c		$a + d$	$c - (a + d)$

FIG. 1343.—Ledger balance.

To illustrate: Totaling each side of each account in the ledger illustrated above, and taking the difference between the two totals, we obtain the "balance" of the account, that is, the sum necessary to be entered on the smaller side to make the two sides balance or be equal. (Since, with letters instead of symbols, we cannot tell which side of an account has the greater total, we can only arbitrarily set the balance down on one side as in Fig. 1343.)

The sum of the balances is $(a + d) - (b + c) + b + c - (a + d)$, which expression plainly reduces to zero.

CHAPTER XIV

BUSINESS STATISTICS (CONCLUDED)

Cost Keeping. Comparisons

COST KEEPING

1401. While a prospective casual creditor's interest in the status of a business may not lead him to carry his investigation beyond the yearly income sheet, the owner's interest, being keener, prompts him to carry the analysis through further stages.

For example, Mr. Doe is not content to know merely how much or how little he has made during a year: he wants to know, when he has completed a contract, how he has come out on it. The most ignorant of contractors would have that much curiosity.

It may be well to add that, unlike the casual creditor, the professional creditor, as the banker to whom Doe continually applies for funds to finance his contracts, may be quite as keen as Doe himself in the financial statistics of progress on current contracts.

1402. For the convenient determination of the profit or loss reaped from a contract, it is only necessary that on taking the contract an account be opened with it, entitled say "Contract No. 127," or perhaps "Smith Garage Contract."

All expenditures made on account of the contract would be charged (debited) to the account, and all payments received for work performed on the contract would be credited to the account. For example, the first pay-roll payment (say $82.50) made in connection with the contract, and the first payment (say $100) received on contract price would show on the account as in Fig. 1402.

Dr. Smith Garage Contract Cr.

	Cash, pay-roll		82.50		Cash, received		100.00

Fig. 1402.

The difference between left side total and right side total, after all entries have been made, gives the profit or loss on the contract.

1403. A knowledge of the profit or loss on a completed contract is of itself a satisfaction, and, combined with a recollection of all circumstances and conditions surrounding the work on that contract, is of course a valuable aid to the judgment in estimating the cost of work on a proposed contract, though it comes too late to aid in managing the work on that completed contract.

For example, if Mr. Doe has recently completed, at a cost of $922.46, a 100 cu. yd. concrete job that he undertook on contract for $912.10, knowing that he lost 10 cents a yard will enable him to estimate more judiciously the next job on which he purposes to bid; and make him scan his mental picture of the past work for defects in management; but the late knowledge of the loss could be of no effect in helping to prevent that loss.

1404. Any knowledge as to loss or gain on a piece of work, in order to be of any help in the management of that particular piece of work, must obviously be obtained from time to time during the progress of that work.

1405. But how is such knowledge to be obtained during the progress of the work? A clue to the answer of this question is found in the diagram of analysis of unit cost of product, shown in Fig. 1405 (a reproduction of Fig. 629).

Suppose that Doe based his bid price, for the work under contract, on a cost per cubic yard of concrete ("unit cost of product" in Fig. 1405) calculated from the ultimate elements (Fig. 1405) evaluated from previous experience. It follows that Doe can learn whether he is making money or losing money on a particular element—for example the quantity of cement per cubic yard of concrete ("units of material B per unit of product," say, in Fig. 1405); or the man-hours required to mix and place 1 cu. yd. of concrete ("man-hours per unit of product" for "operation 2," say, in Fig. 1405); or the price of cement or wage rate for any operation—by comparing the actual quantity of that element going into each cubic yard of concrete with the estimated quantity used for that element in calculating the cost per cubic yard.

For example, if Doe figured on paying $3.40 per bbl. for cement and finds he must pay $3.42, the effect per barrel on profit on a cubic yard of concrete is to depress it by just $0.02.

If he figured that it would require 1.60 bbl. of cement per cubic yard of concrete, and finds that on the work it runs 1.58 bbl. per cubic yard, the effect of this element is to raise the profit per cubic yard by the cost of 0.02 bbl. of cement.

- Unit cost of product
  - Cost of materials (per unit of product)
    - Cost of material A (per unit of product)
      - Units of material A (per unit of product)
      - (×)
      - Unit cost of material A
        - Unit price of material A
        - (+)
        - Other costs per unit of material A
    - (+)
    - Cost of material B (per unit of product)
      - (Same subdivisions as for A, above)
    - (+)
    - . . . .
  - (+)
  - Cost of labor (per unit of product)
    - Cost of operation 1 (per unit of product)
      - Man-hours (per unit of product)
      - (×)
      - Wage rate
    - (+)
    - Cost of operation 2 (per unit of product)
      - Man-hours (per unit of product)
      - (×)
      - Wage rate
    - (+)
    - . . . .
  - (+)
  - Cost of plant (per unit of product)
    - Assembling, setting up, dismantling, dispersing (per unit of product)
    - (+)
    - Depreciation (per unit of product)
    - (+)
    - Supplies (per unit of product)
  - (+)
  - General expense (per unit of product)

FIG. 1405.—An analysis of unit cost of product.

If he figured that it would require 3 man-hr. to mix and place a cubic yard of concrete and he finds on the job that it is taking 2.7 man-hr., the effect of this element on the profit per cubic yard is to raise it by 0.3 multiplied by the wage-rate.

1406. Admitting that Doe can determine whether he is making or losing on any element during the progress of the work by comparing, periodically and element by element, the actual magnitude of each with the anticipated or scheduled magnitude on which his contract price was based; it still remains for him to determine the actual magnitude of each element.

How is he to ascertain, for example, how many bags of cement are going into a cubic yard of concrete, on the average, during a particular day? And how is he to find the number of man-hours of labor going, on the average, during a particular day, into mixing and placing 1 cu. yd.?

1407. Obviously the quantity of cement which goes into a cubic yard of concrete, on the average, during a particular day's work is to be learned by (1) actually tallying or counting the bags of cement used during the day, (2) measuring the linear dimensions, and from them calculating the cubic yards, of concrete completed during the day, and (3) dividing the number of bags by the number of cubic yards.

It is plain too that the number of man-hours going into the mixing and placing of concrete, on the average, during a particular day's work may be ascertained in a similar manner: by (1) noting the number of men engaged in mixing and placing concrete on that day, keeping count of the hours of the day spent in mixing and placing, and taking their product, which is the total man-hours put into the day's output of concrete; and (2) dividing the day's output (the number of cubic yards, ascertained as above described) of concrete into the total of man-hours.

1408. We saw in §1402 that keeping an account with each contract is a convenient way of finding the loss or gain from the completed contract. We are now in position to appreciate the fact that keeping a subsidiary account with each of the divisions or subdivisions of total cost of contract may be a convenient means of keeping the run of the elementary costs as they arise.

For example, Doe might find it convenient to set up an account for lumber, another for sand, another for mixing and placing, and so on.

1409. Keeping a systematic record of tally and count of the elements of cost, such as quantities of materials and labor which go day by day into product, in such a manner as to enable one readily to determine the average quantity of each element entering into each unit of product during a unit of time—as a day or a week—is called cost finding or **cost keeping.**

1410. The method or **system** of keeping costs for any construction job or production plant, to be satisfactory, must be devised to meet the needs of the particular case; and as the needs vary from one field of enterprise to another, and even from enterprise to enterprise in the same general field, it will rarely happen that a system of cost keeping which works well with one enterprise will fit as well the needs of another.

But whatever the form of organization and of product, there is one fundamental controlling **principle**: The "cost elements" which are sought to be disclosed by any system of cost keeping, must be costs of processes, sequences of processes, transactions, etc., that will be repeated in many successive jobs under substantially similar conditions. These "unit processes" may be mechanically simple or complex, but they must be statistically simple; they must often recur or they are useless as cost bases. Therefore a good cost-keeping system must be founded upon an analysis which is a resolution of the operations of an industry into their elementary processes so far as (1) it is possible to find repetitions, and (2) it will pay to obtain the cost data.

The first requisite for devising a cost system for an enterprise is such familiarity with the organization of the enterprise and its method of operation as will enable the deviser to make an analysis that will disclose the cost data, direct and indirect, that are needful to the management. A study of systems working satisfactorily in similar enterprises ought to supply suggestions as to working out details, but cannot take the place of the first requisite named above.

It goes without saying that a cost system cannot be satisfactory if the cost of operating it is equal to or greater than the value of the benefits that are derived from it, or if its operation unduly impedes construction or production. To minimize its operating cost, the system should be so devised as to require a minimum of transcribing of original records—payrolls, bills of material, etc.—and to permit the making of original records by persons having the fullest knowledge of the facts to be recorded.

1411. If all employees were sufficiently intelligent, educated, and reliable, the ideal way of obtaining labor records, for example, would be to have each of them fill out and turn in, each day, a blank form containing spaces for all pertinent primary facts, and also spaces for desired derived quantities. Figure 1411a shows how such a card might look when turned in.

It might be cheaper and offer smaller risk of error, to have all calculated quantities (columns (*f*) – (*i*)) filled out by the office rather than by the workman. The daily collection of workmen's cards for a day would be sorted into bunches by job number; then each bunch would be sorted into piles by feature number; next, each pile would be sorted into groups by detail of feature; and finally each group would be sorted into batches by nature of effort. Then the totals in each batch—total units of achievement, total hours, total pay—would be found and entered on a labor-summary form, something like Fig. 1411b.

JOB OR ORDER NO.—182	FEATURE NO.—18	WORKMAN'S NO.—23 NAME—J. O. Smith
WORKMAN'S DAILY REPORT	DATE—June 25, 1920	OCCUPATION—Carpenter

	Detail of feature (*a*)	Nature of effort (*b*)	Units of achievement (*c*)	Hours worked (*d*)	Pay per hour (*e*)	Total pay (*f*)
1	Foundation	Erecting forms	0.200 MBF	3	1.00	3.00
2	Temporary office	Laying floor	0.220 MBF	5	1.00	5.00
3						

	Rate of labor output (units of achievement per hour) (*g*)	Man-hours per unit of achievement (*h*)	Unit cost of achievement (*i*)	Conditions affecting cost (*j*)
1	0.067 MBF	15 hr./M	$15/M	Delayed 20 m. by surveyors
2	0.044 MBF	22.8 hr./M	$22.80/M	
3				

FIG. 1411a.—Daily report card (for ideal conditions).

Thus would columns (*k*), (*l*), (*m*), and (*n*) be filled without cost for transcription. Then the averages for columns (*o*), (*p*), and (*q*) would be calculated and entered.

At the week end, the workmen's cards would be sorted by workman's name or number into bundles, and the total hours and pay for each bundle would be a particular workman's time and pay for the week.

Job or Order No.—182
Feature No.—18
Detail of Feature—Foundations
Nature of Effort—Erecting forms

Labor Summary Day by Day
For Week Ending................

Date (*k*)	Total units of achievement (*l*)	Total hours (*m*)	Total pay (*n*)	Average rate of labor output (*o*)	Average man-hours per unit of achievement (*p*)	Average unit cost of achievement (*q*)	Remarks
1920 Jan. 25	1.7 MBF	23	$23.00	0.074 MBF	13.5 hr./M	$13.50/M	
Totals and averages							

FIG. 1411b.—Weekly labor summary form.

Finally, totals and averages for each kind of effort, each detail of feature, each feature number, and for each job number, would be found and entered on appropriate summary sheets, by the day, by the week, by the month, and for completed work, by taking totals from workman's cards or from summary sheets.

1412. The scheme of keeping the cost of labor, sketched above, would give all requisite data for estimating direct labor cost of future work, and at the same time enable the management to know from day to day, and from week to week, the direct labor cost of each detail of feature, of each feature, and of each job;

General accounts	Primary accounts	Class code
Construction—*C*	Right of Way 00–09	.... 09 Miscellaneous
Maintenance—*M*	Grade & Roadside 10–19	....
Reconstruction—*R*	Roadway 20–29	.... 27 Top Course
	Ditches & Drains 30–39	....
	Bridges & Culverts 40–49	40 Foundations 41 Abutments .... 45 Pipe Culverts
	....	
Plant & Equipment—*P*	Buildings 70–70.9	....
	Camp equipment 75–75.9	....
	....	
Administration—*A*	Executive 80–89	....
	Maintenance of office 90–91.9	....
	Legal 92–93.9	....
	....	....

Operation code	
—00	Assembling
—01	Backfilling
—02	Blacksmithing
—03	Blasting
....	....
—07	Clearing
—08	Clearing and grubbing
....	....
—18	Excavating earth
....	....
—54	Repairing
....	
....	
....	
—65	Tamping
—66	Trimming
—67	Washing
—68	Washing and screening
—69	Wasting Materials
—70	Water-proofing
—71	Working on joints
—72	Wrecking

FIG. 1413.—Classification of items for assignment to accounts.

and furthermore it would give the management the relative efficiencies of the several workmen and of the several machines.

The foregoing scheme for collecting cost data assumes certain favorable conditions, which are not likely to be all present in any actual enterprise. The student will naturally infer, therefore, that cost finding schemes in actual operation differ more or less from that sketched above, and that the differences in each case result from the practical conditions of the case; and he will find the facts in accord with his inference.

1413. It will be seen in the illustrations above that the job and the feature of the job are each given by number. It is the common practice to use **numbers** and **letters,** alone or in combination, to designate jobs, features, and other subdivisions of the work; and this is done merely for convenience of all concerned. To sort cards, for instance, by numbers or letters is far easier than by names or written descriptions; and to sort by **colors** is easier still.

To illustrate the saving of space in forms that can be effected through the use of designating letters and numbers, the samples (Fig. 1413) of a scheme recommended for highway cost keeping, by Tobin and Losh,[1] will serve.

The manner of using the symbols is shown in the following examples: C–45–18 means "construction—pipe culvert—excavating earth," that is, excavating earth to make bed for placing pipe culvert on construction. C–09–08 indicates clearing and grubbing right-of-way on construction. The item, assembling camp equipment, would be designated by P–75–00; and repairing top course of pavement on maintenance, by M–27–54. It will be observed that the second number in a triple symbol stands for a particular operation, the first number stands for the part operated on, and the letter indicates that general account—Construction, or Maintenance, etc.—to which the work belongs.

1414. The foregoing pertains to labor cost records. Any system of cost keeping must provide also for recording the receipt and disbursement of materials in such detail that the costs of the materials entering into each unit of output can be readily found.

1415. Cost-keeping records should be so planned as to interlock at appropriate points with the accounting records, in order that there may be an independent check on the cost keeping.

---

[1] Bulletin No. 660, U. S. Dept. of Agriculture, Sept. 12, 1918: Highway Cost Keeping, by James J. Tobin and A. R. Losh, U. S. Engineer Economists, Reviewed by Halbert P. Gillette, Consulting Cost Engineer. The Supt. of Documents, Government Printing Office, Washington, D. C. (10 cents a copy.)

COMPARISONS

1416. The nature and need of financial statements and of cost keeping have now been indicated. It remains to present a number of important comparisons which can be made by means of financial statements and cost-keeping records. Let us begin with the balance sheet.

1417. It has been previously stated that, provided there have been no intervening contributions or withdrawals of invested capital or distribution of profits, the net income resulting from a year's business can be obtained by subtracting the net assets that appear on the balance sheet of the beginning of the year from the net assets appearing on the balance sheet of the end of the year. In other words, with the foregoing proviso, a **comparison** of the net assets of the two balance sheets gives the net income.

Now confining the attention to a single balance sheet, let us first consider Current Liabilities appearing thereon. The total of this item by itself tells us almost nothing as to the financial status of the enterprise. It may be small when the status is unpromising or it may be large when the status is promising. The size of Current Liabilities begins to take on significance only when we come to regard it in connection with Assets. And as Fixed Assets can not be disposed of to advantage to meet Current Liabilities, we are brought to the necessity of comparing Current Liabilities with Current Assets. So, although Current Liabilities by itself means nothing and Current Assets by itself means nothing, the **ratio** between Current Assets and Current Liabilities is an index of the financial status.

Bankers and the credit men of wholesale houses through continual practice in interpreting balance sheets of those who seek credit of them, and checking each interpretation by following the course of the corresponding business unit, come to set up standard ratios. For example, they may consider 1½ to 1 a satisfactory ratio of Current Assets to Current Liabilities for one kind of enterprise, 2 to 1 a satisfactory ratio for another kind, and so on.

1418. What has been said above in regard to Current Assets and Current Liabilities holds in a general way also for Fixed Assets and Fixed Liabilities.

These ratios may be taken as typical of a number of ratios that experience has shown to be more or less sensitively indicative of financial status.

1419. Similarly with the income sheet. The ratios existing between the items of a single income sheet are of significance to the experienced.

Among these ratios, that known as the **operating ratio** is perhaps as commonly observed and discussed as any other.

The operating ratio pertaining to the diagram Fig. 1322 of analysis of net income would be the ratio between "receipts from contracts" and "direct cost of contracts;" or perhaps some modification of that ratio resulting from the inclusion of a portion of "general expenses" with "direct cost of contracts."

The operating ratio most popularly discussed relates more particularly to railways, where it means the ratio between railway operating expense and railway operating income. The operating ratio varies from below 50 to more than 100%. While operating ratio is significant, it cannot be well evaluated when considered apart from other data. For instance, the operating ratio of a pick-and-shovel gang may be expected to be high compared with that of a steam-shovel plant; and the operating ratio of a hydro-electric plant, lower than the operating ratio of a steam plant.

1420. Another ratio of prime importance is that between "fixed charges" (comprising taxes, interest on borrowed funds, insurance, and other items which remain constant, or nearly constant, regardless of fluctuations in amount of business done) and "operating income" (which is revenue from operation less direct cost of operation). This ratio is of particular interest to prospective and actual creditors as well as to the owners of the business.

1421. If the regular conduct of a business involves the purchase and later disposal of goods—as is the case with the merchant—the question of interest on tied-up capital is of importance, giving rise to the use of a ratio called the "rate of turnover."

The ratio of cost of goods sold during the year to the cost of goods usually kept in stock is commonly called the rate of **turnover.**

The significance of rate of turnover may be brought out by the following simple example: A merchant sold each year for two years, goods that cost him $20,000. During the first year he laid in the entire year's stock at the first of the year, but at the beginning of the second year he laid in only one-half year's stock, replenishing with another half-year's stock at mid-year. During the first year his rate of turnover was one, during the second, two. The first year required a capital of $20,000; the second a capital only half as large—$10,000. The rate of return on investment was therefore twice as great during the second year as during the first. That is, the rates of return on capital were in proportion to the rates of turnover.

The same question arises of course in manufacturing. Carrying an unnecessarily large stock of raw materials, or needlessly protracting the process of manufacture, or unduly piling up finished product ahead of sales, brings its penalty in the form of superfluous interest charges.

1422. Besides the guidance furnished by ratios between items of the same sheet, there is that supplied by the ratios between items of the same name on successive sheets; and a valuable index of promise of an enterprise may be found even in the ratio between ratios, as for example the ratio between last year's operating ratio and the preceding year's operating ratio.

1423. Turning now from comparison of items which pertain to the balance sheet and the income sheet, we may next direct our attention to comparisons by which the manager of operation judges of operating status, progress, and results. The nature and usefulness of such comparisons may be best shown by means of a few typical examples.

1424. Mr. Doe, the contractor for concrete work, knows from his cost-keeping records the average cost of mixing-and-placing a cubic yard of concrete during a certain week. This figure taken by itself means little. Comparison of this figure with the corresponding estimated figure, on which he based his contract price, tells him, as previously pointed out, whether he is making or losing on that particular element during that particular week. But this is not the only useful comparison: another comparison of great value is that between this week's average cost and past averages: if this week's cost is higher than last week's and particularly if considerably higher, Doe is impelled to find the cause of the increase, and, if conditions permit, to eliminate that cause.

1425. In the course of experience Doe learns the number of man-hours required to mix-and-place a cubic yard of concrete under conditions all favorable, and instinctively uses this as a **standard** with which to measure the efficiency of the concrete workers from week to week, or perhaps from day to day. If the results for a given period are not up to standard, Doe knows that the conditions are not all favorable, and can at once discover and ameliorate the unfavorable conditions.

1426. Another comparison which is of value to Doe is that between the daily or weekly output of foreman X's gang and the corresponding output of foreman Y's gang.

1427. In this brief discussion of comparisons we have employed a few **units of comparison,** for instance, the man-hour, the unit cost, and so on. Some units of comparison are peculiar to one kind of enterprise, some to another, and some are of general utility; and all together they are very numerous.

The most common units of comparison used in railroad operation are: the train-mile, the locomotive-mile; the train-mile per mile of road, and so on. And some of these are specialized in two or more ways; for example: freight locomotive miles, freight helper locomotive miles, and so on to include more than a dozen other kinds of locomotive miles.

CHAPTER XV

MISCELLANEOUS FORECASTS

OUTPUT, LIFE, AND PERIOD OF SERVICE

1501. The average daily, monthly, or yearly **output** of a machine, like that of a laborer (§807), depends on many variable conditions. A pumping engine in connection with a standpipe or reservoir may be run almost continuously under nearly uniform conditions, provided the demand for water be sufficient. Such an engine has the advantage of a steady job, and its maximum and minimum average rates of output are not far apart. Typical of structures which operate under the other extreme of conditions is the plant of the grading contractor. The potential capacity of, say, a steam-shovel can be computed from the dimensions of the dipper, time of round trip of dipper, time required to move up, and area of breast of cut. But the shovel can work only when there are empty cars within its reach; and many circumstances interfere with keeping the shovel continuously supplied with these. Besides the interruptions to actual operation, due to more or less frequent lack of cars, there are the interruptions caused by the necessity of backing up, and of moving from cut to cut; and, finally, the intervals, some long, some short, between jobs. There is a wide range between the output under the most favorable conditions which can be reasonably hoped for and that under the most unfavorable conditions which reasonably ought to be prepared for.

So in making an estimate of the future average rate of output of a structure, we must consider not only the conditions which may have an influence on the rate of output while the structure is actually working, but also the conditions which determine the proportion of working time to the time of enforced idleness.

1502. The **life** of a structure may be terminated before the structure is worn out: it may be brought to an end (1) by some change in the requirements of the service (as where a steel bridge is replaced, long before it is worn out, by a heavier bridge because of the increased weight of trains); (2) by discoveries or inventions, which make it economically necessary to replace the structure before it is worn out (as where the motor truck replaces the horse truck); (3) by a change in the fashions, habits, or customs of the people (resulting in the cessation of a certain demand); (4) by governmental declaration (as when a railroad is required by a regulatory commission to substitute steel cars for wooden cars); or (5) by accident.

If we call the life which is terminated by ordinary wear and tear, the normal life, then we may say that often the actual life is much less than the normal life: that circumstances often prevent a structure's continuing in service till worn out. These facts must be neither overlooked nor ignored by the engineer when estimating the actual life of a structure.

The engineer can usually predict with confidence some of the general conditions under which the proposed structure will be operated. Studying past experience with similar structures operating under similar conditions, he will be able to fix upon the high and low limits of life between which the life of the proposed structure may be reasonably expected to fall. Judgment must tell him just what length of life, between these limits, to choose.

The judgments of a number of engineers on longevity of plant parts will be found in Table K (consult Index).

1503. The **period of service** is one of the data necessary for the solution of the problem of economic selection. In some cases the period of service proposed may be definitely fixed; in others it is uncertain. When the period of service is not otherwise determined it must be either estimated or assumed. Of course the same period of service must be used for all the structures which are under comparison at one time for one service. The following points deserve consideration.

The service required may increase with time. The railroad bridge of 30 yr. ago was designed for loads somewhat in excess of those then carried. Train weights were increased from time to time, with the result that before the bridge was worn out it became too light for the changed requirement of the service. It will be accepted without computation that it is unwise to build a structure today just adequate to today's demand, if tomorrow's demand will be greater. On the other hand, a glance at a compound interest table (Table A) will show that it is not wise to build today for that increment of service which will not be in demand before, say, 50 yr. hence. If, then, the requirements of service may be expected to increase with time, the period of service which is adopted as a basis of economic comparison of structures must be a compromise.

If the demand for a service continue beyond the life of the selected structure, at the end of that life the problem of economic selection of structure will again arise. For example, suppose a wooden highway bridge is wisely selected for a stated service, and built and operated. At the end of the life of this bridge the question as to whether the wooden bridge shall be renewed, or replaced by one of different weight or material, will call for answer. If at that time all the conditions and the data be the same as at present, the answer will be ready to hand, the wooden bridge should be renewed; but if some of the conditions or data at that time be different, the problem of economic selection must be solved anew.

INTEREST RATE

1504. Suppose that in a community all loanable funds have been loaned at a certain rate. Now if more loanable funds come into the community there will be competition among the lenders when some of the loans require renewal, and the effect will be some reduction in the interest rate. On the other hand, if the quantity of loanable funds remains constant while there arises the need of additional loans, there will be competition among the borrowers and the interest rate will be forced up. In short, the relation between supply and demand in loanable funds affects the interest rate.

The interest rate on any loan is fixed in part, also, by the **risk** which the lender runs of not getting back his principal entire.

In 1914 when men were willing to lend their money to the U. S. Government at 2%, they were unwilling to lend to a city or well managed and prosperous railroad at less than 4 or 5%, or to a solvent firm or individual at less than 6 or 8%; and like relations exist between the corresponding higher rates of today. The rate is thus seen to be, in a way, proportionate to the degree of risk as judged by the lenders. During panics the actual risk of lenders is greater than usual because of the great number of business failures, and the fancied risk, which is the effective risk as far as rate is concerned, is still greater; and it is only when those, who in a pinch must borrow or fail, offer higher and higher rates, that one fearful lender after another is tempted to accept the risk which has repelled him. In a financial panic the rate is raised because of the abnormal risks run by the lenders, and then it is boosted by those of the lenders who are willing to take all advantage of the desperate need of the borrower.

One effect of risk is that the greater the amount borrowed, relative to the value of the security which the borrower can offer, the higher will be the rate required of him. From this it follows that the rate may be higher on the amount of capital necessary for permanent construction than on the smaller amount required for less durable construction, particularly when the proposed construction will be a considerable part of the total assets of the borrower. For example, if a man about to set up a small manufacturing business seeks capital to erect a building for the purpose, and part of the security which he has to offer lies in his anticipated profits; he is likely to obtain the needed capital at a lower rate for a cheap structure than for an expensive structure. And from this it also follows that capital for the extension or improvement of a structure, which has been successfully operating for some time, can be obtained at a lower rate than that charged for capital to build a structure for a new enterprise. Thus at the time one railroad company is selling 4½% bonds at par for a second track, another company is selling its 5% bonds below par for its first track.

The point to bear in mind, in solving problems in economic selection, is that the factor of interest on capital may throw the decision in favor of the cheap temporary structure as against the costly permanent structure, in spite of the fact that the resultant of all the other factors is to the advantage of the latter; and hence that one type of structure may be the economic choice at a time of high interest rates, and another type of structure may be the economic choice when interest rates are low.

1505. The interest rate on **sinking fund** deposits will depend on the disposition made of the deposits. The annual deposit may be (1) placed in a savings bank or other similar institution which pays a moderate rate of interest; or (2) invested in productive business, either in the business in hand or in some other, and thus probably but not certainly earn a relatively high rate of interest; or (3) handed directly to the party from whom the capital has been borrowed, as an advance payment on the principal, which disposition is equivalent to placing the deposit in an absolutely safe depository to draw compound interest at the rate paid for the capital invested in first cost. In some cases the disposition of future deposits may be fixed by law or agreement. In all other cases it is reasonable, for the purpose of estimating the yearly cost of service of a future structure, to make the rate on deposits the same as the rate on capital invested in first cost of structure.

POPULATION

1506. Structures are planned to meet future demands for service. Demand for service is usually a function of population of the territory served; and, other things equal, will increase when population increases. Therefore the first step in the determination of the demand for a proposed service, to be expected in a given territory during the coming year (or the second, the tenth, or the twenty-fifth year hence), is to estimate the population for this year.

The population of any town, city, county, or state of the United States, for any of the years 1920, 1910, 1900, 1890, and so on back can be found in the Report of the Bureau of the Census for that year.

For the population of any place, for dates other than those indicated above, resort must be had to a special census or to some method of estimation.

1507. If the living conditions remained constant over a given well-populated area, and if no one came to dwell in the area except by birth and none departed except by death, then the population would increase by **geometric progression** (as a sum of money increases under the process of compound interest).

The population of a city, however, cannot be expected to grow by geometric progression. In the first place a city extends its boundaries from time to time, and at each extension takes in a new area with its existing population; and in the second place the population is being continually augmented by immigration as well as by birth, and decreased by emigration as well as by death; and finally living conditions (on which birth and death rates depend) do not remain constant.

1508. The **cruder methods** of estimating population are based on the assumption that there is a constant ratio between population and (1) the number of votes cast at an election; (2) or the number of children of school age; (3) or the number of names in the directory; or (4) the number of residences.

1509. The method preferred by the Bureau of the Census for estimating the population of a city for any year between two consecutive census years is based on the empirical rule that the absolute yearly growth of population is constant.

For example if the population of a city was 28,600 in 1910 and 33,800 in 1920, then the absolute average yearly growth during the 10 yr. period was (33,800 − 28,600)/10 = 520; and the estimated population for 1924 is 33,800 + 520(1924–1920) = 33,800 + 2080 = 35,880.

1510. A **graphical method** of predicting future population, which at once suggests itself to the engineer, is to plot a curve of past populations, using years as abcissas and corresponding populations as ordinates; and to extend the curve into the future. In this way the population for a given future year is read off the curve at its intersection with the ordinate of that year. However, the reliability of the estimate grows rapidly less as we advance along the continued portion of the curve; for as we advance, the greater becomes the probability that some unforseen change in conditions will greatly modify the past variation in rate of growth.

1511. A **second graphical method,** which may be called the Kuichling method, is as follows: Find for each city and each decade the absolute increase in population for the decade; divide this absolute increase by 10, obtaining the average yearly increase; find the per cent ratio of this average yearly increase to the population at the beginning of the decade; and plot this per cent ratio as an ordinate, using as an abcissa the population at the beginning of the decade. Finally, average the percentages which correspond to the same population, and, using the average percentages for ordinates and corresponding populations for abcissas, plot a new curve which expresses the law of growth of such cities.

In 1889 it was predicted by the use of this law that Rochester would have a population of 204,500 in 1900 and of 283,459 in 1910. For the former year the Census gives the population of that city as 162,608 and for the latter year 218,149. Thus time proved the estimate for 1900 to be about 25% too large and the estimate for 1910 to be about 30% too large. These results are given to show that even the most intelligently and carefully prepared predictions of future population may be considerably wide of the mark, and this when the prediction extends no farther into the future than 10 or 20 yr.

1512. A third graphical method which has several times appeared in print has much to commend it. First the known population of the city A, say, for which future population is to be estimated, is plotted on a transparent sheet of paper or cloth—years as abcissa and population as ordinate. Next on a diagram drawn to the same scale are plotted the population curves of several larger cities, B, C, D, . . . all of which have grown under conditions more or less similar to those expected to obtain in the future growth of the city in question. Then the transparent sheet containing the population curve of city A is so placed on the other diagram that the two sets of axes coincide. Next the upper sheet is moved sidewise exactly parallel to the x-axis until the latest point on the curve for city A lies precisely on the curve for city B, and thereupon the curve of city B is traced on the upper sheet. The upper sheet is again moved sidewise as before until the last point of the curve of city A falls precisely on the curve of city C, and thereupon the curve of city C is traced on the upper sheet. And so on, till each of the remaining curves of the lower sheet has been reproduced on the upper sheet. All of the curves on the upper sheet will of course pass through the latest point of curve A; and the prolongation of curve A into the future can now be made with a judgment powerfully aided by the various curves, B, C, D, . . . which show the actual growths of cities B, C, D, . . . both before and after they passed that population which is represented by the common point of all of the curves.

EARNINGS

1513. An estimate of future earnings may be based upon past earnings either of the same structure or of similar structures operated under more or less similar conditions. For example, to estimate the earnings of a proposed street railway in a given town not yet provided with a street railway, we should obtain and study the statistics of earnings of street railways already operating in towns more or less similar in size and character to the given town, and seek to find some law directly usable for our estimate; as, for instance, the ratio of earnings to population. By plotting a curve of population and a curve of earnings for the same town, we have presented to the eye the general relation between population and earnings, for the conditions there obtaining. These conditions include the character of the population, its distribution and occupation; the location of the line with respect to the centers of residence and business districts; and so on. Plotting sets of curves for as many typical towns as data can be obtained for, we are in a position to discover the ratios between earnings and population for a range of conditions. Applying such of these ratios as appear to be applicable to the estimated population year by year of the town for which the railway is proposed, we obtain the most reliable estimate possible of the earnings year by year of the proposed railway.

PRICES AND WAGE RATES

1514. Future unit **prices,** as of building materials, are often fixed in advance by agreement. For example, it is a common practice of the contractor about to bid on a piece of work to contract provisionally with dealers for the requisite materials at specified unit prices.

When future unit prices have not been thus fixed by agreement the estimate of cost of a proposed structure must be based on a forecast of the unit prices that will prevail during the construction period.

The estimator bases forecast of a future unit price on prices past and present, on the known variations therein, on his knowledge of the causes of the variations, and on his forecast of conditions. If we plot the curve of unit price of labor for one locality over a long period we find the curve to be sinuous. The curve rises and falls, but rises in the long run more than it falls. It is said that the price of labor depends on the ratio of supply and demand; that when the demand is strong and supply weak, price is high; and that when demand is weak and supply strong, price is low. This law is helpful in estimating future prices only in so far as we can foresee the future relation between supply and demand. If the curve of unit price of labor, just mentioned, runs back over a period sufficiently long, we can draw an averaging line by eye and note its general characteristics and the extreme variations of the humps and sags from it. Then by projecting this averaging line into the future we obtain the tendency of average price in the future; and from the deviations of the sags and humps from the averaging line of the past, we can predict the probable deviations of the future. The distance into the future to which the price curve can be safely run depends on the length of the past price curve, its characteristics, and our knowledge of the causes of past fluctuations and our ability to foresee the causes of future fluctuations.

The unit price, present or past, of any kind of material which is to be used in building a structure, may be ascertained in one or more of the following ways: (1) from personal knowledge; (2) from consultation with a dealer; (3) from consultation with a purchaser; (4) from catalogs and discount sheets; and (5) from price quotations appearing in the daily newspaper and in the daily, weekly, and monthly trade and technical publications. (For a list of publications which regularly give market quotations, see pp. 274–276.)

The greater part of the foregoing remarks on prices apply also to **wage rates.**

CHAPTER XVI

VALUATION

Fair Value

SALVAGE, SCRAP, AND WEARING VALUES. DEPRECIATION

1601. The following diagram (Fig. 1601) which will be recognized as a fragment of the diagram of Fig. 414, shows that annual depreciation cost is a function of first cost, salvage value, and life of structure.

$$\text{Depreciation (annual cost of)} \begin{cases} \text{First cost} \\ \text{Salvage value} \\ \text{Life} \end{cases}$$

FIG. 1601.—Analysis of annual depreciation cost.

We have devoted several chapters to analyzing first cost, and have given consideration to the element, life. It is the purpose of the following pages to give some consideration to salvage value, or, as it is more commonly called, second-hand price.

1602. The **salvage value** of a structure is the price for which it is sold second-hand. Thus salvage value is a fact and not an opinion. However, in the problems of this book, salvage value is usually a future fact (just as the population, which a city will have any given number of years hence, is a future fact), and must be predicted or estimated.

1603. The depreciation in the value of a structure between two dates is the loss in value suffered by the structure between the dates. Hence the salvage value of a structure at any date is equal to the first cost of the structure less the total depreciation suffered by the structure between the date of first cost and the date of salvage value. Or, in usual terms, salvage value equals first cost less **depreciation.**

It will be well to point out here, as was done in § 403, that depreciation and depreciation cost are two different things, though depreciation cost is at times referred to by the single word "depreciation" While depreciation is the actual loss in value, depreciation cost is, as we have used the term, the annual contribution to the depreciation fund (sinking fund) required to make good on a certain date the total depreciation which the value of the structure has suffered up to that date.

1604. When a structure is no longer capable of performing service as a structure, or when the serviceability which is still possessed by a structure is not in demand, the structure has no value except the values of the materials and parts of which it is composed. The **scrap value** of a structure is the sum of the market prices of its component materials and parts, less the cost of their preparation for and transportation to market.

For example, when an old highway bridge is condemned to make way for its successor, the sounder timbers may be sold for some temporary use and the rods, bolts, and washers sold to be directly used or remelted. If the old bridge sells for $25 to a buyer who takes the bridge away at his own cost, the scrap value of the bridge is $25.

Scrap value may decrease or increase as the structure grows older.

1605. Of the total sale value of a structure at a given time that part which exists by virtue of the capability of the structure to render service as an entirety is the **wearing value** of the structure at that time.

A structure may at the time of offering for sale be in condition to render service as a structure—may have remaining serviceability—yet, if no one can be found to pay anything for such serviceability, the wearing value at that time is zero. Thus when an iron bridge, which is no longer of use to the owner, is offered for sale, the wearing value of the bridge at that time is zero if no one can be found to buy the bridge for use as a bridge, even if the structure is practically new.

1606. The relations which we have found to exist between salvage value, first cost, depreciation, wearing value, and scrap value are clearly shown in Fig. 1606.

$$\text{Salvage value} \left\{ \begin{array}{l} \text{First cost} \\ \quad (-) \\ \text{Depreciation} \end{array} \right.$$

$$\text{Salvage value} \left\{ \begin{array}{l} \text{Wearing value} \\ \quad (+) \\ \text{Scrap value} \end{array} \right.$$

FIG. 1606.—Analysis of salvage value.

1607. Salvage value (second-hand price) is affected by the following **conditions**: (1) reason for selling; (2) market price of like structure new; (3) condition of structure; and if the sale of structure implies removal, the following are to be added to the foregoing conditions: (4) structure standard or special; (5) structure massive or articulate; and (6) location of structure. The bearings of these conditions on salvage value are considered in the following six sub-sections.

1. The price which can be obtained for a second-hand structure depends to a great extent on what the buyer thinks is the owner's **reason for selling.** It is reasonable to expect a higher price for a partly worn engine when it is part of a plant and business to be sold on account of the death or retirement of the owner than when it is offered for sale with the object of putting in its place a new, like engine; because in the latter case, and not in the former, suspicion is cast on the efficiency or durability of the engine offered. Suspicion is the factor chiefly responsible for the fact that the second-hand price of a little-used article in the open market is usually from 30% to 50% less than first cost.

2. A structure which may be bought ready made in the market has a market price—in fact two market prices, a retail price and a wholesale price. When such a structure is bought and installed ready for use, but not yet used, its salvage value is probably a maximum. Such maximum salvage value cannot, except under conditions named below, be expected to exceed the wholesale price which obtains at the time of installation.

If the **market price new** of similar structures is greater (or less) at the time of second-hand sale than when the second-hand structure was bought new, the salvage value may be expected to be higher (or lower) than it would be if the market price new did not change.

The market price new may advance between the date of purchase new and the date of second-hand sale, so far even as to cause the salvage value to exceed first cost. For example, steel rails bought at $16 per ton and used for years have brought a higher figure at second-hand sale, because at the time of second-hand sale, new rails had advanced to $28 per ton. And in the case of land the salvage value is quite often much greater than the first cost.

Market price new is affected by fluctuations in prices of labor and materials, and also by the ratio of supply to demand. Demand may rise and fall with some regularity with the rounds of the seasons, as is the case with agricultural implements and machinery; with less regularity with waves of good and hard times. They may suddenly rise at the outset of a great constructional enterprise, and as suddenly fall at its completion; or change with changes in the tastes and mode of life of the people. Demand for any given type of structure may be cut off partially or entirely by the appearance of an improved type of structure designed for the same kind of service. A structure is said to be "obsolescent" when the demand for it is falling, owing to the introduction of improved structures, and "obsolete" when the demand has ceased entirely.

While in most cases it is out of the question to arrive by formal calculation at the future market price new of a structure, nevertheless it is necessary to take into account the possibilities of market changes, and weigh these along with other conditions when estimating salvage value.

The effect of change in market price new on salvage value is independent of other conditions which affect salvage value.

3. Of two structures otherwise alike the one which is in the better **condition**—which possesses the greater number of units of serviceability—will have the greater salvage value. Thus salvage value may decrease through neglect of structure as well as through use. For example, the salvage value of a harvesting machine which is left unprotected in the field between harvests falls more between than during harvest seasons.

4. (The remarks in this paragraph are pertinent if the sale implies removal of the structure from its present site.) Of two structures otherwise alike, that one which is **special** (*i.e.*, modified to meet peculiar local conditions) will have a lower salvage value than the other which is **standard** (*i.e.*, designed for normal conditions). A half-worn, small, steel highway bridge, if square, may bring a salvage value much greater than scrap value, but if skew, scrap value only. The salvage value of a trestle which carries a railroad across a depression can seldom be greater than scrap value, because the topographic conditions are rarely the same at any two sites.

5. (The remarks of this paragraph have a bearing on salvage value only when the sale implies removal of structure to a new site). If the cost of separating and putting together without injury the readily handled parts of a structure is great, compared with the cost of the parts themselves, the structure is here said to be **massive.** An **articulate** structure is one the parts of which can be separated and rejoined at a small cost compared with the cost of the parts themselves. According to these definitions, structures of earth, stone, concrete, and the like are massive structures, while machines are articulate in a high degree.

The maximum salvage value of a massive structure (where sale implies removal) may be little more than scrap value, except in the case of a structure which can be moved economically as a unit; while the salvage value of an articulate structure may approach, or even in some cases exceed, first cost. For example, the track on an embankment may sell second-hand for more than half its first cost, while the embankment cannot as a rule be sold at all; and a large structure of wood will have a greater salvage value when parts are joined by bolts and nuts than when spiked together.

6. **Location** may have a marked effect on salvage value if sale of structure implies removal. The salvage value of a more or less isolated structure is less than it would be if the structure were situated at its cearest market, by the cost of transportation to such market. Thus a nearly new engine which would bring a good price at an industrial center may, if located at the far end of a rough trail or long mountain road, as at a mine, find no taker when offered as a gift.

PRICE FIXED BY BUYER AND SELLER

1608. Ordinarily in the sale of an article second-hand between private parties, the seller gets the biggest price that he thinks he can get, and the buyer pays the smallest price that he thinks the seller will accept.

When the seller has no competition and the buyer cannot well do without the article, the seller is likely to demand and obtain an abnormally large price. On the other hand, when the buyer has no competition and is under no constraint to buy, and the seller is pressed for money or has no further use for the article, the buyer is likely to obtain the article for a price abnormally low. Under such conditions the price mutually agreed upon will depend not only on whether the seller or buyer is in the tight place, but also on the tightness of the place, and this circumstance is independent of other conditions named farther on

If both buyer and seller have active competition neither buyer nor seller is at a disadvantage, and the price agreed upon may be called a market price. Such market price is usually considered a fair price.

FAIR PRICE FIXED BY THIRD PARTY CHOSEN BY BUYER AND SELLER

1609. Not infrequently there arise cases in which the seller and buyer of a second-hand structure voluntarily agree to accept such price as may be fixed by a third party who is chosen by them to appraise the value of the structure.

For example, one or more members of a firm that owns an industrial plant may wish to retire from the firm, and it may be agreed that the continuing partners shall pay to the retiring partners a sum to be computed from the fractional interest of the retiring partners and the value of the plant as determined by a disinterested appraiser who is acceptable to all parties.

FAIR PRICE FIXED BY COURT OR COMMISSION

1610. Then there is the class of cases which arise from time to time, in which the ownership of an existing public utility, as, for example, a water-supply, is taken over from a private business unit by the municipality which it serves. The transfer of ownership may be made at a price agreed upon by the seller and buyer; or if the utility is the property of a public service corporation, the municipality may acquire controlling or complete ownership by buying up the stock of the public service corporation.

1611. But usually the utility owners and the municipality fail to come to an agreement on price, and then if the municipality does acquire the utility it does so through legal procedure and at a price fixed, not by the utility owners, not by the municipality, but by a government commission or by a court.

1612. The court or commission undertakes in each case to fix a "fair" price. But so far neither court nor commission has laid down a formula by means of which it is possible to calculate in advance what price in any particular case will be named by court or commission as fair.

1613. The practice of court and of commission has been to base the determination of fair price on some or all of the following factors: (1) The original cost and the costs of additions and betterments to date. (2) The cost of reproduction new. (3) Depreciation. (4) The amount and market value of the stock and bonds of the corporation owning the utility. (5) The commercial value of the utility. (6) All other matters deemed by court or commission to be pertinent to the case.

1614. Neither court nor commission has specified what **weight** should be given to each of these factors in order to reach a fair price in any future case. In fact no court or commission has disclosed the weights which it has assigned to the various elements in reaching its fair price in any case.

1615. The **original cost** can be known from the accounting records of the utility owners if these records were clearly made and have been preserved, and the same holds true of costs of additions and betterments. In many cases, however, such rec-

$$\text{First cost}\left\{\begin{array}{l}\text{Investigation cost}\\ \quad(+)\\ \text{Organization cost}\\ \quad(+)\\ \text{Capitalization cost}\\ \quad(+)\\ \text{Creation cost}\end{array}\right.$$

Fig. 1615.—Analysis of first cost.

ords are either incomplete, or dubious, or lost, or destroyed; and in such a case the original cost must be approximated as nearly as may be by the process of estimating first cost heretofore described.

By Fig. 1615 (a reproduction of Fig. 503) we are reminded of the fact that before construction of the actual physical plant is begun there are prerequisite activities, numerous and varied, requiring outlays which together may form a large part of the total first cost.

And it is necessary to realize that the transfer involves not merely the ownership of the physical plant but ownership of the enterprise as a whole. Thus creation cost must be interpreted as including not only the cost of designing and constructing the physical plant but also the cost of creating the business being transacted.

The work of estimating the cost of a structure built a long or a short time ago is not essentially different from the work of estimating the cost of a proposed structure. But in carrying out the details of the work of estimating past cost there may arise certain difficulties which do not confront the estimator of present or future cost.

If the plans and specifications of the structure as planned or as built are not available, then the number of each of the various kinds of units composing the structure must be ascertained by count or by measurement and calculation. And in some cases the count or measurement is impracticable or even impossible because of physical conditions. For instance, to ascertain the character of the lower portion of the substructure of a bridge abutment or pier may be not feasible; or a considerable portion of the total of material placed in an embankment built on yielding ground may have settled below the general level of the original surface; or the riprap placed to protect the side of an embankment from erosive action of stream or lake may in part or altogether have disappeared in the mud.

Nor is this all: the number of man-hours and machine-hours required to build a structure depends to a great extent upon the conditions surrounding the work of building. For example, the number of such hours required under favorable conditions to build a bridge may be doubled or trebled by the occurrence of floods, say; and yet in any particular case there will be in later years nothing to indicate whether the structure was built under favorable or unfavorable conditions.

If the date of building a structure can be learned, the unit prices of that date can usually be determined, with some degree of confidence, from printed records of prices.

What has been written above as to original cost applies also to costs of past additions and of betterments.

1616. The **cost of reproduction new** may be defined as the total of outlays which would have to be made at the present time to create the existing physical plant and volume of business.

Thus the cost of reproduction new may be greater or less or equal to the original cost depending upon the change which has taken place in prices of labor and materials between the time of construction and the time of appraisal.

Suppose a plant was constructed in 1867. The cost of reproduction new determined in 1897 would be much lower than first cost. If the plant were built in 1914 the cost of reproduction new determined in 1920 would be much higher than first cost.

Thus, though first cost remains the same on whatever date determined, the answer to the question, "What is the cost of reproduction New?", will vary from date to date with the variations in prices.

We have spoken of reproduction new as taking place on a date; but the fact is that the physical plant of a public service enterprise cannot be reproduced on a date: it would require months or years. And the development of the present volume of business would take perhaps several years. Hence into the work of estimating there is introduced the complication of price changes during the period of reproduction.

1617. If the existing plant were new it might be held that its fair value is the estimated cost of reproduction; but the existing plant is not new and, supposing the holding named above to be just, the fair value must be the cost of reproduction less **depreciation.** Thus arises the need of considering the amount of depreciation which the plant has suffered.

The depreciation of an existing plant is often determined by expert inspection and judgment. The expert inspects the plant, part by part, and judges the "per cent condition" of the part.

The depreciation is also sometimes determined by one or another of certain so called depreciation formulas (see chap. XVII).

1618. In regard to the amount and **market value** of the outstanding stocks and bonds of a public service corporation at any time, it suffices to say here that the amount will be shown by the books of the corporation, and the market value can be calculated from the amounts of stock and bonds and the prices at which they are selling.

As the market prices of stock change from day to day, and sometimes to a considerable extent, it has been the practice to take as the market price the average market price over a chosen period of time.

1619. The **commercial value** of a public utility property is found by capitalizing the future net earnings.

By net earnings is meant gross earnings less operating cost, depreciation cost, and taxes.

The estimation of future net earnings is of course attended with uncertainty even in the case of a private enterprise, and in the case of a public service enterprise this estimation is rendered still more uncertain by the impossibility of predicting to what extent, and when, the gross earnings or the cost of service, or both, will be affected by changes which will be made in rates, or service, or both, by government.

The rate of interest to be used in capitalizing net earnings is that at which investors are willing to invest their money in the securities of the enterprise.

1620. Besides the "values" which we have been considering as elements to be weighed in determining "fair price" of a public utility property, there may be in any particular case other conditions and circumstances of the past and present life of the property which will be taken into account by the commission or court in coming to a decision as to what is the "fair price."

1621. So far we have considered valuation for the purpose of determining a fair price for a public utility property. Valuation of such property is also undertaken to determine the value as a **basis** (1) for taxation; (2) for issue of securities; and (3) for rate fixing by government.

1622. Since both the stock-and-bond value and the commercial value of a utility are derived from present and prospective rates, neither of these values can consistently be used as a datum for determining a "fair value" upon which to base rates. To use either as a datum would be tantamount to assuming that the rates in force are themselves the "fair rates" to be determined.

With the exceptions noted above, it may be said that some or all of the elements, which have been described as being taken into account by court or commission in determining "fair price," may be taken into account in determining "fair value" for any of the three purposes enumerated above. And the weight given an element when valuing for one purpose may or may not be the same as when valuing for another purpose.

1623. The engineer's part in the work of valuation lies primarily in estimating (1) original cost and costs of betterments and additions, (2) cost of reproduction new, and (3) depreciation. The engineer who is a member of a valuation commission of course takes a hand in weighing the several valuation factors and judging the "fair value."

CHAPTER XVII

VALUATION (CONCLUDED)

Formulas for Depreciation and Salvage Value

1701. Any formula by which it is proposed to compute the depreciation of a structure is usually called a **depreciation formula.** In this chapter we shall discuss five depreciation formulas, namely: (1) the "straight-line" formula, (2) the "sinking-fund" formula, (3) the Matheson formula, (4) the Gillette formula, and (5) the "equal profit ratios" formula. After all of the formulas have been presented they will be applied to a numerical example.

DERIVATION OF FORMULAS

1702. The **straight-line** formula for calculating depreciation is based upon the assumption that the annual depreciation is equal to the total drop in value during the life of the structure, divided by the life expressed in years.

Let A = age, in years, for which salvage value is sought;
L = life of structure, in years;
C = first cost of structure;
C_A = salvage value of structure at age A yr.;
$\%C_A$ = per cent ratio of C_A to C;
C_L = value of structure at end of life of L yr.;
f = yearly depreciation;
$\%f$ = per cent ratio of f to C;
F_A = depreciation of structure up to age A yr.;
$\%F_A$ = per cent ratio of F_A to C.

From the assumption and notation above we obtain the following formulas.

Yearly depreciation is

$$f = \frac{C - C_L}{L}. \qquad (1702a)$$

Yearly depreciation, per cent, is

$$\%f = 100\frac{C - C_L}{CL}. \qquad (1702b)$$

Depreciation up to age A yr. is

$$F_A = A\frac{C - C_L}{L}. \qquad (1702c)$$

Depreciation, per cent, up to age A yr. is

$$\%F_A = 100\frac{A(C - C_L)}{CL}. \qquad (1702d)$$

Salvage value at age A yr. is

$$C_A = C - A\frac{C - C_L}{L}. \qquad (1702e)$$

Salvage value, per cent, at age A yr. is

$$\%C_A = 100\left(1 - A\frac{C - C_L}{CL}\right). \qquad (1702f)$$

1703. The **sinking-fund** formula is based on the assumption that the depreciation up to any age, A, is equal to the amount, at the end of A yr., of the sinking fund established (actually or in imagination) to offset the total life depreciation of the structure; in other words the formula is based on the assumption that depreciation and amount of depreciation fund at any given time are equal.

Let A = age, in years, for which salvage value is sought;
C = first cost of structure;
C_A = salvage value of structure at age A yr.;
$\%C_A$ = per cent ratio of C_A to C;
C_L = value of structure at the end of its life of L yr.;
F_A = depreciation of structure up to age A yr.;
$\%F_A$ = per cent ratio of F_A to C;
d_L = sinking-fund deposit required to redeem $1 at the end of L yr.;
z_A = amount of sinking fund in which $1 is deposited yearly for A yr.

The following formulas are based upon the assumption and notation given above.

Depreciation up to age A yr. is

$$F_A = (C - C_L)d_Lz_A. \qquad (1703a)$$

Depreciation, per cent, up to age A yr. is

$$\%F_A = 100 \frac{C - C_L}{C} d_L z_A. \tag{1703b}$$

Salvage value at age A yr. is

$$C_A = C - (C - C_L) d_L z_A. \tag{1703c}$$

Salvage value, per cent, at age A yr. is

$$\%C_A = 100\left[1 - \frac{(C - C_L) d_L z_A}{C}\right]. \tag{1703d}$$

1704. The **Matheson** formula differs from the straight-line formula in this, that, whereas with the latter the depreciation each year is taken as a constant percentage of first cost, with the former the depreciation each year is taken as a constant percentage of the salvage value at the beginning of that year. For example, if first cost is \$100 and the so-called rate of depreciation is taken as 5%, then by the Matheson formula at age 1 yr. the salvage value is \$100 − \$5 = \$95; during the second year the drop in value is 5% of \$95, or \$4.75, and the salvage value at the end of the second year is \$95 − \$4.75 = \$90.25, and so on. Evidently this method gives a decreasing yearly depreciation; and the salvage value will become zero only at the end of time, if the structure has no scrap value.

Let A = age, in years for which salvage value is sought;
C = first cost of structure;
C_A = salvage value at age A yr.;
$\%C_A$ = per cent ratio of C_A to C;
C_L = value of structure at the end of life of L yr.;
F_A = depreciation of structure up to age A yr.;
$\%F_A$ = per cent ratio of F_A to C;
k = ratio of depreciation in any 1 yr. to the salvage value at the beginning of that year. k is constant.

From the foregoing assumption and notation we construct the following tabulation:

Year	Yearly depreciation	Salvage value at end of the year, C_A	Total depreciation up to the end of the year
1st	kC	$C - kC = C(1 - k)$	kC
2d	$kC(1 - k)$	$C(1 - k) - kC(1 - k) = C(1 - k)^2$	$C - C(1 - k)^2$
3d	$kC(1 - k)^2$	$C(1 - k)^2 - kC(1 - k)^2 = C(1 - k)^3$	$C - C(1 - k)^3$
......	....	....	....
Ath	$kC(1 - k)^{A-1}$	$C(1 - k)^{A-1} - kC(1 - k)^{A-1} = C(1 - k)^A$	$C - C(1 - k)^A$

From the tabulation we write the following formulas:

Depreciation during the Ath yr. $= kC(1 - k)^{A-1}$. (1704a)

Depreciation up to the age A yr. is

$$F_A = C - C(1 - k)^A. \tag{1704b}$$

Depreciation, per cent, up to age A yr. is

$$\%F_A = 100\,[1 - (1 - k)^A]. \tag{1704c}$$

Salvage value at age A yr. is

$$C_A = C(1 - k)^A = C\left(\frac{C_L}{C}\right)^{A/L}. \quad \text{(See Eq. 1704 f)} \tag{1704d}$$

Salvage value, per cent, at age A yr. is

$$\%C_A = 100\,(1 - k)^A. \tag{1704e}$$

Rate of depreciation so called, is

$$k = 1 - \sqrt[A]{\frac{C_A}{C}} = 1 - \sqrt[L]{\frac{C_L}{C}}. \tag{1704f}$$

1705. The **Gillette** formula is based on the following principle: "The owner of a second-hand machine [structure] is entitled to such a price for it as will enable the purchaser to go on with its use and produce each unit of product at as low a cost as the average unit cost of production would be during the entire life of the machine [structure]."[1]

The Gillette formula assumes an analysis of annual service cost which differs from that which we have carried out in previous pages. This special analysis, which will be used throughout this chapter, is shown by the diagram, Fig. 1705.

$$\text{Annual service cost} \left\{ \begin{array}{l} \text{Depreciation cost} \\ \qquad (+) \\ \text{Interest and other fixed charges } (Cr.) \\ \qquad (+) \\ \text{Operation cost } (O) \\ \qquad (+) \\ \text{Maintenance cost } (M) \end{array} \right.$$

FIG. 1705.—A special analysis of annual service cost.

It will be understood that yearly amortization is another name for annual depreciation cost; that interest on invested capital is here, for the first time in the book, counted as an item of annual service cost; and finally that out of the items heretofore embraced by the term "operation," we have made three groups, namely "other fixed charges," "operation," and "maintenance."

---

[1] H. P. Gillette, *Cost Data*, 2nd ed., 1910, p. 36.

The foregoing remarks will serve to make intelligible the following notation:

Let A = age, in years, for which salvage value is sought;
C = first cost of structure;
C_A = salvage value of structure at age A yr.;
$\%C_A$ = per cent ratio of C_A to C;
C_L = value of structure at the end of its life of L yr.;
d_L = yearly deposit in sinking fund required to redeem \$1 in L yr.;
d_{L-A} = yearly deposit in sinking fund required to redeem \$1 in $L - A$ yr.;
F_A = depreciation of structure up to the age A yr.;
$\%F_A$ = per cent ratio of F_A to C;
M = yearly cost of maintenance for L yr.;
M'' = yearly cost of maintenance after first A yr.;
O = yearly cost of operation for L yr.;
O'' = yearly cost of operation after the first A yr.;
r = ratio of yearly interest and other fixed charges to first cost, C;
r'' = ratio of yearly interest and other fixed charges[1] (for second-hand structure) to C_A;
U = yearly output of structure during life;
U'' = yearly output of structure after the first A yr.;
v_L = unit cost of service for life L yr.;
v_{L-A} = unit cost of service for the last $L - A$ yr. of life.

The unit cost of service over the L yr. of life is found by dividing the yearly cost of service by the corresponding yearly output; thus,

$$v_L = \frac{1}{U}[(C - C_L)d_L + Cr + O + M].$$

To him who buys the structure at the age of A yr., the average unit cost of service during the $L - A$ remaining years of life will be

$$v_{L-A} = \frac{1}{U''}[(C_A - C_L)d_{L-A} + C_A r'' + O'' + M''].$$

According to the principle stated at the beginning of this section, v_{L-A} should equal v_L. Hence

---

[1] If it is not practicable to estimate the *ratio* of the item "other fixed charges," to C_A without knowing the value of C_A, then the item, "other fixed charges," should be included in M'', and r'' will then be simply the interest rate on the capital, C_A.

$$\frac{1}{U''}[(C_A - C_L)d_{L-A} + C_A r'' + O'' + M'']$$
$$= \frac{1}{U}[(C - C_L)d_L + Cr + O + M]. \quad (1705a)$$

Solving this equation for C_A we obtain the following:

Salvage value at age A yr. is

$$C_A = \frac{\frac{U''}{U}[(C-C_L)d_L+Cr+O+M]+C_L d_{L-A}-(O''+M'')}{d_{L-A}+r''} \quad (1705b)$$

Salvage value, per cent, at age A yr. is

$$\%C_A = 100\frac{C_A}{C}. \quad (1705c)$$

Depreciation up to age A yr. is

$$F_A = C - C_A. \quad (1705d)$$

Depreciation, per cent, up to age A yr. is

$$\%F_A = 100\,\frac{C - C_A}{C}. \quad (1705e)$$

Special Case I.—When the salvage value at the end of life is zero, and yearly output and yearly operating cost are uniform over the life of the structure, C_L = zero, $U'' = U$, and $O'' = O$; and introducing these values into Eq. 1705b we obtain the following:

Salvage value at the age A yr. is

$$C_A = \frac{C(d_L + r) + M - M''}{d_{L-A} + r''}. \quad (1705f)$$

Salvage value, per cent, at age A yr. is

$$\%C_A = 100\frac{d_L + r + \frac{M - M''}{C}}{d_{L-A} + r''}. \quad (1705g)$$

Eq. 1705g is virtually that to which Mr. Gillette has given the name "unit cost depreciation formula."

Special Case II.—When yearly output, yearly cost of maintenance, yearly cost of operation, and the ratio of interest and other fixed charges to first cost are uniform over the life of the structure, $U'' = U$, $O'' = O$, $M'' = M$, $r'' = r$. Placing these values in Eq. 1705b we obtain

$$C_A = \frac{(C - C_L)d_L + Cr + C_L d_{L-A}}{d_{L-A} + r} \quad (1705h)$$

When, also, the "other fixed charges" are zero and the interest rate is the same for capital and sinking fund, Eq. 1705h reduces to Eqs. 1705i and 1705j as follows. Substituting for d_L its value $\frac{s-1}{s^L-1}$, for d_{L-A} its value $\frac{s-1}{s^{L-A}-1}$ (where s = amount of \$1 for 1 yr.), and for r its value $s - 1$, we find that Eq. 1705h reduces to

$$C_A = C - (C - C_L)\frac{s^A - 1}{s^L - 1}. \tag{1705i}$$

Since $\frac{s^A-1}{s^L-1} = \frac{s-1}{s^L-1} \cdot \frac{s^A-1}{s-1} = d_L z_A$, we obtain from Eq. 1705i the following:

Salvage value at age A yr. is

$$C_A = C - (C - C_L)d_L z_A. \tag{1705j}$$

This equation, as the reader will see, is the "sinking-fund" depreciation formula (Eq. 1703c).

Special Case III.—If to the provisions of special case *II* we add that interest is to be ignored, Eq. 1705h becomes

$$C_A = \frac{(C - C_L)\frac{1}{L} + C_L\frac{1}{L-A}}{\frac{1}{L-A}}. \tag{1705k}$$

Reducing Eq. 1705k, we obtain the following:

Salvage value at age A yr. is

$$C_A = C - \frac{A(C - C_L)}{L}. \tag{1705m}$$

It should be observed that Eq. 1705m is the "straight-line" depreciation formula (Eq. 1702e).

1706. It remains to derive the **equal profit ratios** formula, which was devised by me and first appeared in the earlier edition of this book.

Sometimes the demand for the service of a structure is expected to increase from the beginning of operation, because (1) of the spreading recognition of the convenience of the service, and the people's readjustment of their mode of life to take advantage of it; and (2) because of the growth of the population, as for example in the case of a new waterworks for a small town, or a railroad in an undeveloped territory. Now, for some reasons it is not economical to build a structure to meet simply the service demand of the first year, and for other reasons it is not economical to build to meet the demand expected 50 or 60 yr. hence. Though the problem of determining for what future time it is economical to build now comes under the head of engineering economics and the engineer may be called upon to solve it, the solution of the problem is here passed by as being beyond the scope of this book. But, whatever may be the precise number of years for which a large and complex structure is built, one thing is certain, namely, that during the early years of operation the potential capacity of the structure may not be fully employed, and we may say a part of the invested capital may thus be necessarily idle for a time. And it may be that the yearly earnings from the sale of service during the earlier years will be less than the yearly cost of service. During each of the early years when the earnings do not meet all the cost of service, not only is there no profit, but the owner must borrow money (from himself or others) to pay that portion of cost of service which the earnings do not cover. Thus he must wait for the profit belonging to the earlier years, and pay interest on the money borrowed, until such time as the excess earnings become great enough to reimburse him.

Many structures are planned and built, with the foreknowledge that the yearly earnings of the early years will not pay the corresponding yearly costs of service, on the reasonable expectation that the earnings of later years will exceed the yearly costs by sums which will pay not only a reasonable yearly profit for the later years, but, also, all deferred profits and the money borrowed during the earlier years together with interest thereon.

How is a fair price to be determined for a structure from which the earnings have not been sufficient to pay yearly costs of service and the yearly profits up to the time of the proposed sale? Evidently a price computed by any formula which ignores earnings will be greatly to the disadvantage of the seller, who, unless earnings are taken into account, may be constrained to whistle for his profits while he pays out of his own pocket the debts incurred for money to help to keep the structure operating. It is believed that no formula can give a fair second-hand price unless it recognize earnings; and in this belief a new formula (Eq. 1706e) is offered, based on the following principle:

A fair price for a second-hand structure is such as will cause the ratio of equivalent uniform yearly profit to investment to be the same for the seller as for the buyer.

Let L = the life of structure, in years;
A = age, in years, for which salvage value is sought;
z_A = amount of an A yr. sinking fund in which \$1 is the end-of-year deposit.

For the seller:

Let b' = equivalent uniform yearly profit for the A yr.;
e' = equivalent uniform yearly earnings for the A yr.;
y_A = equivalent uniform yearly cost of service for the A yr.;
C = first cost of structure (to the seller);
C_A = salvage value of structure at age A (this is the first cost to the buyer);
d_A = end-of-year deposit in the sinking fund required to redeem \$1 at the end of A yr.;
r' = ratio of yearly interest and other fixed charges (for the seller) to the seller's first cost, C;
O' = equivalent uniform yearly cost of operation for the A yr.;
M' = equivalent uniform yearly cost of maintenance for the A yr.

For the buyer:

Let b'' = equivalent uniform yearly profit for the $L - A$ yr.;
e'' = equivalent uniform yearly earnings for the $L - A$ yr.;
y_{L-A} = equivalent uniform yearly cost of service for the $L - A$ yr.;
C_A = first cost of structure (to the buyer);
C_L = salvage value of structure at age L yr.;
d_{L-A} = end-of-year deposit in the sinking fund required to redeem \$1 at the end of $L - A$ yr.;
r'' = ratio of yearly interest and other fixed charges (for the buyer) to the buyer's first cost, C_A;
O'' = equivalent uniform yearly cost of operation for the $L - A$ yr.;
M'' = equivalent uniform yearly cost of maintenance for the $L = A$ yr.

For the seller, whose investment is C, the ratio of equivalent uniform yearly profit to investment is b'/C; and for the buyer, whose investment is C_A, the ratio of equivalent uniform yearly profit to investment is b''/C_A. According to the principle stated above,

$$b'/C = b''/C_A \tag{1706a}$$

From the notation above,

$$b' = e' - y_A,$$

and

$$b'' = e'' - y_{L-A};$$

and the yearly service cost (introducing the primes here for present purposes) is

$$y_A = (C - C_A)d_A + Cr' + O' + M',$$

and, similarly,

$$y_{L-A} = (C_A - C_L)d_{L-A} + C_A r'' + O'' + M''.$$

Substituting these values of b', b'', y_A, and y_{L-A} in Eq. 1706a, we have

$$\frac{e' - [(C - C_A)d_A + Cr' + O' + M']}{C} = \frac{e'' - [(C_A - C_L)d_{L-A} + C_A r'' + O'' + M'']}{C_A}, \tag{1706b}$$

from which we obtain

$$C^2{}_A + C_A z_A[e' - (O' + M') - C(d_A - d_{L-A} + r' - r'')] = Cz_A[e'' - (O'' + M'') + C_L d_{L-A}]. \tag{1706c}$$

Let $2g$ = the coefficient of C_A, and m = the right-hand member of the equation; then

$$C^2{}_A + 2gC_A = m. \tag{1706d}$$

Completing the square

$$C^2{}_A + 2gC_A + g^2 = m + g^2$$

Taking the square root and transposing, we obtain the following formula for the salvage value:

$$C_A = \sqrt{m + g^2} - g, \tag{1706e}$$

where

$$g = (\tfrac{1}{2})z_A[(e' - (O' + M') - C(d_A - d_{L-A} + r' - r'')], \tag{1706f}$$

and

$$m = Cz_A\,[e'' - (O'' + M'') + C_L d_{L-A}], \tag{1706g}$$

and the other letters have the significations given above. Values of z_A may be taken from Table C, and values of d_A and d_{L-A} from Table E.

Special Case.—For this special case two assumptions are made. (1) No profits are made on the undertaking other than those made in the form of interest on first cost. (2) The earnings are proportional to the output.

From assumption (1), b' is zero and b'' is zero. Therefore, from notation, $e' - y_A =$ zero, and $e'' - y_{L-A} =$ zero. Consequently

$$e' = y_A, \qquad (1706h)$$

and

$$e'' = y_{L-A}. \qquad (1706i)$$

From assumption (2), letting U' and U'' be the yearly output for the seller and buyer respectively, $e''/e' = U''/U'$, or,

$$e'' = \frac{U''}{U'}e'. \qquad (1706j)$$

Putting this value of e'' in Eq. 1706i, we have

$$\frac{U''}{U'}e' = y_{L-A};$$

which, since $e' = y_A$ (Eq. 1706h), may be written

$$\frac{U''}{U'}y_A = y_{L-A},$$

whence

$$y_A/U' = y_{L-A}/U'' \qquad (1706k)$$

Eq. 1706k shows that under the two assumptions of this special case, the unit cost of service is uniform over the whole life; and because of this we may write,

$$y_{L-A}/U'' = y_L/U \qquad (1706m)$$

where $y_L =$ equivalent uniform yearly cost of service during the whole life of L yr., and $U =$ equivalent uniform yearly output for the whole life. Substituting for y_{L-A} and y_L their values in terms of amortization, interest and other fixed charges, operation, and maintenance, we obtain,

$$\frac{1}{U''}[(C_A - C_L)d_{L-A} + C_Ar'' + O'' + M'']$$
$$= \frac{1}{U}[(C - C_L)d_L + Cr + O + M]. \qquad (1706n)$$

Equation 1706n is the same as Eq. 1705a. Thus we find that the equal profit ratios formula reduces to the Gillette formula, for the special case in which the profits, other than those paid in the form of interest on first cost, are zero and the earnings are proportional to the output.

COMPARISON OF FORMULAS

Example 1

1707. Let us solve, by using in turn each of the five formulas, the following example:

Example 1.—We have the following data pertaining to a structure. Required the salvage value at the age of 5 yr.

First cost	$10000
Life	10 yr.
Salvage value at end of life	0
Interest rate on capital and amortization fund	5%
Yearly cost of operation and maintenance:	
During the first 5 yr.	$1000
During the last 5 yr.	$2000
Yearly output (units of service):	
During the first 5 yr.	1000
During the last 5 yr.	2000
Yearly earnings from sale of service:	
During the first 5 yr.	$3000
During the last 5 yr.	$6000

Solution by Straight-line Formula.—This formula (Eq. 1702e) is

$$C_A = C - A\frac{C - C_L}{L}.$$

For this example $C = 10{,}000$, $A = 5$, $C_L = 0$, and $L = 10$; hence the salvage value is

$$C_A = 10{,}000 - 5\frac{10{,}000 - 0}{10}$$
$$= \$5000. \quad \textit{Answer.}$$

The reader will observe that this formula ignores interest, output, and cost of operation and maintenance, as well as earnings.

Solution by Sinking-fund Formula.—This formula (Eq. 1703c):

$$C_A = C - (C - C_L)\, d_L z_A.$$

For this example

$$C = 10{,}000,\ C_L = 0,\ \text{A} = 5,\ L = 10;$$

$d_L = d_{10} = 0.0795$ (opposite "10," under "5%," Table E), and $z_A = z_5 = 5.526$ (opposite "5," under "5%," Table C).

Hence $$C_A = 10{,}000 - (10{,}000 - 0)\,(0.0795)\,(5.526)$$
$$= \$5606.83. \quad \textit{Answer.}$$

It will be observed that this formula ignores output, interest on capital, and cost of operation and maintenance, as well as earnings.

Solution by Matheson Formula.—This formula (Eq. 1704d) is

$$C_A = C\left(\frac{C_L}{C}\right)^{A/L}.$$

In this example $C = 10{,}000$, $L = 10$, $A = 5$. According to the data, C_L is zero, but if we substitute zero for C_L in the formula we shall come out with $C_A =$ zero. The basis of the Matheson formula implies that salvage value can be zero only at an infinite age. Therefore to defer to the letter of the implication and at the same time practically to keep to the spirit of the data, let us say that $C_L = \$0.01$.

Hence

$$\begin{aligned} C_A &= 10{,}000\left(\frac{0.01}{10{,}000}\right)^{5/10} = 10{,}000(0.000001)^{1/2} \\ &= 10{,}000(0.001) \\ &= \$10. \quad \textit{Answer.} \end{aligned}$$

It will be noticed that the Matheson formula does not take into account output, interest, cost of operation and maintenance, or earnings, and to this extent is in the same class as the straight-line formula.

Solution by Gillette Formula.—This formula (Eq. 1705b) is

$$C_A = \frac{(U''/U)[(C - C_L)d_L + Cr + O + M] + C_L d_{L-A} - (O'' + M'')}{d_{L-A} + r''}.$$

In this example

$U'' = 2000,$

$U = (1000 \times 5 + 2000 \times 5)/10 = 1500,$

$L = 10,\ A = 5,\ C = 10{,}000,\ C_L =$ zero,

$d_L = d_{10} = 0.0795$ (opposite "10," under "5%," Table E),

$r = r'' = 0.05,$

$O + M = (1000 \times 5 + 2000 \times 5)/10 = 1500,$

$d_{L-A} = d_5 = 0.181$ (opposite "5," under "5 %," Table E),

$O'' + M'' = 2000.$

Hence

$$\begin{aligned} C_A &= \frac{\frac{2000}{1500}[(10000-0)(.0795)+10000(.05)+1500]+0(.181)-2000}{0.181 + 0.05} \\ &= \frac{1.333(795 + 500 + 1500) - 2000}{0.231} \\ &= \$7474.74. \quad \textit{Answer.} \end{aligned}$$

The reader will notice that the Gillette formula takes account of all elements of cost of service, but does not recognize earnings.

Solution by Equal Profit Ratios Formula.—This formula (Eq. 1706e) is

$$C_A = \sqrt{m + g^2} - g$$

where (Eq. 1706f)

$$g = (\tfrac{1}{2})z_A[e' - (O' + M') - C(d_A - d_{L-A} + r' - r'')],$$

and (Eq. 1706g)

$$m = Cz_A[e'' - (O'' + M'') + C_L d_{L-A}]$$

In this example

$$L = 10, \quad A = 5,$$

$z_A = z_5 = 5.526$ (opposite "5," under "5%," Table C),

$e' = 3000,$

$O' + M' = 1000,$

$C = 10{,}000,$

$d_A = d_{L-A} = d_5 = 0.181$ (opposite "5," under "5 %," Table E),

$r' = r'' = 0.05,$

$e'' = 6000,$

$O'' + M'' = 2000,$

$C_L =$ zero.

Therefore

$$g = (\tfrac{1}{2})(5.526)[3000 - 1000 - 10{,}000(0.181 - 0.1810 + .05 - 0.05)] = 5526;$$

$$g^2 = 30{,}536{,}676$$

$$m = 10{,}000(5.526)[6000 - 2000 + 0(0.181)] = 221{,}040{,}000$$

$$m + g^2 = 251{,}576{,}676$$

$$\sqrt{m + g^2} = 15{,}861.17,$$

$$g = 5{,}526.00,$$

and $C_A = \$10{,}335.17$. *Answer.*

It is observable that the equal profit ratios formula takes account of the earnings as well as of all the elements of cost of service. The output enters only indirectly—through the earnings.

It may be interesting to see what will be the ratio of yearly profit to investment for the seller and for the buyer for the salvage value computed by each of the five formulas above. For the seller the yearly cost of service is

$$y_A = (C - C_A)d_A + Cr' + O' + M',$$

and the ratio of yearly profit, b', to investment, C, will be (Eq. 1706a)

$$b'/C = (e' - y_A)/C$$

where e' = seller's yearly earnings.

For the buyer, the yearly cost of service will be

$$y_{L-A} = (C_A - C_L)d_{L-A} + C_A r'' + O'' + M'',$$

and the ratio of his yearly profit, b'', to his investment, C_A, will be

$$b''/C_A = (e'' - y_{L-A})/C_A,$$

where e'' = buyer's yearly earnings.

When the salvage value is computed by the straight-line formula

$$\begin{aligned} y_A &= (10{,}000 - 5000)\ (0.181) + 10{,}000(0.05) + 1000 \\ &= 2405, \end{aligned}$$

and the profit ratio for the seller is

$$\begin{aligned} b'/C &= (e' - y_A)/C = (3000 - 2405)/10{,}000 \\ &= 5.95\%. \end{aligned}$$

Likewise

$$\begin{aligned} y_{L-A} &= (5000 - 0)\ (0.181) + 5000(0.05) + 2000 \\ &= 3155, \end{aligned}$$

and the profit ratio for the buyer is

$$\begin{aligned} b''/C_A &= (e'' - y_{L-A})/C_A = (6000 - 3155)/5000 \\ &= 56.9\%. \end{aligned}$$

In the same way the profit ratios have been computed for each of the salvage values found by means of the four other formulas

TABLE 1707.—YEARLY PROFIT RATIOS RESULTING FROM SALVAGE VALUES COMPUTED BY FIVE DIFFERENT FORMULAS FOR EXAMPLE 1

Formula	Salvage value at age 5 yr.	Ratio of yearly profit to investment, for the	
		Seller, per cent	Buyer, per cent
Straight-line	$5000.00	5.95	56.9
Sinking-fund	5606.83	7.05	48.2
Matheson	10.00	−3.08	39,976.9
Gillette	7474.74	10.4	30.4
Equal profit ratios	10335.17	15.61	15.61

above. All the salvage values and corresponding profit ratios are given in Table 1707.

It must be borne in mind that the profit ratios given in the table above are for the particular case stated at the beginning of this section; and that were the data of the example changed throughout, the ratios would be changed.

Example 2

1708. Let us apply each of the same five formulas in turn to the following case:

Example 2.—The data of this example are as follows:

First cost	$75
Life	10 yr.
Salvage value at end of life	$5
Interest on capital	5%
Interest on amortization fund	4%
Yearly cost of operation and maintenance	$10
Yearly output (units of service)	300
Yearly earnings (from sale of service)	$30

By means of each of the five formulas used in Example 1 above, the salvage value of the present example was computed for each age, 1 yr., 2 yr., and so on, up to 10 yr. (The computations, being similar to those of Example 1 above, are omitted.) The results for each formula were then plotted (Fig. 1708) using years for abscissas and the corresponding salvage values as ordinates. The plotted points were then connected by lines as shown. Thus from the Matheson curve, for example, the salvage value of the structure for which data are given above, at age 5 yr., is about 26% of the first cost.

It should be remembered that the curves of Fig. 1708 result from the data of this particular example. Were we to change the salvage value at end of life from $5 to zero, or any other value, all five curves would be changed. If, however, we should change only the earnings, only one of the curves—that for equal profit ratios—would be changed.

For this example the profit ratios (Table 1708) have been computed on the basis of salvage value at age 6 yr., by the method illustrated in Example 1.

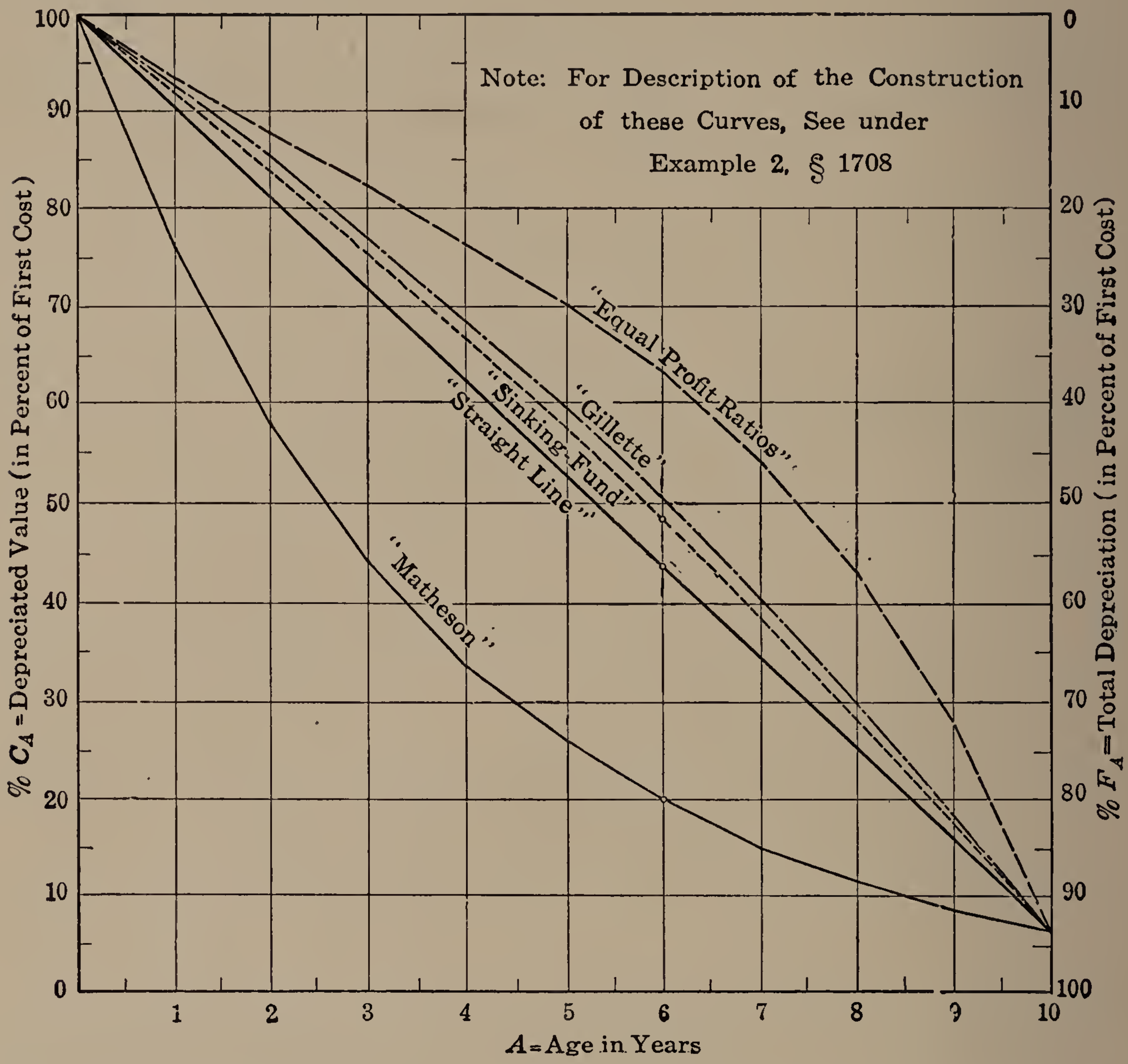


FIG. 1708.—Depreciation formulas compared by application to a particular case.

TABLE 1708.—YEARLY PROFIT RATIOS RESULTING FROM SALVAGE VALUES COMPUTED BY FIVE DIFFERENT FORMULAS FOR EXAMPLE 2

Formula	Salvage value at age 6 yr.	Ratio of yearly profit to investment, for the	
		Seller, per cent	Buyer, per cent
Straight-line	$33.00	13.2	35.7
Sinking-fund	36.32	13.9	29.8
Matheson	14.80	9.6	114.5
Gillette	37.69	14.2	27.8
Equal profit ratios	47.42	16.1	16.1

Example 3

1709. Now apply the five formulas to the following data. *Example* 3.—This example is the same as Example 2, above, except that this has not uniform output and earnings.

First cost	$75
Life	10 yr
Salvage value at end of life	$5
Interest on capital	5%
Interest on amortization funds	4%
Yearly cost of operation and maintenance	$10
Yearly output (units of service):	
During the first 6 yr	100
During the last 4 yr	500
Yearly earnings (from sale of service):	
During the first 6 yr	$10
During the last 4 yr	$50

For the data of this example the salvage values computed by the five formulas used in Example 1, together with the corresponding profit ratios, are given in Table 1709.

TABLE 1709.—YEARLY PROFIT RATIOS RESULTING FROM SALVAGE VALUES COMPUTED BY FIVE DIFFERENT FORMULAS FOR EXAMPLE 3

Formula	Salvage value at age 6 yr.	Ratio of yearly profits to investment, for the	
		Seller, per cent	Buyer, per cent
Straight-line	$ 33.00	−13.5	96.2
Sinking-fund	36.32	−12.8	84.8
Matheson	14.80	−17.1	250.0
Gillette	101.00	0.2	12.2
Equal profit ratios	123.59	4.8	4.8

It will be noticed that the salvage values above for the first three formulas are the same as for the same formulas in the preceding example.

Remarks

1710. For reasons given in §1706, the equal profits ratio formula is believed to be based directly on the correct principle underlying all logical derivations of salvage value, and is expressed in general terms. As is shown in §1706, this formula reduces to the Gillette formula if two assumptions are made. Furthermore, as was shown in §1705, the straight-line and sinking-fund formulas are two forms to which the Gillette formula reduces for certain special conditions. It follows that these four formulas—equal profit ratios, Gillette, sinking-fund, and straight-line—have the same rational basis, and differ from one another only in the number of special assumptions which must be made. The equal profit ratios formula is the most general, and the straight-line the least. If the principle underlying the equal profit ratios formula be accepted as fair, logically the salvage values computed by this formula must be accepted as fair; and the salvage values computed by the Gillette, sinking-fund, and straight-line formulas must be accepted as fair when, and only when, the special conditions described in §§1705 and 1706 can be assumed to hold.

The Matheson formula is based on the idea that yearly depreciation grows less and less as the age of the structure increases. This is the law of second-hand market price, which usually takes the greatest drop during the first year of life of structure, and drops less and less, year by year, thereafter. Hence this formula may be useful in predicting second-hand market price. But, as before noted, second-hand market price is greatly depressed, especially for structures nearly new, by the factor of suspicion. Since this factor is non-existent in cases where salvage value is determined by a disinterested party, the use of the Matheson formula is not justified for such determination.

The new formula, called equal profit ratios formula, is thus shown to furnish the most general solution of the problem of salvage value. It is important, however, that we bear in mind the fact that *no formula can take into account all the conditions and circumstances which may have legitimate influence on the salvage value of a structure;* and that, accordingly, the salvage value computed for any case by this formula must be used with judgment; that is, must be considered in connection with all such pertinent factors as are not represented in the formula. In other words, the computed salvage value must be considered coordinately with all irreducible data. The formula assumes that all financial and engineering plans for the creation and operation of the structure have been made and carried out with reasonable intelligence and absolute honesty and good faith. It is the duty of the appraiser to eliminate from the data the effects of any fraud or incompetence on the part of the promoters, constructors, or operators, before applying the formula.

CHAPTER XVIII

ERRORS IN ESTIMATING

SIMPLE AND COMPOUND ESTIMATES

1801. When we estimate the area of a field by simply looking at it and without consciously estimating dimensions, the result is a **simple** estimate of the area. But if we estimate the length or breadth or both, and obtain the area by taking the product of the dimensions, the result is a **compound** estimate of the area. A compound estimate is a quantity derived from one or more quantities which have been estimated.

Let A be the actual, and a the estimated value of a quantity; let e be the absolute and r the relative error of a. Then the **absolute** error is

$$e = a - A;$$

and the **relative** error is

$$r = (a - A)/A = e/A;$$

or, approximately,

$$r = e/a.$$

Example.—A distance is estimated to be 250 ft. Later it is found, by careful measurement that the distance is 263.2 ft. What is the absolute error of the estimate? What is the relative error? The absolute error is $e = a - A = 250 - 263.2 = -13.2$ ft. The relative error is $r = e/A = -13.2/263.2 = -0.0502$; or approximately, $r = e/a = -13.2/250 = -0.0528$. For all practical purposes the approximate relative error is as serviceable as the actual. In fact, in this case it would be just as well to say "the relative error is minus 5%."

1802. The engineer is not satisfied to accept an estimate, whether made by himself or another, without knowing something of its quality. He wants to know about how far the true value may lie to one side or the other of the estimate. He wants to know the **limit** of error.

Until we obtain the true value of a quantity of which we have an estimate, we must look to past experience for information concerning the limit of error in the estimate.

A man can acquire, in a few years, from those comparisons of estimated and measured quantities which are thrust upon him, a fair knowledge of the limit of error in his estimate of a given kind of quantity under given conditions; but he can acquire the same knowledge in a

TABLE 1802.—FORMULAS FOR LIMITS OF ABSOLUTE AND RELATIVE ERRORS OF COMPOUND ESTIMATES

Notation:

$a_1, a_2, \ldots, a_n$ = estimated values of quantities;
$e_1, e_2, \ldots, e_n$ = limits of absolute error in $a_1, a_2, \ldots, a_n$;
$r_1, r_2, \ldots, r_n$ = limits of relative error in $a_1, a_2, \ldots, a_n$.

Compound estimate formed by	Limit of absolute error	Limit of relative error
Addition: $a_1 + a_2 + \ldots + a_n$	$\pm(e_1 + e_2 + \ldots + e_n)$	$\pm\dfrac{e_1 + e_2 + \ldots + e_n}{a_1 + a_2 + \ldots + a_n}$ or, $\pm\dfrac{a_1r_1 + a_2r_2 + \ldots + a_nr_n}{a_1 + a_2 + \ldots + a_n}$.
Subtraction: $a_1 - a_2$	$\pm(e_1 + e_2)$	$\pm\dfrac{e_1 + e_2}{a_1 - a_2}$ or, $\pm\dfrac{a_1r_1 + a_2r_2}{a_1 - a_2}$
Multiplication: $a_1 \cdot a_2 \cdot \ldots \cdot a_n$	....	$\pm[(e_1/a_1) + (e_2/a_2) + \ldots + (e_n/a_n)]$ . . . (approx.) or, $\pm(r_1 + r_2 + \ldots + r_n)$ . . . (approx.)
Division: a_1/a_2	....	$\pm[(e_1/a_1) + (e_2/a_2)]$ . . . (approx.) or, $\pm(r_1 + r_2)$ . . . (approx.)
Involution: $a^{1/n}$	....	$\pm(e/a)/n$. . . (approx.) or, $\pm r/n$ . . . . . . (approx.)
Evolution: a^n	....	$\pm n(e/a)$. . . . (approx.) or, $\pm nr$. . . . . (approx.)

much shorter time if he will take the extra trouble of estimating every quantity which he is about to measure, and noting the sign and amount of the error discovered by the measurement.

The limits of error in a compound estimate will depend of course on the limits of error of the component simple estimates. In Table 1802 will be found formulas for calculating the limit of error in a compound estimate obtained by addition, subtraction, multiplication, division, involution, or evolution.

COMPENSATION OF ERRORS

1803. An estimate may be wide of the truth because of some blunder or bias; or because of some illusion, of which latter the following is a familiar and typical example: The road running before us slopes upward toward the foot of the mountain, but appears to be level or even to slope downward. All estimates of slope of such a road by men looking along it toward the mountain will be in error with the same sign. But passing beyond all such cases, experience has shown that in the long run (1) positive and negative errors occur with equal frequency; and (2) larger errors occur less frequently than smaller ones. For example, let the probability be ½ that a simple estimate will be in error by more than a given percentage of the limiting error; then the probability that the errors in two simple estimates will each be greater than the given percentage of the limiting error is ¼; and that the two errors will also be of like sign is ⅛; the probability that the error in each of three estimates will be of like sign and greater than the given percentage of the limiting error is 1/32; and so on.

Thus we see that, while error in a simple estimate may be very small and is more likely to be small than large, the probability of its being near the limit is great enough to make it prudent to be prepared to find it there; but, due to the neutralizing effect of positive and negative errors—that is, due to the law of **compensation** of errors—the probability that the error in a compound estimate is near its limiting size grows rapidly less as the number of simple estimates which enter therein is increased.

Thus where a compound estimate is the resultant of numerous simple estimates, its *limit* of error lies in the land of the improbable, and the **probable**[1] error is likely to be more serviceable.

---

[1] The probable error of an estimated quantity is such that the chance of its exceeding the actual error is equal to the chance of its being smaller than the actual error.

Formulas for probable error of a compound estimate, in terms of the probable errors of the component estimates, are given in Table 1803.

TABLE 1803.—FORMULAS FOR PROBABLE ERRORS IN COMPOUND ESTIMATES

Notation:

$a_1, a_2, \ldots, a_n$ = estimated values of quantities;
$p_1, p_2, \ldots, p_n$ = the probable errors of $a_1, a_2, \ldots, a_n$, respectively.

Compound estimate formed by	Probable error of the compound estimate
Addition: $a_1 + a_2 + \ldots + a_n$	$(p_1^2 + p_2^2 + \ldots + p_n^2)^{1/2}$
Subtraction: $a_1 - a_2$	$(p_1^2 + p_2^2)^{1/2}$
Multiplication: $a_1 \times a_2$	$(a_1^2p_2^2 + a_2^2p_1^2)^{1/2}$
Division: a_1/a_2	$(1/a_2^2)(a_1^2p_2^2 + a_2^2p_1^2)^{1/2}$
Involution: $a^{1/n}$	$(p_1/n)a_1^{(1-n)/n}$
Evolution: a^n	$np_1a_1^{n-1}$
Any combination of estimated quantities: $A = f(a_1, a_2, \ldots, a_n)$	$P = [(dA/da_1)^2p_1^2 + (dA/da_2)^2p_2^2 + \ldots + (dA/da_n)^2p_n^2]^{1/2}$

1804. The **practical conclusion** to be drawn from the facts presented above is this: A compound estimate of a whole, formed by summing the estimates of the parts, is more reliable than a simple estimate of the whole, provided of course that the estimator has the same knowledge of parts as he has of the whole.

And from this principle follows another:

The greater the number of parts into which a whole is divided for the purpose of estimate, the greater is the reliability of the compound estimate of the whole.

But the compensation of errors is not the only, and under some conditions may be the least, advantage which comes from estimating a whole by parts. When the estimator divides a whole into parts for the purposes of estimate, the lines are drawn, consciously or unconsciously, in such a way as to permit his taking all possible advantage of whatever special skill he may have in estimating certain kinds of quantities. For example, in the case of a house the estimator, if a mason, will probably part the masonry from the rest whether he subdivides the rest or not; if a plumber, segregate the plumbing; and so on.

Let the whole be divided into unlike parts. Let each part be estimated in subdivisions by an estimator who is specially qualified to estimate that part. Add together all the estimates. This is the rule for making the most reliable estimate of the whole.

ERROR IN FORECASTING FUTURE COST

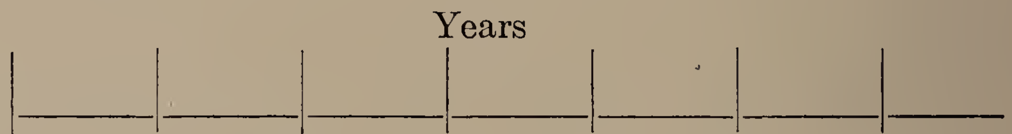
1805. None but a gambler would contract now, either as builder or owner, for the erection of a structure say 20 yr. hence, at a price estimated now. And our present estimate of the cost of a structure grows less and less reliable as the date of erecting the structure is advanced on the calendar. An engineer would count as unprofitable the time spent in predicting the cost of a building to be erected 50, or 100 yr. in the future. In fact he does not undertake such prediction, but instead, when he has occasion to use the cost of a structure to be built in the distant future, as when he is considering the cost of a renewal, he unblushingly assumes that the cost of each renewal will be the same as the estimated cost of the proposed original structure. This practice he can justify on the ground that no other *one* figure for a renewal can be shown to have a greater probability than the figure for the original cost, and the adoption of the latter figure involves a minimum of effort.

The student may well ask: If so uncertain a figure for future cost must be combined with the estimated cost for the near future in calculations for investment choice, why take any extra trouble to make the estimate of cost of the proposed original structure closely approximate? This practice of using a closely approximate figure in combination with a very uncertain figure is not as inconsistent as at first glance it appears to be, as will now be shown.

In the first place, the present effect of the error, whatever it may be, in a forecast of cost for a given date, is the present worth of that error. And the fact that the more advanced the date the larger is likely to be the error in the forecast is fairly offset by another fact, namely, that the more advanced the date the smaller is the present effect of the error.

In the second place, the engineer's estimates of future costs are almost always made for the purpose of comparison: he compares one time-series of costs with another time-series of costs. Now as wages and prices tend generally to fluctuate together—to go up together and come down together—the estimated costs of two future structures are likely both to be in error by the same sign—both too high or both too low—and the error in the difference between these erroneous costs is likely to be less than the error in either cost. And finally, whatever residual error there may be in the difference between two estimated costs, the ratio between the present effect of that residual error and the residual error itself grows rapidly less, as we have seen, as the date of the costs is advanced on the calendar.

To illustrate: We estimate that it will cost X dollars to build a specified structure 10 yr. hence, Y dollars to build a like structure 40 yr. hence, and Z dollars 100 yr. hence. Suppose X, Y, and Z are each wide of the mark by the same sum, $1000. Our present concern with the error in X, if money is worth 4% compounded annually, is $675.60 (1000



Present worth	0	10	20	30	40	50	60	
structure A:								
$1000	$1000							(Original)
456			$1000					(1st renewal)
208					$1000			(2nd renewal)
$1664 Present worth of series A								
Structure B:								
$1500	$1500							(Original)
462				$1500				(Renewal)
$1962 Present worth of series B								
$1664 Present worth of series A (brought down)								
$ 298 = Present difference (on basis of assumed uniform costs)								

Fig. 1805a.—Time schedule showing two assumed series of structure costs. (Contrast with Fig. 1805b.)

times the present worth of $1 due 10 yr. hence); with the error in *Y* as little as $208.30; and with the error in *Z*, only $19.80.

Consider also the following case: It is estimated that of two structures proposed for a 60 yr. service, structure A will cost $1000 and last 20 yr., and structure B will cost $1500 and last 30 yr. A would require two renewals in the 60 yr., and B one renewal.

First, let us assume, as in practice, that each renewal cost will be the same as the corresponding original cost, as indicated on the time schedule, Fig. 1805a; and compare the two series of costs by comparing their present worths calculated for the date of the original cost.

Second, let us suppose that costs will actually rise at a uniform rate from the twentieth year to the fortieth, becoming double on the latter date. The corresponding correct values of the A and B series are then as shown on the time schedule, Fig. 1805b.

Years

Present worth	0	10	20	30	40	50	60	
structure A:								
$1000	$1000							(Original cost)
456			$1000					(1st renewal)
417					$2000			(2nd renewal)
$1873	Present worth of series A							
Structure B:								
$1500	$1500							(Original)
694				$2250				(Renewal)
$2194	Present worth of series B							
1873	Present worth of series A (brought down)							
$ 321	= Present difference (on basis of actual rising costs)							

FIG. 1805b.—Time schedule showing two actual series of structure costs. (Compare with Fig. 1805a.)

On examining the two schedules (Figs. 1805a, 1805b) we note the following: (1) Assuming uniform costs results in an error of (2000 − 1000 =) $1000 in the cost of the second renewal of A; but the present effect of that error is only (417 − 208 =) $209. (2) Likewise in the case of B: the assumption of uniform costs results in an error of (2250 − 1500 =) $750 in the cost of renewal; but the present effect of the error is only (694 − 462 =) $232. (3) On the assumption of uniform costs: the present worth of the A series is only (1873 − 1664 =) $209, or 100 ($^{209}/_{1873}$) = 11% too small; and the present worth of the B series is only (2194 − 1962 =) \$232, or $100(232/2194)$ = 10%+ too small. (4) Finally the assumption that the costs are uniform over the whole time results in a difference between the present worths of the A and B series that is only (321 − 298 =) \$23, or $100(23/321)$ = 7%+ too small.

Thus the original errors—50% in the cost of renewal of A and 33⅓% in the cost of renewal of B—arising from the assumption of constant costs over the period, result in an error of only 7%+ in the difference between the present worths of the A and B series. Now it is important to note that this resulting 7%+ or \$23 error in this difference is no more than would arise from an error of $100(23/1000)$ = 2.3% in the estimate of first cost of A, or from an error of $100(23/1500)$ = 1.5% in the estimate of first cost of B; and an error of 1.5%, or of even 2.3%, in an estimate of first cost, is a small error for even a very carefully prepared estimate.

CHAPTER XIX

ENGINEERING REPORTS

Report [verb] = To bear or bring back as an answer; relate, as what has been discovered by a person sent to examine, explore, or investigate . . . . Report [noun] = . . . ; a statement or relation of facts given in reply to inquiry, or by a person authorized to examine and bring or send information.—*Century Dictionary.*

INTRODUCTION

1901. The purpose of this chapter is to help the young engineer to write a report.

1902. The all-controlling fact to be born in mind when constructing a report is this: *A report is an answer to a question,* or consists of a series of answers corresponding to a series of questions.

Any misgivings that may arise in the student's mind as to his ability to construct a report are attributable, I think, to his having the notion that he is entering upon work entirely beyond his experience.

That such a notion is altogether mistaken should become clear to him when he has laid a firm grip on the cardinal fact that a report is an answer to a question, and when he recalls the further fact that all his life a great part of his talking and writing has been given to aswering questions, actual or implied. It is only necessary for him to fully appreciate this fact to turn misgivings into confidence.

I have known very intelligent students who have, until rightly questioned, considered the chief characteristics of an engineering report to be that it is about 8 in. wide, 13 in. top to bottom, from 5 to 50 pages thick, and constructed by some process known only to the eminent engineer.

1903. On countless occasions the reader, as answerer, has fitted answer to question, and on other countless occasions, as questioner, has received answer to question; and in so doing has necessarily acquired skill that is great in itself, though it may be relatively little compared with the corresponding skill of another person.

From the reader's own experience, then, he ought to be able to extract some general principles of the formation of reports, which, as pointed out above, are answers. At any rate he can readily test the following statements by reference to his own experience.

1904. An answer in its essence may be good, bad, or indifferent; but with this we are not here concerned.

The answer first exists in the answerer's mind as an idea or a group of related ideas.

Giving the answer consists in transferring the idea or group of ideas from the mind of the answerer to the mind of the questioner. (In this chapter we are interested not in the essence of the answer but in the mechanics of giving the answer.)

1905. Mark this **axiom:** The perfect method of giving an answer is that which (1) permits the transfer of the idea or group of related ideas from the mind of the answerer to the mind of the questioner with the least loss of vividness and proportion, and (2) permits the transfer to be effected with minimum effort on the part of the questioner.

1906. The amount of detail required to make an adequate answer depends upon circumstances.

Suppose my employer directs me to find out if Mr. Smith will call on him tomorrow morning at 10 o'clock. I find out in a moment; and, observing that my employer has been waiting for the answer, I report "He will." This is a perfect report.

Again, suppose that, receiving the same direction as before, it is an hour before I can get an answer to the question, and during that time my employer gives his attention to various persons and matters. Soon after getting the answer, I report to my employer "Mr. Smith will call at 10 o'clock tomorrow." This is a satisfactory report.

Now, if not only the question, but also the idea in the answer, is precisely the same in the one case as in the other, and if the report, "He will," is satisfactory in the first case, why will the report, "He will," not be satisfactory in the second case? And why will not the report, "Mr. Smith will call at 10 o'clock tomorrow morning," be satisfactory in the first case if it is satisfactory in the second? In the first case I know that my employer, when I am about to report, holds the question clearly in his mind; the report, "He will," is therefore adequate; and any additional words would be superfluous. To report in this case "Mr. Smith will call at 10 o'clock tomorrow" would not only waste words but would have a tendency by its fullness to give a first impression to the effect that the answer was not affirmative. In the second case, where the employer's mind has been busy with other matters, to report "He will" would be likely to do no more than cause him to drop his jaw and raise his eyes and say, at least to himself, "Who will what?"

In these two examples we have a clear illustration of the fact that though in two cases the question may be precisely the same and the answers may be precisely the same in essence, a difference in circumstances in the two cases may require a marked difference in the amount of detail necessary to the perfect answer.

1907. An answer may be given in writing, orally, or even by gesture.

1908. The **gesture** may be and often is the most efficient means of conveying an answer. As ordinarily used the gesture is entirely adequate, transfers the idea constituting the answer without diminution of vividness or change of proportion.

I hear an automobile passing my house and ask a child to step to the window and see in which direction it is going; and the child, after looking (and observing that I am regarding him, and remembering that I am quite deaf), jerks his thumb in the direction taken by the automobile. The jerk of the thumb is under the circumstances a perfect answer, a perfect report. Why is it perfect? Because it is entirely adequate, it is precise and concise, and, under the circumstances, it is more quickly, more certainly, and more easily interpreted than a report in any other form could possibly be.

1909. The **oral** transfer of an idea is habitually aided by gesture. The extent of this aid becomes apparent on talking to an unseen person, as a person in the dark or in the next room or at the telephone. Without the help of gesture we struggle somewhat to choose word and phrase with greater care in order to effect the same result. Of course in talking to an unseen person one is bothered also by not being able to read currently in the person's face the effect of the talk to him addressed.

Voice modulation, which with gesture aids in idea-transfer between persons face to face, becomes when speaking to an unseen person the sole aid given to the bare words. The unnatural tones used by many persons in telephoning are no doubt due to the effort to supply a substitute for gesture; and due also to a suspicion that the person addressed is not hearing or is not giving attention.

1910. In giving an answer in writing the answerer is without the aid of gesture or tone, and must necessarily give greater care to the choice of words and phrases than when speaking if he would transfer his idea as successfully as when speaking. His only aid is such as can be gained from the printer's art.

1911. Circumstances control the decision as to choice of means of communicating an answer. As remarked above, gesture frequently accompanies oral delivery; and in many cases the delivery of a written answer is accompanied by brief oral epitome.

1912. The purpose of this chapter is, as announced in the beginning, to be useful to the student or engineer in writing his first reports. The preceding discussion in this chapter has been given in the hope of bringing the student to make continual use of his own experience in fitting answers to questions, when composing a report or discussing the construction of a report. If the foregoing paragraphs of the chapter shall lead the student to test the reasonableness of each of the following paragraphs by referring it to his own experience, and to avoid altogether any attempt to memorize or blindly follow any of the paragraphs, the foregoing paragraphs will have justified their insertion.

THE WRITTEN REPORT

1913. Under this head will be given (1) certain general principles, elaborated from the axiom of §1905; (2) specifications drawn from the general principles; and (3) a schedule of procedure for preparing a report.

The general principles should be well digested by the student; the specifications and the procedure should be read first critically as an aid to appreciating the general principles, and later referred to, when writing a report, for such hints and suggestions as they may offer.

General Principles

1914. An engineer, on completing an investigation for a client (who of course may be the engineer's official superior), prepares a report on the investigation and submits the report to the client.

The primary purpose of the report is to furnish the client with the **result** of the investigation—*the answer to the question.*

With the result the report must (1) present sufficient description of the **object** of the investigation to re-establish the object of the investigation—the question—vividly in the client's mind; and (2) give sufficient information regarding the **conditions** and **methods** of the investigation (*a*) to enable the client justly to judge how well the question has been understood and how thoroughly investigated by the writer, and (*b*) also to enable the client to have every step of the original investigation intelligently checked.

The secondary purpose of the report is to present information pertaining to the investigation to others than the client.

1915. Whether the investigation is of great importance, as when made to determine the best source of water supply for a great city, or of relatively trifling importance, as when merely to find the approximate cost of a proposed culvert, the report of the investigation (1) should contain all information that probable users of the report will expect to find in it, and (2) should be so written and arranged, in whole and in each part, that any probable user can with minimum effort (*a*) read through understandingly such portions as interest him, and (*b*) refer to the report for any desired particulars.

1916. We cannot write a satisfactory report unless while writing we bear continually in mind the particular men or classes of men who will read or consult the report.

What classes of men will probably read the report? Readers may be classified according to their interests in the report: (1) readers who, interested in the investigation only as a whole, want the briefest intelligible statement of the results; (2) readers who are interested also in the partial results and in the chief processes which produced them; (3) readers who wish to look into the smallest details that have an effect on the final result.

Any user may belong to one class at one time and to another at another time in considering the same report. For example, the manager receiving when pressed for time the engineer's report, of say twenty typed pages, is in class 1; but the next day, with sufficient time, he steps into class 2 and reads connectedly the more important parts of the body of the report; and a few days later, perhaps, he enters class 3 by setting another engineer to check the calculations both in the body of the report and in the appendices.

For the first class, the briefest intelligible abstract is sufficient; for the second, the body of the report is needed; for the third, the body of the report and appendices. A further classification may be made: (4) users who read the report as a whole; and (5) users who refer to the report for details—data, processes, or results.

For the convenience of class 5 each element of the report must be presented as a distinctly separate part bearing a plain label, and the position of each particular part with respect to the others must be correct, and, furthermore, must be indicated by either an analytical table of contents or an index, or by both.

For the convenience of class 4, on the other hand, the quality of continuity is essential. Both classes can be satisfied by the same report if each important head or sub-head is immediately followed or preceded by suitable connective words or sentences.

By way of illustrating the foregoing points, let us run over the probable users of some report, say a favorable report on a complete investigation to determine the financial promise of a proposed irrigation system. Among the probable users of the report may be: (*a*) the promoter (class 1); (*b*) the men with whom the promoter seeks to form a company (class 1); (*c*) the banker who is approached for construction capital (class 1); (*d*) the engineer employed by the banker to give an opinion on the engineering features of the plan (classes 2 and 4); (*e*) the engineer's assistants (class 3); (*f*) the lawyer employed by the banker to pass on the legal points involved (class 3); (*g*) the expert on lands and crops and their irrigation—employed by the banker—(class 3); (*h*) the engineer, lawyer, or other expert who files a copy of the report for reference (class 5).

1917. Besides the ideas which the writer wishes to transfer from his own mind to the mind of his readers, the report should contain certain connectives—words, phrases, sentences, paragraphs, perhaps in extreme cases several paragraphs—inserted here and there with the sole purpose of making easy for the readers the **transition** from one idea to another and from one group of ideas to another.

When plodding through a report, as when tramping through a dense forest, one requires to know from time to time where he is with respect to the way as a whole. To know this he must know where he came from, the road he has traveled, the road he is about to take, and his destination. A report is not well written if the writer, at each change of subject, has not flashed a light back over the way he has come and has not flashed another light forward over the way he is to go.

1918. The results of an investigation constitute the answer to a problem. In chap. II it was pointed out that to find the answer to a problem involved two steps: (1) analyzing the problem, and (2) solving the problem; and that analyzing a problem consists in dividing and subdividing a problem into major and minor component problems, each complete in itself.

If follows therefore that the part of a report which deals with any sub-problem should be constructed on the same principles that control the construction of the report as a whole, and should similarly present the *results*, together with the object, conditions, and methods, of the investigation made in finding the answer to the sub-problem.

It is easy to go to absurd extremes in applying this principle; but it is not difficult to avoid doing so if the writer will keep clearly in mind the essentials of §§1905 and 1915.

Specifications for a Report

1919. The following specifications undertake to bring the axiom of §1905 and the foregoing general principles to bear on each of the major parts of a report. It is particularly important that every statement in these specifications be interpreted in the light of the axiom and the general principles.

Since every complex problem is composed of a number of parts each of which is itself a problem, it follows that each part of a report pertaining to a sub-problem may be considered as a sub-report on that sub-problem. Hence, for each sub-problem, as for the original problem, the following specifications hold.

1920. The **statement** of the problem usually consists of two parts, the first of which is a brief description no longer than necessary to make complete identification easy.

The statement of a sub-problem should be introduced by indicating to the reader the relation the sub-problem bears to the main problem or to preceding or succeeding sub-problems. In other words, the report writer should undertake to orient the reader as he takes up a new part of the report.

1921. All pertinent **data** in hand at the outset of the investigation may constitute the second part of the statement of the problem; but generally only major data are included in the statement, and the details are left to be introduced as needed.

Data may be classified thus: (1) data at hand, that is, data taken over as a part of the specification of the problem; (2) data from authority, that is, data obtained from reading or from consultation; (3) assumed data; and (4) data obtained by special investigation auxiliary to the main investigation. Each datum, wherever introduced in the report, should be appropriately labeled with its class, and its source given.

Besides indispensable data, which can hardly be overlooked in writing the report, there may be data of a lower order—pertinent though not indispensable—which may escape the attention of the report writer. Among such data may be listed the following: (5) geographical, geological, topographical, meteorological, political, historical, and sociological data; (6) all facts bearing on the validity of evidence presented in or with the report, for example, such evidence as certificates, appended sub-reports, legal or other, mine samples and assays.

A part of the work—sometimes the most difficult part—of solving the problem may consist (1) in determining what data, other than those in hand are necessary to the solution and (2) in finding them.

Of course data found in the process of analyzing or solving the problem would not appear in the statement.

1922. The logic of the **process** followed in passing from the data to the result—from question to answer—should be presented, leaving details to appear in an appendix. (See §1926.)

If the process is expressed by formula, the formula should be stated and its notation explained. If the formula is quoted the name of its author and the reference to the publication from which it was taken should be given; if original its derivation should be presented in an appendix to the report.

1923. If the process includes two or more coordinate parts the results of these should be recapitulated in a **summary.** The last part of the process then consists in combining the data of the summary into one final **result.**

1924. Often the final result of an investigation is an **opinion** based on premises consisting of both irreducible data and definite calculated quantities. The report writer should present his reasoning from premises to opinion in such fashion as to leave the reader who accepts the premises no alternative but to accept the opinion.

There are two good reasons why the report writer should put special effort on the presentation of the premises, the opinion, and the reasoning leading from the one to the other. In the first place, a failure of his best effort to show a clear logical path from premises to opinion is a pretty fair indication that the opinion is not to be trusted by even himself, and that, consequently, he should rebuild the opinion from the premises. In the second place, a perfectly clear setting forth of premises, logical reasoning, and opinion gives not only to the client the assurance that the opinion is sound, but also to all readers of the report most effective notice of the expectional value of the services of the man who wrote it.

1925. The **tabular form** of statement should be employed, so far as practicable, in presenting data, calculations, and results; otherwise the **statement form.** Quantities are hard to find in running text.

1926. Certain kinds of matter, not of interest to the reader of the report as a whole, yet pertinent, should be added as **appendices.** Placed in the body of the report, such matter is merely a hindrance to the progress of the reader.

Among the matters which are appropriately placed in appendices are the following:

1. Derivation of a formula.
2. Tabulated quantities including data, calculations, and results. For example, in solving a problem it may be necessary to calculate the volume of an embankment from cross-sections. This calculation makes a great array of data in groups exactly alike; many sets of calculations, all of the same kind; and a multitude of minor results, each in nature like every other. In such a case the general reader will care nothing whatever for any details of data, of calculations, or of results—except the final result, the volume of the fill. Therefore, placing all this matter in the middle of the report is like putting a hurdle in a pedestrian's path. Why then not omit the details? Because some reader of the report will very likely wish to check over the details. Both the general reader and the particular reader can be well served by putting the details in the appendix.
3. Original notes or records of observations, and original documents. For example, in a mining report, the record sheets of samples, assay, and calculations of the value, recovery, etc., should go in as an appendix. Logs of scientific experiments and engineering tests appear as appendices. Legal documents, certificates of title, etc., are proper material for an appendix.
4. Sub-reports. These may be reports submitted by others to whom special questions or sub-problems have been given for answer, opinion, or special investigation—legal, financial, mathematical, physical, chemical, geological, and so on.

Of course, reference to an appendix will be made whenever in the body of the report appendix material is used.

1927. The briefest statement, quickly clear to the client, of the gist of the conclusion—the result of the investigation—the *answer to the question*—should be written, either as a part of the letter of transmittal, or as a separate statement on the title page of the report or on the page following the title page. This statement should be written in non-technical language, as far as practicable.

The writer of the report should bear in mind the primary purpose of the report. The *client is waiting for the answer* to the problem. He is likely to have no interest whatever in the process by which the answer is reached except in so far as a knowledge of the process used may help him or his assistants to judge of the reliability of the answer. It is poor policy to keep the client waiting an unnecessary moment or to make it necessary for him to hunt through the report for the answer. The writer of the report may be certain that while the client is turning through the pages in search of the answer the client's thoughts are distinctly not complimentary to the writer.

1928. The nature of the matter of a report may be such as to tempt the weak or unscrupulous, for financial advantage, to alter the report in some way, such as removing a page or inserting a substitute; and, in case it is, the writer should present in the report certain **evidence of genuineness.** He should put his signature on every page of the report, including maps, original records, etc.; state on the title page that every page bears his signature; and in the table of contents give the page numbers corresponding to the listed parts.

Procedure for Preparing a Report

1929. The following schedule of steps to be taken in preparing a report is intended primarily for the guidance of the student who is preparing a report on an investigation carried on under the general direction of an instructor. However, this schedule should be helpful as well to the young engineer who is preparing his first reports in the practice of his profession.

While reading each of the following directions, the report writer should hold in mind the axiom of §1905; otherwise he may misinterpret the directions.

(*A*) Make an outline for the report, using such terms as appear in your analysis and in the headings of your log (*i.e.*, your running record) of solution. Attempt to make plain the relation between parts by indenting. Show the outline to the instructor. (*B*) Make a second outline, improving as far as may be on the (1) terms and (2) arrangement of the first. Ask the instructor to criticize your work. (*C*) Make a third outline for the report, using the same terms as in the second outline, but so prepared as to show (1) center heads, (2) side heads and (3) the "type" to be used for each head. Submit the third outline to the instructor for review. (*D*) In making the first draft of the report as directed below, have before you (1) the "third outline" as approved by the instructor, (2) your analysis, (3) your log of solution, and (4) any models of arrangement or expression that you consider helpful to you. Follow the outline with respect to type and position of heads and sub-heads. Follow the log of solution as to matter. Write two or three pages of your report, and show these to the instructor before writing further. (*E*) Complete the first draft of the report, and submit it to the instructor for criticism. (*F*) Write a brief abstract of the conclusion of the report—the answer to the question. Submit the abstract to the instructor for criticism. Prefix the abstract to the manuscript of the report. (*G*) Write a letter of transmittal, and lay it on the abstract. (*H*) Write a table of contents. (That part of the table of contents pertaining to the body of the report should be based on the third outline.) Lay the table of contents on top of the letter of transmittal. (*I*) Make a title page. (*J*) Put any appendices after the body of the report. (*K*) Make a fair copy, that is, the final draft, of the report. If you type the fair copy, it is kind to make a carbon copy for the office file for the benefit of your successors. (*L*) Put folded drawings at the back of the written pages of the report. (*M*) Bind the whole in a manila cover. The binding should be strong and neat, and at the same time permit easy handling of the pages and drawings when the report is being read. (*N*) Place a title on the outside of the front cover, like that of the title page. (*O*) Hand in the completed report.

SUMMARY

1930. A certain few of the ideas presented above in this chapter are applicable at every step in preparing a report. These are, of course, the most valuable ideas of the chapter; and the student will find it profitable to make them his own—not by learning them by heart but by assimilating them with his own experience in making and in receiving answers. Here are the ideas:

1. A report is the answer to a question.
2. The answer to a question is an idea or a group of related ideas.
3. The answer first exists in the mind of the answerer (the report writer).
4. To give the answer is to transfer the idea or group of ideas from the mind of the answerer to the mind of the questioner (client).
5. A perfect method of giving an answer is that which (*a*) transfers the idea or related ideas from mind to mind with the least loss of vividness and proportion, and (*b*) effects the transfer with minimum effort on the part of the questioner (axiom §1905).

The section on General Principles consists for most part of corollaries of the axiom of §1905 (reproduced in (5) above) stated with particular reference to written reports; the Specifications go to a certain extent into the particulars of the organizations of written reports; and the section on Procedure gives the sequence of routine steps to be taken by the student in preparing a report on an assigned problem.

1931. In conclusion the following is offered as a test which the report writer can profitably apply to the first draft of his report:

Picture in turn the men, for whom a report is written, each reading the report in the presence of the report writer. If none of these men feels impelled at any point of his reading to turn to the writer for a little elucidation, or for further explanation, or for additional facts or reasons, or for assistance in finding what he wants, or for questioning the genuineness of any part of the evidence presented, or to criticise any of the mechanical features of the report, the report may be considered to be without material fault, and the report writer may well believe that his most intelligent readers will not soon forget him and his work.

APPENDIX A

SPECIMEN ARTICLES OF INCORPORATION, CERTIFICATE OF INCORPORATION, AND BY-LAWS

NOTE.—These specimens are like the originals from which they are taken except for a partial change of name of corporation made by due process of law, and the substitution of one name and address.

[Specimen of Articles of Incorporation]

ARTICLES OF INCORPORATION OF BRADFORD, KIMBALL & CO.

KNOW ALL MEN BY THESE PRESENTS

THAT WE, the undersigned, all of whom are citizens of the State of California, have this day voluntarily associated ourselves together for the purpose of forming a corporation under the laws of the State of California, and we do hereby certify first, that the name of this corporation shall be BRADFORD, KIMBALL & CO.

1. That the purposes for which said corporation is formed are:

(*a*) To buy and sell municipal corporation and other bonds, shares of capital stock, both preferred and common, warrants, certificates of indebtedness and any and all other securities, and generally to transact any and everything that pertains to the business of bonds or stocks or dealing in securities, either directly for said corporation or indirectly as broker for others;

(*b*) To buy or otherwise acquire, own, hold, borrow, improve and develop all kinds of real and personal property whatsoever;

(*c*) To borrow and lend money and execute mortgage and deeds of trust to secure the repayment of money so borrowed and to do any and all things incidental to the same;

(*d*) To contribute in any manner to the expense of promoting, contracting, acquiring, improving or maintaining, any work or enterprise by whomsoever owned;

(*e*) To make contracts of any kind relating to any kind of property or business, in either an independent or representative capacity;

(*f*) To promote, do, acquire, hold and dispose of, anything incident to, or necessary, convenient or proper to carry out any of the things or purposes aforesaid, or which from time to time may be declared by the board of directors of this company then in office to be necessary, useful, incidental or auxiliary to accomplish any of the purposes of the corporation, or to promote its welfare;

(*g*) To do one or more of said things in any part of the United States of America and in any part of the world.

2. That the place where the principal business of said corporation is to be transacted is the City and County of San Francisco, State of California.

3. The term for which said corporation is to exist is fifty (50) years from date of its incorporation.

4. That the number of directors of said corporation shall be three (3) and the names and residences of its directors who are to serve for the first year and until the election of their successors are as follows, to wit:

Names	Residences
C. B. BRADFORD	510 Warren Street, San Mateo, California.
JOHN SMITH	1240 Carlisle Ave., San Bonno, California.
SHERMAN KIMBALL	1101 Emerson Street, Palo Alto, California.

5. The amount of the capital stock of said corporation is One Hundred Thousand (100,000) Dollars, and the number of shares into which it is divided is One Thousand (1000) Shares of the par value of One Hundred (100) Dollars each.

6. The amount of said capital stock which has been actually subscribed is THREE HUNDRED (300) DOLLARS, and following are the names of the persons by whom the same has been subscribed:

Names	Number of Shares	Amount
C. B. BRADFORD	1	$100.00
JOHN SMITH	1	$100.00
SHERMAN KIMBALL	1	$100.00

IN WITNESS WHEREOF, we have hereunto set our hands and seals at San Francisco, California. this 3rd day of *February*, 1920.

(Signed) C. B. BRADFORD (SEAL)
(Signed) JOHN SMITH (SEAL)
(Signed) SHERMAN KIMBALL (SEAL)

STATE OF CALIFORNIA, }
CITY AND COUNTY OF SAN FRANCISCO, } ss:

On this 3rd day of February, A. D. 1920, before me, M. V. COLLINS (Signed), a Notary Public in and for the City and County of San Francisco, State of California, duly commissioned and sworn, personally appeared C. B. BRADFORD, JOHN SMITH and SHERMAN KIMBALL, known to me to be the persons described in, whose names are subscribed to, and who each acknowledged to me that they executed the foregoing instrument for the purposes therein mentioned.

IN WITNESS WHEREOF, I have hereunto set my hand and affixed my offical seal at my office in the City and County of San Francisco, State of California, the day and year in this certificate first above written.

SEAL

M. V. COLLINS (Signed)

Notary Public in and for the City and County of San Francisco, State of California.

[Specimen of Certificate of Incorporation]

STATE OF CALIFORNIA

DEPARTMENT OF STATE

I, Frank C. Jordan, Secretary of State of the State of California, do hereby certify that a copy of the Articles of Incorporation of...........BRADFORD, KIMBALL & CO........... certified by the County Clerk of the CITY AND County of SAN FRANCISCO as a copy of such Articles filed in his office, was filed in this office on the 4th day of FEBRUARY A.D. 1920, which Articles and the copy thereof contained the required Statement of Facts, to wit: The name of the corporation as aforesaid; the purpose for which it is formed; the place where its principal business is to be transacted; the term for which it is to exist; the number of its directors or trustees, and the names and residences of those who are appointed for the first year; the amount of its capital stock, and the number of shares into which it is divided, and the par value thereof; the amount of its capital stock actually subscribed and by whom.

GREAT SEAL OF
THE STATE OF
CALIFORNIA

IN WITNESS WHEREOF, I have hereunto set my hand and affixed the Great Seal of the State of California this 4th day of
FEBRUARY, 1920.
FRANK C. JORDAN
Secretary of State
By FRANK H. CORY Deputy

[Specimen of By-Laws]

BY-LAWS
of the
BRADFORD, KIMBALL & CO.

THE NAME of the corporation shall be the

BRADFORD, KIMBALL & CO.

Article I
Corporate Seal

The Corporate Seal shall consist of a cricle having on its circumference the words "Bradford, Kimball & Co., San Francisco, California, Incorporated February 4th, 1920."

Article II
Corporate Powers

The corporate powers are vested in a Board of THREE Directors, who must be Stockholders, each holding not less than one share of stock in his own name on the books of the Corporation, and TWO shall constitute a quorum.

Article III
Election of Directors

The Directors shall be elected by ballot, at the annual meeting of the Stockholders, to serve for one year, and until their successors are elected. Their term of office to begin immediately after election.

Article IV

Vacancies

Vacancies in the Board shall be filled at a meeting of the Board by the remaining Directors. Those elected shall hold office until the next annual meeting of the Stockholders.

Article V

Power of Directors

Directors have power:

1st. To call special meetings of the Stockholders. They must call a meeting at any time, upon the written request of Stockholders holding one-third of the capital stock issued.

2nd. To appoint and remove (except Directors), officers, agents and employees, fix their duties and compensation, and require from them security for faithful service.

3rd. To manage the business of the Corporation, to make for it the rules and regulations and By-Laws.

4th. To incur indebtedness for the conduct of business, the terms and amount of which shall be entered on the minutes of the Board. The President and Secretary shall sign officially all written promises to pay debts.

Article VI

Duties of Directors

The Board of Directors shall:

1st. Cause to be kept a record of its meetings, and of the proceedings of the Stockholders, and cause a full statement at the regular meeting of the Stockholders to be made in writing, showing in detail the assets and liabilities of the Corporation, and generally the condition of its affairs.

2nd. Declare dividends out of the profits, when such profits shall, in the opinion of the Directors, warrant the same.

3rd. Repeal and amend the By-Laws and adopt new By-Laws at their discretion, as provided in Article XVI.

4th. Supervise all officers, agents and employees and see that their duties are properly performed. To cause to be issued to Stockholders, certificates of stock for the amount of their holdings.

Article VII

Officers

The officers shall be a President, First Vice-President, Second Vice-President, Secretary and Treasurer, to be elected by and hold office at the pleasure of the Board of Directors. The compensation and tenure of office of all officers (other than Directors) shall be fixed by said Directors.

Article VIII

President

The Board of Directors, at their first regular meeting, shall elect one of their number President; and if at any time the President is unable to act, a Vice-President shall perform his duties; and if a Vice-President is unable to act, they shall appoint some other member of the Board to his place. The President, or in his absence the Director, appointed as above provided:

1st. Shall preside over all meetings of the Stockholders and Directors.

2nd. Shall sign, as President, all certificates of stock, and all contracts and other instruments of writing which have been first approved by the Board of Directors.

3rd. Shall call the Directors together whenever he deems it necessary, and shall, subject to the advice of the Directors, have direction of the affairs of the Corporation, and shall discharge such other duties as may be required by the By-Laws.

The President, or two of the Directors, may call special meetings of the Directors at any time, and notice shall be given of such called meetings by leaving a written or printed notice at the last known place of business or of residence of each Director. Each Director must, at the first meeting after his election, cause to be entered in the minutes his address, to which all notices may be directed until similar notice and entry of change of residence has by him been given in writing. Such service of notice, shall be entered on the minutes of the Corporation; and the said minutes upon being read and approved at a subsequent meeting of the Board, shall be conclusive upon the question of service.

Article IX

Secretary

1st. The Secretary shall keep a record of the proceedings of the Board of Directors and of the Stockholders.

2nd. Shall keep the corporate seal and the book of blank certificates of stock, fill up and countersign all certificates, and make the corresponding entries in the margin of such book on such issuance; and he shall affix said seal to all papers requiring a seal.

3rd. Shall keep a Transfer Book, and a Stock Ledger in debit and credit form showing the number of shares issued to and transferred by any Stockholder, and the dates of such issuance and transfer.

4th. Shall keep proper Account Books, countersign all checks drawn upon the Treasurer and discharge such other duties as pertain to his office and as are prescribed by the Board of Directors.

5th. Shall serve all notices required either by law or by the By-Laws of the Company; and in case of his absence, inability, refusal or neglect so to do, then such notices may be served by any person thereunto directed by the President or Vice-President of the Company.

Article X

The Treasurer

The Treasurer shall receive and keep all the funds of the Corporation and pay them out only on the check of the President or either one of the Vice-Presidents and himself as treasurer, or else on the check of any two of the three officers to wit: President and first and second Vice-Presidents.

Article XI

Books and Papers

Such books and such papers as may be placed on file by vote of the Stockholders or Directors shall at all times in business hours, be subject to the inspection of the Board of Directors and of any Stockholder.

Article XII

Certificates of Stock

Certificates of stock shall be of such form and device as the Board of Directors may direct; and shall be signed by the President and countersigned by the Secretary, and express on its face its number, date of issuance, the number of shares for which, and the person to whom it is issued.

The Certificate Book shall contain a margin on which shall be entered the number, date, number of shares, and name of the person expressed in the corresponding certificate.

Article XIII

Transfer of Stock

Shares of the Corporation may be transferred at any time by the holders, or by power of attorney, or by their legal representatives, by endorsement on the certificate of Stock. But no transfer is valid until the surrender of the surrendered certificate and a receipt is given by the Secretary.

A surrendered certificate shall be cancelled before a new one is issued in lieu thereof, and the Secretary shall preserve the certificate so cancelled as a voucher. If, however, a certificate is lost or destroyed, the Board of Directors may order a new certificate issued as is by law required or permitted.

Article XIV

Meetings

The annual meeting of the Stockholders may be held in San Francisco, California on the first Monday in January, in each year and shall be called by a notice printed in one or more newspapers, published in the City and County of San Francisco as the Directors may direct, for at least once a week for two weeks last preceding the day of meeting.

No meeting of Stockholders shall transact business, unless a majority of stock is represented, except to adjourn from day to day, or until such time as may be deemed proper.

At such annual meeting, Directors for the ensuing year shall be elected by ballot, in the manner as provided in the Civil Code, to serve for one year, and until their successors are elected. If, however, for want of a quorum or other cause, a Stockholders' meeting shall not be held on the day above named, or should the Stockholders fail to complete their elections, or such other business as may be presented for their consideration, those present may adjourn from day to day, until the same shall be accomplished in the manner provided in the Civil Code.

Article XV

Voting

At all corporate meetings, each Stockholder, either in person or by proxy, shall be entitled to as many votes as he owns shares of stock, in the manner required by the Civil Code. Such proxy shall be in writing, and filed with the Secretary.

Article XVI

Amendments

The By-Laws may be altered or amended at any meeting of the Stockholders in the manner required by the Civil Code, or at a meeting of the Board of Directors at their discretion by a majority vote.

KNOW ALL MEN BY THESE PRESENTS

That we, the undersigned, being the holders and owners of three (3) shares of the Capital Stock of the

BRADFORD, KIMBALL & CO.

viz.: the issuance and delivery to us of said shares being subject to the approval of the Corporation Commissioner of the State of California hereby assent to the foregoing By-Laws, and adopt the same as the By-Laws of the said Corporation.

IN WITNESS WHEREOF, we have hereunto subscribed our names, this 5th day of February A.D., nineteen hundred and twenty

C. B. BRADFORD
JOHN SMITH
SHERMAN KIMBALL
} Stockholders.

KNOW ALL MEN BY THESE PRESENTS

That we, the undersigned, Directors and Secretary of the Corporation known as and called the BRADFORD, KIMBALL & CO. do hereby certify that the above and foregoing By-Laws were adopted as the By-Laws of said Corporation, on the 5th day of February A.D. nineteen hundred and twenty; and that the same do now constitute the By-Laws of said Corporation.

{SEAL OF THE CORPORATION}

C. B. BRADFORD
JOHN SMITH
SHERMAN KIMBALL
} Directors.

ROBT. G. CRAIG
Secretary of the BRADFORD, KIMBALL & CO.

APPENDIX B

TABLES OF FORMULAS AND VALUES

TABLE PAGE

I. Simple interest formulas 228

II. Compound interest formulas 229

III. Formulas for uniform series of payments, wherein 1 is one of given elements 231

IV. Formulas for uniform series of payments, in which payment is made at *end* of each k yr. interval 233

V. Formulas for uniform series of payments, in which payment is made at the *beginning* of each k yr. interval 235

VI. Formulas for uniform series of payments, in which payment is made at the *beginning of each and at the end of the last* k yr. interval 237

VII. Formulas for equivalent uniform annual Costs, Income, Yield, and Profit *on insert sheet facing* 298

(*A*) Compound amount of 1 for n periods (accumulation factor) 239

(*B*) Present worth of 1 due n periods hence (discount factor) 241

(*C*) Amount of uniform series (series accumulation factor) 243

(*D*) Present worth of uniform series (series discount factor) 245

(*E*) Annual payment in uniform series (backward spreading factor) . . 247

(*F*) Annual payment in uniform series (forward spreading factor) 249

(*G*) Days of the year 250

(*H*) Simple interest (365 da. year) 252

(*J*) Simple interest (360 da. year) 256

(*K*) Structure life table 259

TABLE I.—SIMPLE INTEREST FORMULAS

Given	Required	Formula	
Principal, P Rate, i Time (years), n	Interest, I Amount, S	$I = Pin$ (Tables H and J) $S = P(1 + in)$	(1) (2)
Rate, i Time (years), n Interest, I	Principal, P	$P = I/(in)$	(3)

TABLE I.—SIMPLE INTEREST FORMULAS (*Continued*)

Given	Required	Formula
Rate, i Time (years), n Amount, S	Principal, P	$P = S/(1 + in)$ (4)
Interest, I Principal, P Rate, i	Time (years), n	$n = I/(Pi)$ (5)

TABLE II.—COMPOUND INTEREST FORMULAS

Given	Required	Formula
Rate, i Interest periods in one year, h Time (years), n Principal, \$1	Amount, s_n Interest, I	$s_n = (1 + i/h)^{hn}$ (18a) (Table A) $I = s_n - 1$ (19a)
Rate, i Interest periods in one year, h Time (years), n Amount, \$1	Present worth, p_n	$p_n = \dfrac{1}{(1 + i/h)^{hn}}$ (20b) (Table B)
Interest periods in one year, h Time (years), n Principal, \$1 Amount, s_n	Rate, i	$i = h[(s_n)^{1/hn} - 1]$ (21)
Rate, i Interest periods in one year, h Principal, \$1 Amount, s_n	Time (years), n	$n = \dfrac{\log s_n}{h \log(1 + i/h)}$ (22)
Rate ("nominal"), i Interest periods in one year, h	"Effective" rate (compound interest on \$1 for one year), e	$e = (1 + i/h)^h - 1$ (15)
"Effective" rate, e Interest periods in one year, h	Rate ("nominal"), i	$i = h[(1 + e)^{1/h} - 1]$ (17)

USE OF TABLE III

(Table III need be used only in case the conditions of the problem are beyond the range of Tables C, D, E, and F.)

When one of the given sums—present worth, deposit, or amount—is other than \$1, solve the problem first on the assumption that it is \$1, and multiply the result by the number of dollars in the given sum.

Example.—What is the amount of the sinking fund in which \$1 is deposited at the end of each year for 20 years, if interest is at 6 per cent compounded semi-annually?

The formula for this case is (Eq. 31, Table III),

$$z_n = \frac{s^n - 1}{s - 1}.$$

Substituting for s its value in terms of i and h, we have

$$z_n = \frac{(1 + i/h)^{hn} - 1}{(1 + i/h)^h - 1}.$$

In this example, $i = 0.06$, $h = 2$, and $n = 20$. Therefore

$$z_n = \frac{(1 + 0.06/2)^{2\times 20} - 1}{(1 + 0.06/2)^2 - 1} = \frac{{}^{1}1.03^{40} - 1}{{}^{1}1.03^{2} - 1}$$

$$= \frac{3.262 - 1}{1.061 - 1}$$

$$= \$37. \quad \textit{Answer.}$$

---

[1] 1.03^{40} is evidently the compound amount of \$1 for 40 years at 3 per cent compounded annually; and 1.03^2 is the compound amount of \$1 for two years at 3 per cent compounded annually. Therefore the values of 1.03^{40} and 1.03^2 can be taken directly from Table A.

TABLE III.—FORMULAS FOR UNIFORM SERIES OF PAYMENTS

For cases in which one of the given elements is \$1, and the payment is made at the *end* of each year.

n = life of sinking fund, in years.
s = amount of \$1 for one year
$= (1 + i/h)^h$.
i = rate on deposits.
h = number of interest periods in one year.

Given	Required	Formula
Deposit, \$1	Amount, z_n	$z_n = \frac{s^n - 1}{s - 1}$. (Table C) (31)
Deposit, \$1	Present worth, w_n	$w_n = \frac{s^n - 1}{s - 1} \cdot \frac{1}{s^n}$. (Table D) (32)
Deposit, \$1 Life (years), $n = \infty$	Present worth, w_∞	$w_\infty = \frac{1}{s - 1}$ (32b)
Deposit, \$1 Present worth of m-year sinking fund, w_m Present worth of $(m+n)$-year sinking fund, w_{m+n}	Present worth of n-year sinking fund deferred m years, w'_n	$w'_n = w_{m+n} - w_m$ (32c) $= \frac{s^n - 1}{s - 1} \cdot \frac{1}{s^{n+m}}$ (32d)
Amount, \$1	Deposit, d_n	$d_n = \frac{s - 1}{s^n - 1}$. (Table E) (33)
Present worth, \$1	Deposit, a_n	$a_n = \frac{(s - 1)s^n}{s^n - 1}$. (Table F) (34)
Present worth, \$1 Life (years), $n = \infty$	Deposit, a_∞	$a_\infty = s - 1$ (34b)
Deposit, \$1 Amount, z_n	Life (years), n	$n = \frac{\log [z_n(s - 1) + 1]}{\log s}$ (34c)
Deposit, \$1 Present worth, w_n	Life (years), n	$n = \frac{\log \left[\frac{1}{1 - (s - 1)w_n}\right]}{\log s}$ (34d)
Amount, \$1 Deposit, d_n	Life (years), n	$n = \frac{\log \left[\frac{s - 1}{d_n} + 1\right]}{\log s}$ (34e)
Present worth, \$1 Deposit, a_n	Life (years), n	$n = \frac{\log \left[\frac{a_n}{a_n - (s - 1)}\right]}{\log s}$ (34f)

USE OF TABLE IV

The formulas of Table IV assume that the interest period is not longer than the deposit interval; that is, the formulas do not hold when the deposit interval is shorter than the interest period. For example, Eq. 27 will give the amount of the sinking fund in which D is deposited at the end of each half-year if interest is compounded semi-annually, or quarterly, or oftener, but will *not* give the amount if interest is compounded annually.

This table need be resorted to only when the conditions of the problem are beyond the range of Tables C, D, E, and F.

Example.—If at the end of each half-year for five years, we deposit $100 in a savings bank which pays interest at the rate of 4 per cent compounded quarterly, what will be the accumulated amount at the end of the fifth year?

The formula to be used in this case is (Eq. 27, Table IV),

$$Z = D\frac{s^n - 1}{s^k - 1}, \text{ where } s = (1 + i/h)^h, \; s^n = (1 + i/h)^{hn},$$
$$s^k = (1 + i/h)^{hk}.$$

Substituting, we have

$$Z = D\frac{(1 + i/h)^{hn} - 1}{(1 + i/h)^{hk} - 1}.$$

In this example, $D = 100$, $h = 4$, $n = 5$, and $k = 1/2$. Therefore

$$Z = 100\frac{(1 + 0.04/4)^{4\times 5} - 1}{(1 + 0.04/4)^{4\times ½} - 1} = 100\,\frac{1.01^{20} - 1}{1.01^2 - 1} = 100\,\frac{1.220 - 1}{1.020 - 1}$$
$$= \$1100\pm. \text{ *Answer.*}$$

Note.—1.01^{20} is the amount of $1 for 20 years at 1 per cent compounded annually, and 1.01^2 is the amount of $1 for two years at 1 per cent compounded annually. As Table A does not give amounts for the rate of 1 per cent, the values of 1.01^{20} and 1.01^2 were computed by logarithms.

TABLE IV.—FORMULAS FOR UNIFORM SERIES OF PAYMENTS

In which payment is made at the *end* of each k-year period.

n = life of sinking fund.
k = number of years in one deposit-interval
= or $> 1/h$.
s = amount of \$1 for one year
$= (1 + i/h)^h$.
i = rate.
h = number of interest periods in one year.

Given	Required	Formula
Deposit, D	Amount, Z	$Z = D\dfrac{s^n-1}{s^k-1}$ (27)
Deposit, D	Present worth, W	$W = D\dfrac{s^n-1}{s^k-1}\cdot\dfrac{1}{s^n}$ (28)
Amount, Z	Deposit, D	$D = Z\dfrac{s^k-1}{s^n-1}$ (29)
Present worth, W	Deposit, D	$D = W\dfrac{s^k-1}{s^n-1}\cdot s^n$ (30)
Deposit, D Life, n years Deferred m years	Present worth, W'	$W' = D\dfrac{s^n-1}{s^k-1}\cdot\dfrac{1}{s^{m+n}}$ (30b)
Amount, Z Deposit, D	Life, n	$n = \dfrac{\log[(Z/D)(s^k-1)+1]}{\log s}$ (30c)
Present worth, W Deposit, D	Life, n	$n = \dfrac{\log\left[\dfrac{1}{1-(W/D)(s^k-1)}\right]}{\log s}$ (30d)
Deposit, D Life, $n = \infty$	Present worth, W	$W = D\dfrac{1}{s^k-1}$ (30e)
Present worth, W Life, $n = \infty$	Deposit, D	$D = W(s^k-1)$ (30f)
Deposit, D Life, $n = \infty$ Deferred m years	Present worth, W'	$W' = D\dfrac{1}{s^k-1}\cdot\dfrac{1}{s^m}$ (30g)

USE OF TABLE V

The formulas of Table V assume that the interest period is not longer than the deposit interval; that is, the formulas do not hold when the deposit interval is shorter than the interest period. For example, Eq. 30h will give the amount of the sinking fund in which D is deposited at the end of each half-year if interest is compounded semi-annually, or quarterly, or oftener, but will *not* give the amount if interest is compounded annually.

Example.—If at the *beginning* of each half-year for five years, we deposit $100 in a savings bank which pays interest at the rate of 4 per cent compounded quarterly, what will be the accumulated amount at the end of the fifth year?

For this case the formula is (Eq. 30, Table V),

$Z = D\dfrac{s^n - 1}{s^k - 1}s^k$, where $s = (1 + i/h)^h$, $s^n = (1 + i/h)^{hn}$, and $s^k = (1 + i/h)^{hk}$. Substituting, we have

$$Z = D\frac{(1 + i/h)^{hn} - 1}{(1 + i/h)^{hk} - 1}(1 + i/h)^{hk}.$$

In this example, $D = 100$, $h = 4$, $n = 5$, and $k = 1/2$. Therefore

$$Z = 100\frac{(1+0.04/4)^{4\times 5} - 1}{(1+0.04/4)^{4\times ½} - 1}(1+0.04/4)^{4\times ½} = 100\frac{1.01^{20} - 1}{1.01^2 - 1}1.01^2$$

$$= 100\frac{1.220 - 1}{1.020 - 1}1.020$$

$$= \$1122\pm. \quad \textit{Answer.}$$

Note.—1.01^{20} is the amount of $1 for 20 years at 1 per cent compounded annually, and 1.01^2 is the amount of $1 for two years at 1 per cent compounded annually. Since Table A does not give amounts for the rate of 1 per cent, the values of 1.01^{20} and 1.01^2 were computed by logarithms.

In which payment is made at the *beginning* of each k-year period.

TABLE V.—FORMULAS FOR UNIFORM SERIES OF PAYMENTS

n = life of sinking fund.
k = number of years in one deposit interval
= or $> 1/h$.
s = amount of \$1 for one year
$= (1 + i/h)^h$.
i = rate.
h = number of interest periods in one year.

Given	Required	Formula
Deposit, D	Amount, Z	$Z = D \frac{s^n - 1}{s^k - 1} s^k$ (30h)
Deposit, D	Present worth, W	$W = D \frac{s^n - 1}{s^k - 1} \cdot \frac{s^k}{s^n}$ (30i)
Amount, Z	Deposit, D	$D = Z \frac{s^k - 1}{s^n - 1} \cdot \frac{1}{s^k}$ (30j)
Present worth, W	Deposit, D	$D = W \frac{s^k - 1}{s^n - 1} \cdot s^{n-k}$ (30k)
Deposit, D Life, n years Deferred m years	Present worth, W'	$W' = D \frac{s^n - 1}{s^k - 1} \cdot \frac{s^k}{s^{n+m}}$ (30l)
Amount, Z Deposit, D	Life, n	$n = \frac{\log\left[(Z/D)\frac{s^k - 1}{s^k} + 1\right]}{\log s}$ (30m)
Present worth, W Deposit, D	Life, n	$n = \frac{\log\left[\frac{1}{1 - (W/D)\left(1 - \frac{1}{s^k}\right)}\right]}{\log s}$ (30n)
Deposit, D Life, $n = \infty$	Present worth, W	$W = D \frac{s^k}{s^k - 1}$ (30o)
Present worth, W Life, $n = \infty$	Deposit, D	$D = W \frac{s^k - 1}{s^k}$ (30p)
Deposit, D Life, $n = \infty$ Deferred m years	Present worth, W'	$W' = D \frac{s^k}{s^k - 1} \cdot \frac{1}{s^m}$ (30q)

USE OF TABLE VI

The formulas of Table VI assume that the interest period is not longer than the deposit interval; that is, the formulas do not hold when the deposit interval is shorter than the interest period. For example, Eq. 30r will give the amount of the sinking fund in which D is deposited at the end of each half-year if interest is compounded semi-annually, or quarterly, or oftener, but will *not* give the amount if interest is compounded annually.

Example.—If at the *beginning of each* half-year and at the *end of the last* half-year of a five-year period, we deposit $100 in a savings bank which pays interest at the rate of 4 per cent compounded quarterly, what will be the accumulated amount at the end of the fifth year?

For this example we use Eq. 30r of Table VI:

$Z = D\dfrac{s^{n+k} - 1}{s^k - 1}$, where $s = (1 + i/h)^h$, $s^{n+k} = (1 + i/h)^{h(n+k)}$ and $s^k = (1 + i/h)^{hk}$. Substituting, we have

$$Z = D\frac{(1 + i/h)^{h(n+k)} - 1}{(1 + i/h)^{hk} - 1}.$$

In this example, $D = 100$, $h = 4$, $n = 5$, and $k = 1/2$. Therefore

$$Z = 100\frac{(1 + 0.04/4)^{(5+\frac{1}{2})4} - 1}{(1 + 0.04/4)^{4\times\frac{1}{2}} - 1} = 100\frac{1.01^{22} - 1}{1.01^2 - 1}$$

$$= 100\frac{1.245 - 1}{1.020 - 1}$$

$$= \$1225\pm.$$ *Answer.*

Note.—1.01^{22} is the amount of $1 for 22 years at 1 per cent. compounded annually, and 1.01^2 is the amount of $1 for two years at 1 per cent compounded annually. Since Table A does not give compound amount for the rate 1 per cent, the values of 1.01^{22} and 1.01^2 were computed by logarithms.

TABLE VI.—FORMULAS FOR UNIFORM SERIES OF PAYMENTS

In which payment is made at *beginning of each* and at the *end of the last* k-year period.

n = life of sinking fund.
k = number of years in one deposit interval
= or $> 1/h$.
s = amount of \$1 for one year
= $(1 + i/h)^h$.
i = rate.
h = number of interest periods in one year.

Given	Required	Formulas
Deposit, D	Amount, Z	$Z = D\frac{s^{n+k} - 1}{s^k - 1}$ (30r)
Deposit, D	Present worth, W	$W = D\frac{s^{n+k} - 1}{s^k - 1} \cdot \frac{1}{s^n}$ (30s)
Amount, Z	Deposit, D	$D = Z\frac{s^k - 1}{s^{n+k} - 1}$ (30t)
Present worth, W	Deposit, D	$D = W\frac{s^k - 1}{s^{n+k} - 1} \cdot s^n$ (30u)
Deposit, D Life, n years Deferred m years	Present worth, W'	$W' = D\frac{s^{n+k} - 1}{s^k - 1} \cdot \frac{1}{s^{n+m}}$ (30v)
Amount, Z Deposit, D	Life, n	$n = \frac{\log\left[(Z/D)\frac{s^k - 1}{s^k} + \frac{1}{s^k}\right]}{\log s}$ (30w)
Present worth, W Deposit, D	Life, n	$n = \frac{\log\left[\frac{1}{s^k - (W/D)(s^k - 1)}\right]}{\log s}$ (30x)
Deposit, D Life, $n = \infty$	Present worth, W	$W = D\frac{s^k}{s^k - 1}$ (30y)
Present worth, W Life, $n = \infty$	Deposit, D	$D = W\frac{s^k - 1}{s^k}$ (30z)
Deposit, D Life, $n = \infty$ Deferred m years	Present worth, W'	$W' = D\frac{s^k}{s^k - 1} \cdot \frac{1}{s^m}$ (30zz)

USE OF TABLE A

Table A gives the value of s^n of Eq. 18a, Table II, for given values of i, h, and n.

When interest is compounded annually.

Example 1.—What is the amount of $1 for 15 years at 7 per cent compounded annually? Under "7 per cent" and opposite "15" of Table A, we find $2.759, which is the required amount.

Example 2.—What is the amount of $147.60 for 15 years at 7 per cent compounded annually? As in the example above, we find the amount of $1 for the given conditions to be $2.759; and multiplying this by the principal, we obtain 147.60 × 2.759 = $407.22±. *Answer.*

When interest is compounded semi-annually, or quarterly, etc.

Example 3.—What is the amount of $1 for 13 years at 5 per cent compounded semi-annually? Here there are 2 interest periods in the year; therefore $h = 2$, $i/h = 0.05/2 = 0.025$, and $nh = 13 \times 2 = 26$. Opposite "26," under "21/2 per cent," find, in Table A, $1.900 which is the amount required.

When the number of interest periods is not given in the Table.

Example 4.—What is the amount of $1 for 82 years at 6 per cent compounded annually? In Table A, opposite 41 (which is 1/2 of 82) and under 6 per cent we find 10.90. The amount for 82 years is $10.90^2 = \$118.81 \pm$. *Answer.* (Explanation: $s^n = (s^{n/2})^2 = (s^{n/3})^3 = (s^{n/4})^4$, etc.)

Example 5.—What is the amount of $1 for 40 years at 8 per cent compounded quarterly? The number of interest periods is $hn = 4 \times 40 = 160$, which is beyond the reach of the Table. $i/h = 0.08/4 = 0.02$. In Table A, opposite "40" and under "2 per cent," find 2.208. The required amount is

$$2.208^4 = \$22.69. \quad \textit{Answer.}$$

Table A.—Compound Amount (s^n) of 1

For *n* years when interest is compounded annually at

n	2%	2–½%	3%	3–½%	4%	4–½%	5%	6%	7%	8%
1	1.020	1.025	1.030	1.035	1.040	1.045	1.050	1.060	1.070	1.080
2	1.040	1.051	1.061	1.071	1.082	1.092	1.103	1.124	1.145	1.166
3	1.061	1.077	1.092	1.109	1.125	1.141	1.158	1.191	1.225	1.260
4	1.082	1.104	1.126	1.148	1.170	1.193	1.216	1.262	1.311	1.360
5	1.104	1.131	1.159	1.188	1.217	1.246	1.276	1.338	1.403	1.469
6	1.126	1.160	1.194	1.229	1.265	1.302	1.340	1.419	1.501	1.587
7	1.149	1.189	1.230	1.272	1.316	1.361	1.407	1.504	1.606	1.714
8	1.172	1.218	1.267	1.317	1.369	1.422	1.477	1.594	1.718	1.851
9	1.195	1.249	1.305	1.363	1.423	1.486	1.551	1.689	1.838	1.999
10	1.219	1.280	1.344	1.411	1.480	1.553	1.629	1.791	1.967	2.159
11	1.243	1.312	1.384	1.460	1.539	1.623	1.710	1.898	2.105	2.332
12	1.268	1.345	1.426	1.511	1.601	1.696	1.796	2.012	2.252	2.518
13	1.294	1.379	1.469	1.564	1.665	1.772	1.886	2.133	2.410	2.720
14	1.319	1.413	1.513	1.619	1.732	1.852	1.980	2.261	2.579	2.937
15	1.346	1.448	1.558	1.675	1.801	1.935	2.079	2.397	2.759	3.172
16	1.373	1.485	1.605	1.734	1.873	2.022	2.183	2.540	2.952	3.426
17	1.400	1.522	1.653	1.795	1.948	2.113	2.292	2.693	3.159	3.700
18	1.428	1.560	1.702	1.857	2.026	2.208	2.407	2.854	3.380	3.996
19	1.457	1.599	1.754	1.923	2.107	2.308	2.527	3.026	3.617	4.316
20	1.486	1.639	1.806	1.990	2.191	2.412	2.653	3.207	3.870	4.661
21	1.516	1.680	1.860	2.059	2.279	2.520	2.786	3.400	4.141	5.034
22	1.546	1.722	1.916	2.132	2.370	2.634	2.925	3.604	4.430	5.437
23	1.577	1.765	1.974	2.206	2.465	2.752	3.072	3.820	4.741	5.871
24	1.608	1.809	2.033	2.283	2.563	2.876	3.225	4.049	5.072	6.341
25	1.641	1.854	2.094	2.363	2.666	3.005	3.386	4.292	5.427	6.848
26	1.673	1.900	2.157	2.446	2.772	3.141	3.556	4.549	5.807	7.396
27	1.707	1.948	2.221	2.532	2.883	3.282	3.733	4.822	6.214	7.988
28	1.741	1.996	2.288	2.620	2.999	3.430	3.920	5.112	6.649	8.627
29	1.776	2.046	2.357	2.712	3.119	3.584	4.116	5.418	7.114	9.317
30	1.811	2.098	2.427	2.807	3.243	3.745	4.322	5.743	7.612	10.06
31	1.848	2.150	2.500	2.905	3.373	3.914	4.538	6.088	8.145	10.87
32	1.885	2.204	2.575	3.007	3.508	4.090	4.765	6.453	8.715	11.74
33	1.922	2.259	2.652	3.112	3.648	4.274	5.003	6.841	9.325	12.68
34	1.961	2.315	2.732	3.221	3.794	4.466	5.253	7.251	9.978	13.69
35	2.000	2.373	2.814	3.334	3.946	4.667	5.516	7.686	10.68	14.79
36	2.040	2.433	2.898	3.450	4.104	4.877	5.792	8.147	11.42	15.97
37	2.081	2.493	2.985	3.571	4.268	5.097	6.081	8.636	12.22	17.25
38	2.122	2.556	3.075	3.696	4.439	5.326	6.385	9.154	13.08	18.63
39	2.165	2.620	3.167	3.825	4.616	5.566	6.705	9.704	13.99	20.12
40	2.208	2.685	3.262	3.959	4.801	5.816	7.040	10.29	14.97	21.72
41	2.252	2.752	3.360	4.098	4.993	6.078	7.392	10.90	16.02	23.46
42	2.297	2.821	3.461	4.241	5.193	6.352	7.762	11.56	17.14	25.34
43	2.343	2.892	3.565	4.390	5.400	6.637	8.150	12.25	18.34	27.37
44	2.390	2.964	3.671	4.543	5.617	6.936	8.557	12.99	19.63	29.56
45	2.438	3.038	3.782	4.702	5.841	7.248	8.985	13.76	21.00	31.92
46	2.487	3.114	3.895	4.867	6.075	7.574	9.434	14.59	22.47	34.47
47	2.536	3.192	4.012	5.037	6.318	7.915	9.906	15.47	24.05	37.23
48	2.587	3.271	4.132	5.214	6.571	8.271	10.40	16.39	25.73	40.21
49	2.639	3.353	4.256	5.396	6.833	8.644	10.92	17.38	27.53	43.43
50	2.692	3.437	4.384	5.585	7.107	9.033	11.47	18.42	29.46	46.90
55	2.972	3.889	5.082	6.633	8.646	11.26	14.64	24.65	41.32	68.91
60	3.281	4.400	5.892	7.878	10.52	14.03	18.68	32.99	57.95	101.3
65	3.623	4.978	6.830	9.357	12.80	17.48	23.84	44.14	81.27	148.8
70	4.000	5.632	7.918	11.11	15.57	21.78	30.43	59.08	114.0	218.6
75	4.416	6.372	9.179	13.20	18.95	27.15	38.83	79.06	159.9	321.2
80	4.875	7.210	10.64	15.68	23.05	33.83	49.56	105.8	224.2	472.0
85	5.383	8.157	12.34	18.62	28.04	42.16	63.25	141.6	314.5	693.5
90	5.943	9.229	14.30	22.11	34.12	52.54	80.73	189.5	441.1	1019.0
95	6.562	10.44	16.58	26.26	41.51	65.47	103.0	253.5	618.7	1497.0
100	7.245	11.81	19.22	31.19	50.50	81.59	131.5	339.3	867.7	2200.

USE OF TABLE B

Table B gives the value of p^n of Eq. 20b, Table II, for given values of i, h, and n.

When interest is compounded annually.

Example 1.—What is the present worth of $1 due 17 years hence, = $0.0929+. (Explanation: $p^n = (p^{n/2})^2 = (p^{n/3})^3$, and so on.) interest being at 5 per cent compounded annually? Opposite "17" and under "5 per cent" of Table B, we find $0.4363, which is the required present worth.

Example 2.—What is the present worth of $100 under the conditions above? For the given conditions we find, as in Example 1, the present worth of $1 to be $0.4363; and multiply this by 100, obtaining the required present worth, $43.63.

When interest is compounded semi-annually, or quarterly, etc.

Example 3.—What is the present worth of $1 due 10 years hence, interest being at 8 per cent compounded quarterly? Here $h = 4$, and $n = 10$; hence the number of interest periods is $hn = 40$. $i/h = 0.08/4 = 0.02$. Opposite "40" and under "2 per cent" of Table B, we find $0.4529, which is the required present worth.

When the number of interest periods is not given in the table.

Example 4.—What is the present worth of $1 due 60 years hence, with interest at 4 per cent compounded semi-annually? Here the number of interest periods is $hn = 2 \times 60 = 120$, which is not in the table. 60, which is one-half of 120, is in the table. $i/h = 0.04/2 = 0.02$. Opposite "60" and under "2 per cent," in Table B, we find 0.3048. The required present worth is 0.3048^2.

TABLE B.—PRESENT WORTH (p^n) OF 1

Due *n* years hence, when interest is compounded annually at

n	2%	2-½%	3%	3-½%	4%	4-½%	5%	6%	7%	8%
1	0.9804	0.9756	0.9709	0.9662	0.9615	0.9569	0.9524	0.9434	0.9346	0.9259
2	0.9612	0.9518	0.9426	0.9335	0.9246	0.9157	0.9070	0.8900	0.8734	0.8573
3	0.9423	0.9286	0.9151	0.9019	0.8890	0.8763	0.8638	0.8396	0.8163	0.7938
4	0.9238	0.9060	0.8885	0.8714	0.8548	0.8386	0.8227	0.7921	0.7629	0.7350
5	0.9057	0.8839	0.8626	0.8420	0.8219	0.8025	0.7835	0.7473	0.7130	0.6806
6	0.8880	0.8623	0.8375	0.8135	0.7903	0.7679	0.7462	0.7050	0.6663	0.6302
7	0.8706	0.8413	0.8131	0.7860	0.7599	0.7348	0.7107	0.6651	0.6227	0.5835
8	0.8535	0.8207	0.7894	0.7594	0.7307	0.7032	0.6768	0.6274	0.5820	0.5403
9	0.8368	0.8007	0.7664	0.7337	0.7026	0.6729	0.6446	0.5919	0.5439	0.5002
10	0.8203	0.7812	0.7441	0.7089	0.6756	0.6439	0.6139	0.5584	0.5083	0.4632
11	0.8043	0.7621	0.7224	0.6849	0.6496	0.6162	0.5847	0.5268	0.4751	0.4289
12	0.7885	0.7436	0.7014	0.6618	0.6246	0.5897	0.5568	0.4970	0.4440	0.3971
13	0.7730	0.7254	0.6810	0.6394	0.6006	0.5643	0.5303	0.4688	0.4150	0.3677
14	0.7579	0.7077	0.6611	0.6178	0.5775	0.5400	0.5051	0.4423	0.3878	0.3405
15	0.7430	0.6905	0.6419	0.5969	0.5553	0.5167	0.4810	0.4173	0.3624	0.3152
16	0.7284	0.6736	0.6232	0.5767	0.5339	0.4945	0.4581	0.3936	0.3387	0.2919
17	0.7142	0.6572	0.6050	0.5572	0.5134	0.4732	0.4363	0.3714	0.3166	0.2703
18	0.7002	0.6412	0.5874	0.5384	0.4936	0.4528	0.4155	0.3503	0.2959	0.2502
19	0.6864	0.6255	0.5703	0.5202	0.4746	0.4333	0.3957	0.3305	0.2765	0.2317
20	0.6730	0.6103	0.5537	0.5026	0.4564	0.4146	0.3769	0.3118	0.2584	0.2145
21	0.6598	0.5954	0.5375	0.4856	0.4388	0.3968	0.3589	0.2942	0.2415	0.1987
22	0.6468	0.5809	0.5219	0.4692	0.4220	0.3797	0.3418	0.2775	0.2257	0.1839
23	0.6342	0.5667	0.5067	0.4533	0.4057	0.3634	0.3256	0.2618	0.2109	0.1703
24	0.6217	0.5529	0.4919	0.4380	0.3901	0.3477	0.3101	0.2470	0.1971	0.1577
25	0.6095	0.5394	0.4776	0.4231	0.3751	0.3327	0.2953	0.2330	0.1842	0.1460
26	0.5976	0.5262	0.4637	0.4088	0.3607	0.3184	0.2812	0.2198	0.1722	0.1352
27	0.5859	0.5134	0.4502	0.3950	0.3468	0.3047	0.2678	0.2074	0.1609	0.1252
28	0.5744	0.5009	0.4371	0.3817	0.3335	0.2916	0.2551	0.1956	0.1504	0.1159
29	0.5631	0.4887	0.4243	0.3687	0.3207	0.2790	0.2429	0.1846	0.1406	0.1073
30	0.5521	0.4767	0.4120	0.3563	0.3083	0.2670	0.2314	0.1741	0.1314	0.0994
31	0.5412	0.4651	0.4000	0.3442	0.2965	0.2555	0.2204	0.1643	0.1228	0.0920
32	0.5306	0.4538	0.3883	0.3326	0.2851	0.2445	0.2099	0.1550	0.1147	0.0852
33	0.5202	0.4427	0.3770	0.3213	0.2741	0.2340	0.1999	0.1462	0.1072	0.0789
34	0.5100	0.4319	0.3660	0.3105	0.2636	0.2239	0.1904	0.1379	0.1002	0.0730
35	0.5000	0.4214	0.3554	0.3000	0.2534	0.2143	0.1813	0.1301	0.0937	0.0676
36	0.4902	0.4111	0.3450	0.2898	0.2437	0.2050	0.1727	0.1227	0.0875	0.0626
37	0.4806	0.4011	0.3350	0.2800	0.2343	0.1962	0.1644	0.1158	0.0818	0.0580
38	0.4712	0.3913	0.3252	0.2706	0.2253	0.1878	0.1566	0.1092	0.0765	0.0537
39	0.4619	0.3817	0.3158	0.2614	0.2166	0.1797	0.1491	0.1031	0.0715	0.0497
40	0.4529	0.3724	0.3066	0.2526	0.2083	0.1719	0.1420	0.0972	0.0668	0.0460
41	0.4440	0.3633	0.2976	0.2440	0.2003	0.1645	0.1353	0.0917	0.0624	0.0426
42	0.4353	0.3545	0.2890	0.2358	0.1926	0.1574	0.1288	0.0865	0.0583	0.0395
43	0.4268	0.3458	0.2805	0.2278	0.1852	0.1507	0.1227	0.0816	0.0545	0.0365
44	0.4184	0.3374	0.2724	0.2201	0.1780	0.1442	0.1169	0.0770	0.0509	0.0338
45	0.4102	0.3292	0.2644	0.2127	0.1712	0.1380	0.1113	0.0727	0.0476	0.0313
46	0.4022	0.3211	0.2567	0.2055	0.1646	0.1320	0.1060	0.0685	0.0445	0.0290
47	0.3943	0.3133	0.2493	0.1985	0.1583	0.1263	0.1009	0.0647	0.0416	0.0269
48	0.3865	0.3057	0.2420	0.1918	0.1522	0.1209	0.0961	0.0610	0.0389	0.0249
49	0.3790	0.2982	0.2350	0.1853	0.1463	0.1157	0.0916	0.0575	0.0363	0.0230
50	0.3715	0.2909	0.2281	0.1791	0.1407	0.1107	0.0872	0.0543	0.0339	0.0213
55	0.3365	0.2572	0.1968	0.1508	0.1157	0.0888	0.0683	0.0406	0.0242	0.0145
60	0.3048	0.2273	0.1697	0.1269	0.0951	0.0713	0.0535	0.0303	0.0173	0.0099
65	0.2761	0.2009	0.1464	0.1069	0.0781	0.0572	0.0419	0.0227	0.0123	0.0067
70	0.2500	0.1776	0.1263	0.0900	0.0642	0.0459	0.0329	0.0169	0.0088	0.0046
75	0.2265	0.1569	0.1089	0.0758	0.0528	0.0368	0.0258	0.0127	0.0063	0.0031
80	0.2051	0.1387	0.0940	0.0638	0.0434	0.0296	0.0202	0.0095	0.0045	0.0021
85	0.1858	0.1226	0.0811	0.0537	0.0357	0.0237	0.0158	0.0071	0.0032	0.0014
90	0.1683	0.1084	0.0699	0.0452	0.0293	0.0190	0.0124	0.0053	0.0023	0.0010
95	0.1524	0.0958	0.0603	0.0381	0.0241	0.0153	0.0097	0.0039	0.0016	0.0007
100	0.1380	0.0846	0.0520	0.0321	0.0198	0.0123	0.0076	0.0029	0.0012	0.0005

USE OF TABLE C

Table C gives the value of z_n of Eq. 31, Table III, for given values of i, h, and n.

When the deposit is made at the end of each year and interest is compounded annually.

Example 1.—At the end of each year for 20 years we deposit \$1 in a savings bank which pays 4 per cent compounded annually. What is the amount of the sinking fund thus created? In Table C, opposite "20" and under "4 per cent," we find \$29.78, which is the required amount.

Example 2.—If the deposit were \$100, instead of \$1, in the foregoing example, the required amount would be $100 \times 29.78 = \$2978 \pm$.

When the deposit is made at the end of each half-year and interest is compounded semi-annually.

Example 3.—If we deposit \$1 at the end of each half-year for 10 years, and interest on deposits is 8 per cent compounded semi-annually, what is the amount of the series of deposits? $i/h = 0.08/2 = 0.04$; and $hn = 2 \times 10 = 20$. In Table C, opposite "20" and under "4 per cent," we find \$29.78, which is the required amount.

When the deposit is made at the end of each interest period, whether the interest period be one-half year, or one-fourth year, or one-sixth year, find the required amount in Table C, opposite hn and under $100i/h$ per cent.

TABLE C.—AMOUNT OF SERIES (z^n) OR SERIES ACCUMULATION FACTOR

Amount of n yr. series of year-end payments of 1 each, with interest compounded annually at ——

n	2%	2-½%	3%	3-½%	4%	4-½%	5%	6%	7%	8%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.020	2.025	2.030	2.035	2.040	2.045	2.050	2.060	2.070	2.080
3	3.060	3.076	3.091	3.106	3.122	3.137	3.153	3.184	3.215	3.246
4	4.122	4.153	4.184	4.215	4.246	4.278	4.310	4.375	4.440	4.506
5	5.204	5.256	5.309	5.362	5.416	5.471	5.526	5.637	5.751	5.867
6	6.308	6.388	6.468	6.550	6.633	6.717	6.802	6.975	7.153	7.336
7	7.434	7.547	7.662	7.779	7.898	8.019	8.142	8.394	8.654	8.923
8	8.583	8.736	8.892	9.052	9.214	9.380	9.549	9.897	10.26	10.64
9	9.755	9.955	10.16	10.37	10.58	10.80	11.03	11.49	11.98	12.49
10	10.95	11.20	11.46	11.73	12.01	12.29	12.58	13.18	13.82	14.49
11	12.17	12.48	12.81	13.14	13.49	13.84	14.21	14.97	15.78	16.65
12	13.41	13.80	14.19	14.60	15.03	15.46	15.92	16.87	17.89	18.98
13	14.68	15.14	15.62	16.11	16.63	17.16	17.71	18.88	20.14	21.50
14	15.97	16.52	17.09	17.68	18.29	18.93	19.60	21.02	22.55	24.21
15	17.29	17.93	18.60	19.30	20.02	20.78	21.58	23.28	25.13	27.15
16	18.64	19.38	20.16	20.97	21.82	22.72	23.66	25.67	27.89	30.32
17	20.01	20.86	21.76	22.71	23.70	24.74	25.84	28.21	30.84	33.75
18	21.41	22.39	23.41	24.50	25.65	26.86	28.13	30.91	34.00	37.45
19	22.84	23.95	25.12	26.36	27.67	29.06	30.54	33.76	37.38	41.45
20	24.30	25.54	26.87	28.28	29.78	31.37	33.07	36.79	41.00	45.76
21	25.78	27.18	28.68	30.27	31.97	33.78	35.72	39.99	44.87	50.42
22	27.30	28.86	30.54	32.33	34.25	36.30	38.51	43.39	49.01	55.46
23	28.84	30.58	32.45	34.46	36.62	38.94	41.43	47.00	53.44	60.89
24	30.42	32.35	34.43	36.67	39.08	41.69	44.50	50.82	58.18	66.76
25	32.03	34.16	36.46	38.95	41.65	44.57	47.73	54.86	63.25	73.11
26	33.67	36.01	38.55	41.31	44.31	47.57	51.11	59.16	68.68	79.95
27	35.34	37.91	40.71	43.76	47.08	50.71	54.67	63.71	74.48	87.35
28	37.05	39.86	42.93	46.29	49.97	53.99	58.40	68.53	80.70	95.34
29	38.79	41.86	45.22	48.91	52.97	57.42	62.32	73.64	87.35	104.0
30	40.57	43.90	47.58	51.62	56.08	61.01	66.44	79.06	94.46	113.3
31	42.38	46.00	50.00	54.43	59.33	64.75	70.76	84.80	102.1	123.3
32	44.23	48.15	52.50	57.33	62.70	68.67	75.30	90.89	110.2	134.2
33	46.11	50.35	55.08	60.34	66.21	72.76	80.06	97.34	118.9	146.0
34	48.03	52.61	57.73	63.45	69.86	77.03	85.07	104.2	128.3	158.6
35	49.99	54.93	60.46	66.67	73.65	81.50	90.32	111.4	138.2	172.3
36	51.99	57.30	63.28	70.01	77.60	86.16	95.84	119.1	148.9	187.1
37	54.03	59.73	66.17	73.46	81.70	91.04	101.6	127.3	160.3	203.1
38	56.11	62.23	69.16	77.03	85.97	96.14	107.7	135.9	172.6	220.3
39	58.24	64.78	72.23	80.72	90.41	101.5	114.1	145.1	185.6	238.9
40	60.40	67.40	75.40	84.55	95.03	107.0	120.8	154.8	199.6	259.1
41	62.61	70.09	78.66	88.51	99.83	112.8	127.8	165.0	214.6	280.8
42	64.86	72.84	82.02	92.61	104.8	118.9	135.2	176.0	230.6	304.2
43	67.16	75.66	85.48	96.85	110.0	125.3	143.0	187.5	247.8	329.6
44	69.50	78.55	89.05	101.2	115.4	131.9	151.1	199.8	266.1	356.9
45	71.89	81.52	92.72	105.8	121.0	138.8	159.7	212.7	285.7	386.5
46	74.33	84.55	96.50	110.5	126.9	146.1	168.7	226.5	306.8	418.4
47	76.82	87.67	100.4	115.4	132.9	153.7	178.1	241.1	329.2	452.9
48	79.35	90.86	104.4	120.4	139.3	161.6	188.0	256.6	353.3	490.1
49	81.94	94.13	108.5	125.6	145.8	169.9	198.4	273.0	379.0	530.3
50	84.58	97.48	112.8	131.0	152.7	178.5	209.3	290.3	406.5	573.8
55	98.59	115.6	136.1	160.9	191.2	227.9	272.7	394.2	575.9	848.9
60	114.1	136.0	163.1	196.5	238.0	289.5	353.6	533.1	813.5	1253.
65	131.1	159.1	194.3	238.8	295.0	366.2	456.8	719.1	1147.	1847.
70	150.0	185.3	230.6	288.9	364.3	461.9	588.5	967.9	1614.	2720.
75	170.8	214.9	272.6	348.5	448.6	581.0	756.7	1301.	2270.	4003.
80	193.8	248.4	321.4	419.3	551.2	729.6	971.2	1747.	3189.	5887.
85	219.1	286.3	377.9	503.4	676.1	914.6	1245.	2343.	4479.	8656.
90	247.2	329.2	443.3	603.2	828.	1145.	1595.	3141.	6287.	12724.
95	278.1	377.7	519.3	721.8	1013.	1433.	2041.	4209.	8824.	18702.
100	312.2	432.5	607.3	862.6	1238.	1791.	2610.	5638.	12382.	27485.

USE OF TABLE D

Table D gives values of w_n of Eq. 32, Table III, for given values of i, h, and n.

When the payment is made (or avoided) at the end of each year, and interest is compounded annually.

Example 1.—What can we afford to pay now in order to effect a saving of \$1 at the end of each year for seven years, if money is worth 8 per cent compounded annually? In Table D, opposite "7" and under "8 per cent," we find \$5.206, which is the required present sum.

Example 2.—What is the capitalized value of a yearly expense of \$100 for 25 years, if interest is 6 per cent compounded annually? Opposite "25" and under "6 per cent," Table D, we find \$12.78, which multiplied by 100 gives \$1278±, which is the capitalized value sought.

When the payment is made (or avoided) at the end of each interest period which is less than one year.

Example 3.—What principal is required to yield an annuity of \$200, one-half of which is to be paid at the end of each half-year, for 14 years, if interest is 6 per cent compounded semi-annually? $i/h = 0.06/2 = 0.03$, $hn = 2 \times 14 = 28$. Opposite "28" and under "3 per cent," Table D, we find \$18.76; and this multiplied by 100 gives \$1876±, which is the required principal.

TABLE D.—PRESENT WORTH OF SERIES (w_n) OR SERIES DISCOUNT FACTOR

Present worth of n yr. series of year-end payments of 1 each, with interest compounded annually at ———

n	2%	2-½%	3%	3-½%	4%	4-½%	5%	6%	7%	8%
1	0.9804	0.9756	0.9709	0.9662	0.9615	0.9569	0.9524	0.9434	0.9346	0.9259
2	1.942	1.927	1.913	1.900	1.886	1.873	1.859	1.833	1.808	1.783
3	2.884	2.856	2.829	2.802	2.775	2.749	2.723	2.673	2.624	2.577
4	3.808	3.762	3.717	3.673	3.630	3.588	3.546	3.465	3.387	3.312
5	4.713	4.646	4.580	4.515	4.452	4.390	4.329	4.212	4.100	3.993
6	5.601	5.508	5.417	5.329	5.242	5.158	5.076	4.917	4.767	4.623
7	6.472	6.349	6.230	6.115	6.002	5.893	5.786	5.582	5.389	5.206
8	7.325	7.170	7.020	6.874	6.733	6.596	6.463	6.210	5.971	5.747
9	8.162	7.971	7.786	7.608	7.435	7.269	7.108	6.802	6.515	6.247
10	8.983	8.752	8.530	8.317	8.111	7.913	7.722	7.360	7.024	6.710
11	9.787	9.514	9.253	9.002	8.760	8.529	8.306	7.887	7.499	7.139
12	10.58	10.26	9.954	9.663	9.385	9.119	8.863	8.384	7.943	7.536
13	11.35	10.98	10.63	10.30	9.986	9.683	9.394	8.853	8.358	7.904
14	12.11	11.69	11.30	10.92	10.56	10.22	9.899	9.295	8.745	8.244
15	12.85	12.38	11.94	11.52	11.12	10.74	10.38	9.712	9.108	8.559
16	13.58	13.06	12.56	12.09	11.65	11.23	10.84	10.11	9.447	8.851
17	14.29	13.71	13.17	12.65	12.17	11.71	11.27	10.48	9.763	9.122
18	14.99	14.35	13.75	13.19	12.66	12.16	11.69	10.83	10.06	9.372
19	15.68	14.98	14.32	13.71	13.13	12.59	12.09	11.16	10.34	9.604
20	16.35	15.59	14.88	14.21	13.59	13.01	12.46	11.47	10.59	9.818
21	17.01	16.18	15.42	14.70	14.03	13.40	12.82	11.76	10.84	10.02
22	17.66	16.77	15.94	15.17	14.45	13.78	13.16	12.04	11.06	10.20
23	18.29	17.33	16.44	15.62	14.86	14.15	13.49	12.30	11.27	10.37
24	18.91	17.88	16.94	16.06	15.25	14.50	13.80	12.55	11.47	10.53
25	19.52	18.42	17.41	16.48	15.62	14.83	14.09	12.78	11.65	10.67
26	20.12	18.95	17.88	16.89	15.98	15.15	14.38	13.00	11.83	10.81
27	20.71	19.46	18.33	17.29	16.33	15.45	14.64	13.21	11.99	10.94
28	21.28	19.96	18.76	17.67	16.66	15.74	14.90	13.41	12.14	11.05
29	21.84	20.45	19.19	18.04	16.98	16.02	15.14	13.59	12.28	11.16
30	22.40	20.93	19.60	18.39	17.29	16.29	15.37	13.76	12.41	11.26
31	22.94	21.40	20.00	18.74	17.59	16.54	15.59	13.93	12.53	11.35
32	23.47	21.85	20.39	19.07	17.87	16.79	15.80	14.08	12.65	11.43
33	23.99	22.29	20.77	19.39	18.15	17.02	16.00	14.23	12.75	11.51
34	24.50	22.72	21.13	19.70	18.41	17.25	16.19	14.37	12.85	11.59
35	25.00	23.15	21.49	20.00	18.66	17.46	16.37	14.50	12.95	11.65
36	25.49	23.56	21.83	20.29	18.91	17.67	16.55	14.62	13.04	11.72
37	25.97	23.96	22.17	20.57	19.14	17.86	16.71	14.74	13.12	11.78
38	26.44	24.35	22.49	20.84	19.37	18.05	16.87	14.85	13.19	11.83
39	26.90	24.73	22.81	21.10	19.58	18.23	17.02	14.95	13.26	11.88
40	27.36	25.10	23.11	21.36	19.79	18.40	17.16	15.05	13.33	11.92
41	27.80	25.47	23.41	21.60	19.99	18.57	17.29	15.14	13.39	11.97
42	28.23	25.82	23.70	21.83	20.19	18.72	17.42	15.22	13.45	12.01
43	28.66	26.17	23.98	22.06	20.37	18.87	17.55	15.31	13.51	12.04
44	29.08	26.50	24.25	22.28	20.55	19.02	17.66	15.38	13.56	12.08
45	29.49	26.83	24.52	22.50	20.72	19.16	17.77	15.46	13.61	12.11
46	29.89	27.15	24.78	22.70	20.88	19.29	17.88	15.52	13.65	12.14
47	30.29	27.47	25.02	22.90	21.04	19.41	17.98	15.59	13.69	12.16
48	30.67	27.77	25.27	23.09	21.20	19.54	18.08	15.65	13.73	12.19
49	31.05	28.07	25.50	23.28	21.34	19.65	18.17	15.71	13.77	12.21
50	31.42	28.36	25.73	23.46	21.48	19.76	18.26	15.76	13.80	12.23
55	33.17	29.71	26.77	24.26	22.11	20.25	18.63	15.99	13.94	12.32
60	34.76	30.91	27.68	24.94	22.62	20.64	18.93	16.16	14.04	12.38
65	36.20	31.96	28.45	25.52	23.05	20.95	19.16	16.29	14.11	12.42
70	37.50	32.90	29.12	26.00	23.39	21.20	19.34	16.38	14.16	12.44
75	38.68	33.72	29.70	26.41	23.68	21.40	19.48	16.46	14.20	12.46
80	39.74	34.45	30.20	26.75	23.92	21.57	19.60	16.51	14.22	12.47
85	40.71	35.10	30.63	27.04	24.11	21.70	19.68	16.55	14.24	12.48
90	41.59	35.67	31.00	27.28	24.27	21.80	19.75	16.58	14.25	12.49
95	42.38	36.17	31.32	27.48	24.40	21.88	19.81	16.60	14.26	12.49
100	43.10	36.61	31.60	27.66	24.50	21.95	19.85	16.62	14.27	12.49

USE OF TABLE E

Table E gives the value of d_n of Eq. 33, Table III, for given values of i, h, and n.

When the deposit is made at the end of each year and interest is compounded annually.

Example 1.—What sum must be deposited at the end of each year in a savings bank paying 4 per cent compounded annually, in order that the accumulation at the end of 18 years may be $1? Opposite "18" and under "4 per cent," Table E, we find $0.0390, which is the required deposit.

Example 2.—What end-of-year deposit is required to redeem $2000 at the end of seven years, if interest on deposits is 5 per cent compounded annually? Opposite "7" and under "5 per cent ," Table E, we find $0.1228, which multiplied by 2000 gives $245.60 ±, which is the deposit required.

When the deposit is made at the end of each interest period which is less than one year.

Example 3.—What deposit is required at the end of each half-year to redeem $1 at the end of 16 years, if interest on deposits is 4 per cent compounded semi-annually? $i/h = 0.04/2 = 0.02$. $hn = 2 \times 16 = 32$. Opposite "32" and under "2 per cent," Table E, we find $0.0226, which is the required deposit.

Table E.—Annual Payment d_n or Backward Spreading Factor

Annual year-end payment in an n yr. series that amounts to 1 with interest compounded annually at ———

n	2%	2-½%	3%	3-½%	4%	4-½%	5%	6%	7%	8%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.4950	0.4938	0.4926	0.4914	0.4902	0.4890	0.4878	0.4854	0.4831	0.4808
3	0.3268	0.3251	0.3235	0.3219	0.3203	0.3188	0.3172	0.3141	0.3111	0.3080
4	0.2426	0.2408	0.2390	0.2373	0.2355	0.2337	0.2320	0.2286	0.2252	0.2219
5	0.1922	0.1902	0.1884	0.1865	0.1846	0.1828	0.1810	0.1774	0.1739	0.1705
6	0.1585	0.1565	0.1546	0.1527	0.1508	0.1489	0.1470	0.1434	0.1398	0.1363
7	0.1345	0.1325	0.1305	0.1285	0.1266	0.1247	0.1228	0.1191	0.1156	0.1121
8	0.1165	0.1145	0.1125	0.1105	0.1085	0.1066	0.1047	0.1010	0.0975	0.0940
9	0.1025	0.1005	0.0984	0.0964	0.0945	0.0926	0.0907	0.0870	0.0835	0.0801
10	0.0913	0.0893	0.0872	0.0852	0.0833	0.0814	0.0795	0.0759	0.0724	0.0690
11	0.0822	0.0801	0.0781	0.0761	0.0741	0.0722	0.0704	0.0668	0.0634	0.0601
12	0.0746	0.0725	0.0705	0.0685	0.0666	0.0647	0.0628	0.0593	0.0559	0.0527
13	0.0681	0.0660	0.0640	0.0621	0.0601	0.0583	0.0565	0.0530	0.0497	0.0465
14	0.0626	0.0605	0.0585	0.0566	0.0547	0.0528	0.0510	0.0476	0.0443	0.0413
15	0.0578	0.0558	0.0538	0.0518	0.0499	0.0481	0.0463	0.0430	0.0398	0.0368
16	0.0537	0.0516	0.0496	0.0477	0.0458	0.0440	0.0423	0.0390	0.0359	0.0330
17	0.0500	0.0479	0.0460	0.0440	0.0422	0.0404	0.0387	0.0354	0.0324	0.0296
18	0.0467	0.0447	0.0427	0.0408	0.0390	0.0372	0.0355	0.0324	0.0294	0.0267
19	0.0438	0.0418	0.0398	0.0379	0.0361	0.0344	0.0327	0.0296	0.0268	0.0241
20	0.0412	0.0391	0.0372	0.0354	0.0336	0.0319	0.0302	0.0272	0.0244	0.0219
21	0.0388	0.0368	0.0349	0.0330	0.0313	0.0296	0.0280	0.0250	0.0223	0.0198
22	0.0366	0.0346	0.0327	0.0309	0.0292	0.0275	0.0260	0.0230	0.0204	0.0180
23	0.0347	0.0327	0.0308	0.0290	0.0273	0.0257	0.0241	0.0213	0.0187	0.0164
24	0.0329	0.0309	0.0290	0.0273	0.0256	0.0240	0.0225	0.0197	0.0172	0.0150
25	0.0312	0.0293	0.0274	0.0257	0.0240	0.0224	0.0210	0.0182	0.0158	0.0137
26	0.0297	0.0278	0.0259	0.0242	0.0226	0.0210	0.0196	0.0169	0.0146	0.0125
27	0.0283	0.0264	0.0246	0.0229	0.0212	0.0197	0.0183	0.0157	0.0134	0.0114
28	0.0270	0.0251	0.0233	0.0216	0.0200	0.0185	0.0171	0.0146	0.0124	0.0105
29	0.0258	0.0239	0.0221	0.0204	0.0189	0.0174	0.0160	0.0136	0.0114	0.0096
30	0.0246	0.0228	0.0210	0.0194	0.0178	0.0164	0.0151	0.0126	0.0106	0.0088
31	0.0236	0.0217	0.0200	0.0184	0.0169	0.0154	0.0141	0.0118	0.0098	0.0081
32	0.0226	0.0208	0.0190	0.0174	0.0159	0.0146	0.0133	0.0110	0.0091	0.0075
33	0.0217	0.0199	0.0182	0.0166	0.0151	0.0137	0.0125	0.0103	0.0084	0.0069
34	0.0208	0.0190	0.0173	0.0158	0.0143	0.0130	0.0118	0.0096	0.0078	0.0063
35	0.0200	0.0182	0.0165	0.0150	0.0136	0.0123	0.0111	0.0090	0.0072	0.0058
36	0.0192	0.0175	0.0158	0.0143	0.0129	0.0116	0.0104	0.0084	0.0067	0.0053
37	0.0185	0.0167	0.0151	0.0136	0.0122	0.0110	0.0098	0.0079	0.0062	0.0049
38	0.0178	0.0161	0.0145	0.0130	0.0116	0.0104	0.0093	0.0074	0.0058	0.0045
39	0.0172	0.0154	0.0138	0.0124	0.0111	0.0099	0.0088	0.0069	0.0054	0.0042
40	0.0166	0.0148	0.0133	0.0118	0.0105	0.0093	0.0083	0.0065	0.0050	0.0039
41	0.0160	0.0143	0.0127	0.0113	0.0100	0.0089	0.0078	0.0061	0.0047	0.0036
42	0.0154	0.0137	0.0122	0.0108	0.0095	0.0084	0.0074	0.0057	0.0043	0.0033
43	0.0149	0.0132	0.0117	0.0103	0.0091	0.0080	0.0070	0.0053	0.0040	0.0030
44	0.0144	0.0127	0.0112	0.0099	0.0087	0.0076	0.0066	0.0050	0.0038	0.0028
45	0.0139	0.0123	0.0108	0.0095	0.0083	0.0072	0.0063	0.0047	0.0035	0.0026
46	0.0135	0.0118	0.0104	0.0091	0.0079	0.0068	0.0059	0.0044	0.0033	0.0024
47	0.0130	0.0114	0.0100	0.0087	0.0075	0.0065	0.0056	0.0041	0.0030	0.0022
48	0.0126	0.0110	0.0096	0.0083	0.0072	0.0062	0.0053	0.0039	0.0028	0.0020
49	0.0122	0.0106	0.0092	0.0080	0.0069	0.0059	0.0050	0.0037	0.0026	0.0019
50	0.0118	0.0103	0.0089	0.0076	0.0066	0.0056	0.0048	0.0034	0.0025	0.0017
55	0.0101	0.0087	0.0073	0.0062	0.0052	0.0044	0.0037	0.0025	0.0017	0.0012
60	0.0088	0.0074	0.0061	0.0051	0.0042	3.0035	0.0028	0.0019	0.0012	0.0008
65	0.0076	0.0063	0.0051	0.0042	0.0034	0.0027	0.0022	0.0014	0.0009	0.0005
70	0.0067	0.0054	0.0043	0.0035	0.0027	0.0022	0.0017	0.0010	0.0006	0.0004
75	0.0059	0.0047	0.0037	0.0029	0.0022	0.0017	0.0013	0.0008	0.0004	0.0002
80	0.0052	0.0040	0.0031	0.0024	0.0018	0.0014	0.0010	0.0006	0.0003	0.0002
85	0.0046	0.0035	0.0026	0.0020	0.0015	0.0011	0.0008	0.0004	0.0002	0.0001
90	0.0040	0.0030	0.0023	0.0017	0.0012	0.0009	0.0006	0.0003	0.0002	0.0001
95	0.0036	0.0026	0.0019	0.0014	0.0010	0.0007	0.0005	0.0002	0.0001	0.0000
100	0.0032	0.0023	0.0016	0.0012	0.0008	0.0006	0.0004	0.0002	0.0001	0.0000

USE OF TABLE F

Table F gives the value of a_n of Eq. 34, Table III, for given values of i, h, and n.

When payment is made at the end of each year and interest is compounded annually.

Example 1.—What end-of-year annuity will be produced for 10 years by a principal of $1, if interest is 4 per cent compounded annually? Opposite "10" and under "4 per cent," Table F, we find $0.1233, which is the annuity sought.

Example 2.—What end-of-year payment for a period of five years is the equivalent of $1000 paid now, if interest is 7 per cent compounded annually? Opposite "5" and under "7 per cent," Table F, we find $0.2439, which multiplied by 1000 gives $243.90 ±, which is the quantity sought.

When the payment is made at the end of each interest period which is less than one year.

Example 3.—A present debt of $10,000 is to be paid in 20 equal semi-annual payments, one at the end of each half-year. If interest is 4 per cent, compounded semi-annually, what should each payment be? $i/h = 0.04/2 = 0.02$, $hn = 2 \times 10 = 20$. Opposite "20" and under "2 per cent," Table F, we find 0.0612, which multiplied by 10,000 gives $612 ±, which is the quantity sought.

TABLE F.—ANNUAL PAYMENT a_n OR FORWARD SPREADING FACTOR

Annual year-end payment in an n yr. series whose present worth is 1 with interest compounded annually at ———

n	2%	2-½%	3%	3-½%	4%	4-½%	5%	6%	7%	8%
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700	1.0800
2	0.5150	0.5188	0.5226	0.5264	0.5302	0.5340	0.5378	0.5454	0.5531	0.5608
3	0.3468	0.3501	0.3535	0.3569	0.3603	0.3638	0.3672	0.3741	0.3811	0.3880
4	0.2626	0.2658	0.2690	0.2723	0.2755	0.2787	0.2820	0.2886	0.2952	0.3019
5	0.2122	0.2152	0.2184	0.2215	0.2246	0.2278	0.2310	0.2374	0.2439	0.2505
6	0.1785	0.1815	0.1846	0.1877	0.1908	0.1939	0.1970	0.2034	0.2098	0.2163
7	0.1545	0.1575	0.1605	0.1635	0.1666	0.1697	0.1728	0.1791	0.1856	0.1921
8	0.1365	0.1395	0.1425	0.1455	0.1485	0.1516	0.1547	0.1610	0.1675	0.1740
9	0.1225	0.1255	0.1284	0.1314	0.1345	0.1376	0.1407	0.1470	0.1535	0.1601
10	0.1113	0.1143	0.1172	0.1202	0.1233	0.1264	0.1295	0.1359	0.1424	0.1490
11	0.1022	0.1051	0.1081	0.1111	0.1141	0.1172	0.1204	0.1268	0.1334	0.1401
12	0.0946	0.0975	0.1005	0.1035	0.1066	0.1097	0.1128	0.1193	0.1259	0.1327
13	0.0881	0.0910	0.0940	0.0971	0.1001	0.1033	0.1065	0.1130	0.1197	0.1265
14	0.0826	0.0855	0.0885	0.0916	0.0947	0.0978	0.1010	0.1076	0.1143	0.1213
15	0.0778	0.0808	0.0838	0.0868	0.0899	0.0931	0.0963	0.1030	0.1098	0.1168
16	0.0737	0.0766	0.0796	0.0827	0.0858	0.0890	0.0923	0.0990	0.1059	0.1130
17	0.0700	0.0729	0.0760	0.0790	0.0822	0.0854	0.0887	0.0954	0.1024	0.1096
18	0.0667	0.0697	0.0727	0.0758	0.0790	0.0822	0.0855	0.0924	0.0994	0.1067
19	0.0638	0.0668	0.0698	0.0729	0.0761	0.0794	0.0827	0.0896	0.0968	0.1041
20	0.0612	0.0641	0.0672	0.0704	0.0736	0.0769	0.0802	0.0872	0.0944	0.1019
21	0.0588	0.0618	0.0649	0.0680	0.0713	0.0746	0.0780	0.0850	0.0923	0.0998
22	0.0566	0.0596	0.0627	0.0659	0.0692	0.0725	0.0760	0.0830	0.0904	0.0980
23	0.0547	0.0577	0.0608	0.0640	0.0673	0.0707	0.0741	0.0813	0.0887	0.0964
24	0.0529	0.0559	0.0590	0.0623	0.0656	0.0690	0.0725	0.0797	0.0872	0.0950
25	0.0512	0.0543	0.0574	0.0607	0.0640	0.0674	0.0710	0.0782	0.0858	0.0937
26	0.0497	0.0528	0.0559	0.0592	0.0626	0.0660	0.0696	0.0769	0.0846	0.0925
27	0.0483	0.0514	0.0546	0.0579	0.0612	0.0647	0.0683	0.0757	0.0834	0.0914
28	0.0470	0.0501	0.0533	0.0566	0.0600	0.0635	0.0671	0.0746	0.0824	0.0905
29	0.0458	0.0489	0.0521	0.0554	0.0589	0.0624	0.0660	0.0736	0.0814	0.0896
30	0.0446	0.0478	0.0510	0.0544	0.0578	0.0614	0.0651	0.0726	0.0806	0.0888
31	0.0436	0.0467	0.0500	0.0534	0.0569	0.0604	0.0641	0.0718	0.0798	0.0881
32	0.0426	0.0458	0.0490	0.0524	0.0559	0.0596	0.0633	0.0710	0.0791	0.0875
33	0.0417	0.0449	0.0482	0.0516	0.0551	0.0587	0.0625	0.0703	0.0784	0.0869
34	0.0408	0.0440	0.0473	0.0508	0.0543	0.0580	0.0618	0.0696	0.0778	0.0863
35	0.0400	0.0432	0.0465	0.0500	0.0536	0.0573	0.0611	0.0690	0.0772	0.0858
36	0.0392	0.0425	0.0458	0.0493	0.0529	0.0566	0.0604	0.0684	0.0767	0.0853
37	0.0385	0.0417	0.0451	0.0486	0.0522	0.0560	0.0598	0.0679	0.0762	0.0849
38	0.0378	0.0411	0.0445	0.0480	0.0516	0.0554	0.0593	0.0674	0.0758	0.0845
39	0.0372	0.0404	0.0438	0.0474	0.0511	0.0549	0.0588	0.0669	0.0754	0.0842
40	0.0366	0.0398	0.0433	0.0468	0.0505	0.0543	0.0583	0.0665	0.0750	0.0839
41	0.0360	0.0393	0.0427	0.0463	0.0500	0.0539	0.0578	0.0661	0.0747	0.0836
42	0.0354	0.0387	0.0422	0.0458	0.0495	0.0534	0.0574	0.0657	0.0743	0.0833
43	0.0349	0.0382	0.0417	0.0453	0.0491	0.0530	0.0570	0.0653	0.0740	0.0830
44	0.0344	0.0377	0.0412	0.0449	0.0487	0.0526	0.0566	0.0650	0.0738	0.0028
45	0.0339	0.0373	0.0408	0.0445	0.0483	0.0522	0.0563	0.0647	0.0735	0.0826
46	0.0335	0.0368	0.0404	0.0441	0.0479	0.0518	0.0559	0.0644	0.0733	0.0824
47	0.0330	0.0364	0.0400	0.0437	0.0475	0.0515	0.0556	0.0641	0.0730	0.0822
48	0.0326	0.0360	0.0396	0.0433	0.0472	0.0512	0.0553	0.0639	0.0728	0.0820
49	0.0322	0.0356	0.0392	0.0430	0.0469	0.0509	0.0550	0.0637	0.0726	0.0819
50	0.0318	0.0353	0.0389	0.0426	0.0466	0.0506	0.0548	0.0634	0.0725	0.0817
55	0.0301	0.0337	0.0373	0.0412	0.0452	0.0494	0.0537	0.0625	0.0717	0.0812
60	0.0288	0.0324	0.0361	0.0401	0.0442	0.0485	0.0528	0.0619	0.0712	0.0808
65	0.0276	0.0313	0.0351	0.0392	0.0434	0.0477	0.0522	0.0614	0.0709	0.0805
70	0.0267	0.0304	0.0343	0.0385	0.0427	0.0472	0.0517	0.0610	0.0706	0.0804
75	0.0259	0.0297	0.0337	0.0379	0.0422	0.0467	0.0513	0.0608	0.0704	0.0802
80	0.0252	0.0290	0.0331	0.0374	0.0418	0.0464	0.0510	0.0606	0.0703	0.0802
85	0.0246	0.0285	0.0326	0.0370	0.0415	0.0461	0.0508	0.0604	0.0702	0.0801
90	0.0240	0.0280	0.0323	0.0367	0.0412	0.0459	0.0506	0.0603	0.0702	0.0801
95	0.0236	0.0276	0.0319	0.0364	0.0410	0.0457	0.0505	0.0602	0.0701	0.0800
100	0.0232	0.0273	0.0316	0.0362	0.0408	0.0456	0.0504	0.0602	0.0701	0.0800

TABLE G.—NUMBER OF DAYS IN THE YEAR UP TO AND INCLUDING ANY GIVEN DATE

Day of month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Day of month
1	1	32	60	91	121	152	182	213	244	274	305	335	1
2	2	33	61	92	122	153	183	214	245	275	306	336	2
3	3	34	62	93	123	154	184	215	246	276	307	337	3
4	4	35	63	94	124	155	185	216	247	277	308	338	4
5	5	36	64	95	125	156	186	217	248	278	309	339	5
6	6	37	65	96	126	157	187	218	249	279	310	340	6
7	7	38	66	97	127	158	188	219	250	280	311	341	7
8	8	39	67	98	128	159	189	220	251	281	312	342	8
9	9	40	68	99	129	160	190	221	252	282	313	343	9
10	10	41	69	100	130	161	191	222	253	283	314	344	10
11	11	42	70	101	131	162	192	223	254	284	315	345	11
12	12	43	71	102	132	163	193	224	255	285	316	346	12
13	13	44	72	103	133	164	194	225	256	286	317	347	13
14	14	45	73	104	134	165	195	226	257	287	318	348	14
15	15	46	74	105	135	166	196	227	258	288	319	349	15
16	16	47	75	106	136	167	197	228	259	289	320	350	16
17	17	48	76	107	137	168	198	229	260	290	321	351	17
18	18	49	77	108	138	169	199	230	261	291	322	352	18
19	19	50	78	109	139	170	200	231	262	292	323	353	19
20	20	51	79	110	140	171	201	232	263	293	324	354	20
21	21	52	80	111	141	172	202	233	264	294	325	355	21
22	22	53	81	112	142	173	203	234	265	295	326	356	22
23	23	54	82	113	143	174	204	235	266	296	327	357	23
24	24	55	83	114	144	175	205	236	267	297	328	358	24
25	25	56	84	115	145	176	206	237	268	298	329	359	25
26	26	57	85	116	146	177	207	238	269	299	330	360	26
27	27	58	86	117	147	178	208	239	270	300	331	361	27
28	28	59	87	118	148	179	209	240	271	301	332	362	28
29	29	.....	88	119	149	180	210	241	272	302	333	363	29
30	30	.....	89	120	150	181	211	242	273	303	334	364	30
31	31		90		151		212	243		304		365	31

For leap year add 1 to each number after the number for Feb. 28.

USE OF TABLE H

Example.—A note for $280.50, bearing 6 per cent simple interest, is dated September 22, 1905 and falls due June 17, 1907. What is the simple interest on the note for the whole time?

Solution on the basis of a 365-day year:

September 22, 1905 to September 22, 1906	1 year,
September 22, 1906 to June 17, 1907:	
Days in one year	365
September 22 is, of the year, day number (Table G)	265
Remaining days of 1906 (not a leap year)	100
Days, January 1 to June 17 inclusive (Table G)	168
Days, September 22, 1906 to June 17, 1907	268 days.
Whole time	1 year, 268 days.

Interest at 1 per cent. on $280.50 for 1 year	$2.8050
Interest at 1 per cent. on 200.00 for 260 days (Table H)	1.4246
Interest at 1 per cent. on 80.00 for 260 days (Table H)	0.5698
Interest at 1 per cent. on 0.50 for 260 days (Table H)	0.0035
Interest at 1 per cent. on 200.00 for 8 days (Table H)	0.0438
Interest at 1 per cent. on 80.00 for 8 days (Table H)	0.0175
Interest at 1 per cent. on 0.50 for 8 days (Table H)	0.0001
Interest at 1 per cent. on $280.50 for 1 year, 268 days	$4.8643
which multiplied by	6
gives the interest at 6 per cent on $280.50 for 1 year 268 days	$29.1859,

or, $29.19. *Answer.*

TABLE H.—SIMPLE INTEREST (365-DAY YEAR)

At 1 per cent on

Days	$1000	$2000	$3000
1	0.02740	0.05479	0.08219
2	0.05479	0.10959	0.16438
3	0.08219	0.16438	0.24658
4	0.10959	0.21918	0.32877
5	0.13699	0.27397	0.41096
6	0.16438	0.32877	0.49315
7	0.19178	0.38356	0.57534
8	0.21918	0.43836	0.65753
9	0.24658	0.49315	0.73973
10	0.27397	0.54795	0.82192
20	0.54795	1.09589	1.64384
30	0.82192	1.64384	2.46575
40	1.09589	2.19178	3.28767
50	1.36986	2.73973	4.10959
60	1.64384	3.28767	4.93151
70	1.91781	3.83562	5.75342
80	2.19178	4.38356	6.57534
90	2.46575	4.93151	7.39726
100	2.73973	5.47945	8.21918
110	3.01370	6.02740	9.04110
120	3.28767	6.57534	9.86301
130	3.56164	7.12329	10.68493
140	3.83562	7.67123	11.50685
150	4.10959	8.21918	12.32877
160	4.38356	8.76712	13.15068
170	4.65753	9.31507	13.97260
180	4.93151	9.86301	14.79452
190	5.20548	10.41096	15.61644
200	5.47945	10.95890	16.43836
210	5.75342	11.50685	17.26027
220	6.02740	12.05479	18.08219
230	6.30137	12.60274	18.90411
240	6.57534	13.15068	19.72603
250	6.84932	13.69863	20.54795
260	7.12329	14.24658	21.36986
270	7.39726	14.79452	22.19178
280	7.67123	15.34247	23.01370
290	7.94521	15.89041	23.83562
300	8.21918	16.43836	24.65753
310	8.49315	16.98630	25.47945
320	8.76712	17.53425	26.30137
330	9.04110	18.08219	27.12329
340	9.31507	18.63014	27.94521
350	9.58904	19.17808	28.76712
360	9.86301	19.72603	29.58904

TABLE H.—SIMPLE INTEREST (365-DAY YEAR) (*Continued*)
At 1 per cent on

Days	$4000	$5000	$6000
1	0.10959	0.13699	0.16438
2	0.21918	0.27397	0.32877
3	0.32877	0.41096	0.49315
4	0.43836	0.54795	0.65753
5	0.54795	0.68493	0.82192
6	0.65753	0.82192	0.98630
7	0.76712	0.95890	1.15068
8	0.87671	1.09589	1.31507
9	0.98630	1.23288	1.47945
10	1.09589	1.36986	1.64384
20	2.19178	2.73973	3.28767
30	3.28767	4.10959	4.93151
40	4.38356	5.47945	6.57534
50	5.47945	6.84932	8.21918
60	6.57534	8.21918	9.86301
70	7.67123	9.58904	11.50685
80	8.76712	10.95890	13.15068
90	9.86301	12.32877	14.79452
100	10.95890	13.69863	16.43836
110	12.05479	15.06849	18.08219
120	13.15068	16.43836	19.72603
130	14.24658	17.80822	21.36986
140	15.34247	19.17808	23.01370
150	16.43836	20.54795	24.65753
160	17.53425	21.91781	26.30137
170	18.63014	23.28767	27.94521
180	19.72603	24.65753	29.58904
190	20.82192	26.02740	31.23288
200	21.91781	27.39726	32.87671
210	23.01370	28.76712	34.52055
220	24.10959	30.13699	36.16438
230	25.20548	31.50685	37.80822
240	26.30137	32.87671	39.45205
250	27.39726	34.24658	41.09589
260	28.49315	35.61644	42.73973
270	29.58904	36.98630	44.38356
280	30.68493	38.35616	46.02740
290	31.78082	39.72603	47.67123
300	32.87671	41.09589	49.31507
310	33.97260	42.46575	50.95890
320	35.06849	43.83562	52.60274
330	36.16438	45.20548	54.24657
340	37.26027	46.57534	55.89041
350	38.35616	47.94521	57.53425
360	39.45205	49.31507	59.17808

Table H.—Simple Interest (365-day Year) (*Continued*)

At 1 per cent on

Days	$7000	$8000	$9000
1	0.19178	0.21918	0.24658
2	0.38356	0.43836	0.49315
3	0.57534	0.65753	0.73973
4	0.76712	0.87671	0.98630
5	0.95890	1.09589	1.23288
6	1.15068	1.31507	1.47945
7	1.34247	1.53425	1.72603
8	1.53425	1.75342	1.97260
9	1.72603	1.97260	2.21918
10	1.91781	2.19178	2.46575
20	3.83562	4.38356	4.93151
30	5.75342	6.57534	7.39726
40	7.67123	8.76712	9.86301
50	9.58904	10.95890	12.32877
60	11.50685	13.15068	14.79452
70	13.42466	15.34247	17.26027
80	15.34247	17.53425	19.72603
90	17.26027	19.72603	22.19178
100	19.17808	21.91781	24.65753
110	21.09589	24.10959	27.12329
120	23.01370	26.30137	29.58904
130	24.93151	28.49315	32.05479
140	26.84931	30.68493	34.52055
150	28.76712	32.87671	36.98630
160	30.68493	35.06849	39.45205
170	32.60274	37.26027	41.91781
180	34.52055	39.45205	44.38356
190	36.43836	41.64384	46.84931
200	38.35616	43.83562	49.31507
210	40.27397	46.02740	51.78082
220	42.19178	48.21918	53.24657
230	44.10959	50.41096	56.71233
240	46.02740	52.60274	59.17808
250	47.94521	54.79452	61.64384
260	49.86301	56.98630	64.10959
270	51.78082	59.17808	66.57534
280	53.69863	61.36986	69.04110
290	55.61644	63.56164	71.50685
300	57.53425	65.75342	73.97260
310	59.45205	67.94520	76.43836
320	61.36986	70.13699	78.90411
330	63.28767	72.32877	81.36986
340	65.20548	74.52055	83.83562
350	67.12329	76.71233	86.30137
360	69.04110	78.90411	88.76712

USE OF TABLE J

Example.—A note for $280.50 bearing 6 per cent simple interest is dated Sept. 22, 1905 and falls due June 17, 1907. What is the simple interest on the note for the whole time?

Solution on the basis of a 360-day year:

	Year	Month	Day
June 17, 1907	1907	6	17
Sept. 22, 1905	1905	9	22
Time	1 year	8 months	25 days

Interest at 1 per cent on $280.50 for 1 year	$2.8050
Interest at 1 per cent on 200.00 for 8 months (Table J)	1.3333
Interest at 1 per cent on 80.00 for 8 months (Table J)	0.5333
Interest at 1 per cent on 0.50 for 8 months (Table J)	0.0033
Interest at 1 per cent on 200.00 for 25 days (Table J)	0.1388
Interest at 1 per cent on 80.00 for 25 days (Table J)	0.0556
Interest at 1 per cent on 0.50 for 25 days (Table J)	0.0003
Interest at 1 per cent. on $280.50 for 1 year, 8 months, 25 days	$4.8696
which multiplied by	6
gives the interest at 6 per cent on $280.50 for 1 year, 8 months, 25 days	$29.2176,

or, $29.22. *Answer.*

TABLE J.—SIMPLE INTEREST (360-DAY YEAR)

At 1 per cent on

Days	$1000	$2000	$3000
1	0.02777	0.05555	0.08333
2	0.05555	0.11111	0.16666
3	0.08333	0.16666	0.25000
4	0.11111	0.22222	0.33333
5	0.13888	0.27777	0.41666
6	0.16666	0.33333	0.50000
7	0.19444	0.38888	0.58333
8	0.22222	0.44444	0.66666
9	0.25000	0.50000	0.75000
10	0.27777	0.55555	0.83333
11	0.30555	0.61111	0.91666
12	0.33333	0.66666	1.00000
13	0.36111	0.72222	1.08333
14	0.38888	0.77777	1.16666
15	0.41666	0.83333	1.25000
16	0.44444	0.88888	1.33333
17	0.47222	0.94444	1.41666
18	0.50000	1.00000	1.50000
19	0.52777	1.05555	1.58333
20	0.55555	1.11111	1.66666
21	0.58333	1.16666	1.75000
22	0.61111	1.22222	1.83333
23	0.63888	1.27777	1.91666
24	0.66666	1.33333	2.00000
25	0.69444	1.38888	2.08333
26	0.72222	1.44444	2.16666
27	0.75000	1.50000	2.25000
28	0.77777	1.55555	2.33333
29	0.80555	1.61111	2.41666
Months			
1	0.83333	1.66666	2.50000
2	1.66666	3.33333	5.00000
3	2.50000	5.00000	7.50000
4	3.33333	6.66666	10.00000
5	4.16666	8.33333	12.50000
6	5.00000	10.00000	15.00000
7	5.83333	11.66666	17.50000
8	6.66666	13.33333	20.00000
9	7.50000	15.00000	22.50000
10	8.33333	16.66666	25.00000
11	9.16666	18.33333	27.50000
Year			
1	10.00000	20.00000	30.00000

Table J.—Simple Interest (360-day Year) (*Continued*)
At 1 per cent on

Days	$4000	$5000	$6000
1	0.11111	0.13888	0.16666
2	0.22222	0.27777	0.33333
3	0.33333	0.41666	0.50000
4	0.44444	0.55555	0.66666
5	0.55555	0.69444	0.83333
6	0.66666	0.83333	1.00000
7	0.77777	0.97222	1.16666
8	0.88888	1.11111	1.33333
9	1.00000	1.25000	1.50000
10	1.11111	1.38888	1.66666
11	1.22222	1.52777	1.83333
12	1.33333	1.66666	2.00000
13	1.44444	1.80555	2.16666
14	1.55555	1.94444	2.33333
15	1.66666	2.08333	2.50000
16	1.77777	2.22222	2.66666
17	1.88888	2.36111	2.83333
18	2.00000	2.50000	3.00000
19	2.11111	2.63888	3.16666
20	2.22222	2.77777	3.33333
21	2.33333	2.91666	3.50000
22	2.44444	3.05555	3.66666
23	2.55555	3.19444	3.83333
24	2.66666	3.33333	4.00000
25	2.77777	3.47222	4.16666
26	2.88888	3.61111	4.33333
27	3.00000	3.75000	4.50000
28	3.11111	3.88888	4.66666
29	3.22222	4.02777	4.83333
Months			
1	3.33333	4.16666	5.00000
2	6.66666	8.33333	10.00000
3	10.00000	12.50000	15.00000
4	13.33333	16.66666	20.00000
5	16.66666	20.83333	25.00000
6	20.00000	25.00000	30.00000
7	23.33333	29.16666	35.00000
8	26.66666	33.33333	40.00000
9	30.00000	37.50000	45.00000
10	33.33333	41.66666	50.00000
11	36.66666	45.83333	55.00000
Year			
1	40.00000	50.00000	60.00000

TABLE J.—SIMPLE INTEREST (360-DAY YEAR) (*Continued*)
At 1 per cent on

Days	$7000	$8000	$9000
1	0.19444	0.22222	0.25000
2	0.38888	0.44444	0.50000
3	0.58333	0.66666	0.75000
4	0.77777	0.88888	1.00000
5	0.97222	1.11111	1.25000
6	1.16666	1.33333	1.50000
7	1.36111	1.55555	1.75000
8	1.55555	1.77777	2.00000
9	1.75000	2.00000	2.25000
10	1.94444	2.22222	2.50000
11	2.13888	2.44444	2.75000
12	2.33333	2.66666	3.00000
13	2.52777	2.88888	3.25000
14	2.72222	3.11111	3.50000
15	2.91666	3.33333	3.75000
16	3.11111	3.55555	4.00000
17	3.30555	3.77777	4.25000
18	3.50000	4.00000	4.50000
19	3.69333	4.22222	4.75000
20	3.88888	4.44444	5.00000
21	4.08333	4.66666	5.25000
22	4.27777	4.88888	5.50000
23	4.47222	5.11111	5.75000
24	4.66666	5.33333	6.00000
25	4.86111	5.55555	6.25000
26	5.05555	5.77777	6.50000
27	5.25000	6.00000	6.75000
28	5.44444	6.22222	7.00000
29	5.63888	6.44444	7.25000
Months			
1	5.83333	6.66666	7.50000
2	11.66666	13.33333	15.00000
3	17.50000	20.00000	22.50000
4	23.33333	26.66666	30.00000
5	29.16666	33.33333	37.50000
6	35.00000	40.00000	45.00000
7	40.83333	46.66666	52.50000
8	46.66666	53.33333	60.00000
9	52.50000	60.00000	67.50000
10	58.33333	66.66666	75.00000
11	64.16666	73.33333	82.50000
Year			
1	70.00000	80.00000	90.00000

TABLE K.—STRUCTURE LIFE TABLES[1]

Key to Authorities

A—Wisconsin R. R. Commission.
B—St. Louis Public Service Commission, Union Electric Light & Power Co.
C—Traction Valuation Commission, Chicago Consolidated Traction Co.
D—B. J. Arnold—Appraisal of the Coney Island & Brooklyn Railroad, Feb. 1, 1909.
E—Leonard Metcalf, Transactions American Society Civil Engineers, 1909, p. 24, Vol. LXIV.
F—Henry L. Gray.
G—Arbitrators, Street Lighting Controversy, Atlanta, Ga., 1899.
H—Nathan Hayward, The Bell Telephone Co. of Pennsylvania, Aug. 31, 1912.
I—H. P. Gillette, Everett Railway & Water Co., Jan. 29, 1912.
J—H. P. Gillette, Washington Ry. Appraisal.
K—Henry Floy, 3rd Ave. Case N. Y. City.
L—Prof. M. E. Cooley, Milwaukee 3c case.
M—Beegs, Milwaukee 3c case.
N—M. G. Starret, Milwaukee 3c case.
O—W. D. Pence, Milwaukee 3c case.
P—Union Traction Co. Case Chicago and Union Traction Co., Stone & Webster.
Q—B. J. Arnold, Chicago Appraisals 4 cases.
R—Marwick, Mitchell & Co., Appraisal of a large street railway system, Foster, p. 199.
S—Chicago Traction Commission.
T—Milwaukee Electric Railway & Light Co.
U—George W. Cravens, Industrial Power Plants.
V—Gillette's Handbook of Cost Data.

KIND OF PLANT	ESTIMATED LIFE, YEARS	AUTHORITY
Aerial lines	20	B
Arc lamps	6.7	G
Arc lamps	12.5	B
Arc lamps	15	A
Batteries, storage	10	T
Batteries, storage	10–20	U
Batteries, storage	15	A
Batteries, storage	15–20	P
Batteries, storage	20	B
Batteries, storage	20	K
Batteries, storage	33.3	R
Belting	20	A
Benches (gas plant)	25	A
Bins, storage	10–33.3	Q
Boilers	10	D

---

[1] Taken by permission from *Mechanical and Electrical Cost Data*, Gillette and Dana (N. Y.: McGraw-Hill Book Co., Inc., 1918).

KIND OF PLANT	ESTIMATED LIFE, YEARS	AUTHORITY
Boilers	10–28.6	S
Boilers	11.75–15	O
Boilers	12–16	E
Boilers	13.3	T
Boilers	15–20	P
Boilers	22.2	R
Boilers	25–28.6	C
Boilers	28.6	Q
Boilers	30–40	U
Boilers, fire tube	10	G
Boilers, fire tube	15	B
Boilers, fire tube, electric light plants	15–30	A
Boilers, fire tube, waterworks	20–25	A
Boilers, water tube	20	K
Boilers, water tube, electric light plant	20	A
Boilers, water tube, waterworks	20–25	A
Brakes, air	20	A
Bridges	40	R
Bridges, Howe truss	16.7	J
Breechings, steel	10	Q
Breeching and connections	10–28.6	C
Blowers, centrifugal (gas plant)	15	A
Buildings	33.3	T
Buildings	40	R
Buildings	50–100	U
Buildings, brick	25–50	A
Buildings, brick	66.6	C
Buildings, carhouses	33.3	I
Buildings, car barns	50	A
Buildings, coal sheds and stables, frame	20–25	A
Buildings, dwellings frame	35	A
Buildings, frame	20–50	E
Buildings, frame	50	G
Buildings, gas retort houses, brick	30	A
Buildings, grain elevators	33.3	J
Buildings, masonry	40–50	E
Buildings, miscellaneous	33.3	I, J
Buildings, office 1st class stone and brick	75	A
Buildings, power plant	33.3	I
Buildings, power stations	50	A
Buildings, railroad transportation department	33.3	J
Buildings, roundhouses	33.3	J
Buildings, shops	33.3	I, J
Buildings, shops, 2nd class	50	A
Buildings, snow sheds	25	J
Buildings, stations, fuel and water	33.3	J

Kind of Plant	Estimated Life, Years	Authority
Buildings, stations and waiting rooms.	33.3	I
Buildings, sub-station	33.3	I
Buildings, telephone	24	H
Bulkheading	10	J
Cables	15–25	U
Cables	20	T
Cables	50	S
Cable, aerial exchange	12	H
Cable, aerial exchange loading coils	20	H
Cable, aerial exchange terminal	10	H
Cable, aerial lead covered	12	A
Cable, aerial lead covered	15	A
Cable, aerial toll	12	H
Cable, feeders	25	O
Cable, feeders	66.6	Q
Cables, feeders overhead	33.3	R
Cables, feeder, underground	25	R
Cable, house	13	H
Cable, house terminals	10	H
Cable, submarine	9	H
Cable, underground (u. g.)	16.6	I
Cable, u. g. exchange main	20	H
Cable, u. g. exchange, subsidiary	13	H
Cable, u. g. exchange loading coils	20	H
Cable, u. g. exchange terminals	10	H
Cables, u. g. high tension	20	K
Cable, u. g., lead covered	20	A
Cables, u. g., lead covered	20	B
Cables, u. g., lead covered	25	A
Cable, u. g. toll	25	H
Cable, u. g. toll loading coils	20	H
Cable, u. g. terminals	10	H
Cars, see Rolling Stock.		
Chimney	33.3	C
Chimney, steel	10	D
Chimney, steel	14.3	Q
Chimney, brick	33.3	Q
Coal and ash handling machinery, see Machinery.		
Compressors, air	20–25	C
Concentrators ammonia (gas plant)	15	A
Condensers	10	G
Condensers	15	B
Condensers	20	A, D, K
Condensers	25	C
Condensers (gas plant)	30	A
Conduits	50	A, B
Conduits	100	K

Kind of Plant	Estimated Life, Years	Authority
Conduits (includes manholes)	50	R
Conduit, u. g.	16.6	I
Conduit, u. g., exchange, main	50	H
Conduit, u. g. exchange, subsidiary	15	H
Conduit, u. g. toll	50	H
Conveyors	22.2	R
Cranes	50	Q
Cribbing	10	J
Cross-arms	8–12	A
Crossings, R. R.	12.5	I
Culverts, log or timber	16.7	J
Dams	33.3 and 50	I
Distribution system, electric railway	11.75	M
Distribution system, electric railway	12.5	L
Distribution system, electric railway	14.25	N
Distribution system, electric railway	33.3	I
Docks	33.3	J
Drains, box	16.7	J
Economizers	10–20	Q
Engines	22.2	R
Engines, gas	15	A
Engines, steam	10–33.3	S
Engines, steam	13.3–20	D
Engines, steam	15–20	O, P
Engines, steam	15–25	E
Engines, steam	20	K, T
Engines, steam	20–33.3	B, Q
Engines, steam	20–40	U
Engines, steam, high speed	15	A, B
Engines, steam, high speed	20	G
Engines, steam, slow speed	20	A
Equipment	8	I
Equipment, electrical	11.75	M
Equipment, shop	10	I
Equipment, shop	10–25	U
Equipment, shop	10–33.3	S
Equipment, shop	13.3	T
Equipment, power plant	20	I
Exhausters (gas plant)	25	A
Extractors, tar, P. & A. (gas plant)	40	A
Feeders, see Cable.		
Fences	14.3	J
Fences	20	R
Fences, snow	10	R
Foundations, machinery, same as life of apparatus supported	....	C, K
Foundations, machinery	16.6	Q
Furniture	7	A

KIND OF PLANT	ESTIMATED LIFE, YEARS	AUTHORITY
Furniture and fixtures	12.5	L
Furniture and fixtures	20	I, N, R
Gas connections, cast iron (within the plant)	50	A
Gas (water) machines complete	30	A
Gas mains, cast iron 3 in. and 4 in	50	A
Gas mains, cast iron, 6 in. and larger	75	A
Gas mains, wrought iron and steel under 3 in	20	A
Gas mains, wrought iron and steel over 3 in	30	A
Gas services	20	A
Generators	12.5–33.3	C
Generators	15–20	P
Generators	20	D, K, O, Q
Generator, belted	10–20	S
Generator, belted	13.3	T
Generator, belted	15	U
Generators, direct connected	13.3	T
Generators, direct connected	20	S
Generators, direct connected	25	U
Generators, modern type	20	A
Generators, obsolete type	15	A
Generators, steam-turbo	15	B
Generators, steam-turbo	20	A
Generators, steam-turbo	10	G
Governors, gas (consumer's)	25	A
Governors (gas plant)	5	A
Heaters	16.7–25	C
Heaters	22.2	R
Heaters	33.3	Q
Heaters, feed water, closed	30	A
Heaters, feed water, open	30	A
Holders, (gas plant)	50	A
Horses and wagons, see Teams and Vehicles.		
Hydrants	40–50	E
Hydrants, connections	6.6	I
Locomotives	28–31	V
Machinery, coal and ash handling	5	O
Machinery, coal and ash handling	10	O
Machinery, coal and ash handling	14.3	Q, C
Machinery, coal and ash handling	15–20	P
Machinery, coal and ash handling	20	K
Machinery, electrical	20–30	E
Machinery, fuel oil handling	25	C
Machinery, shop	10	J
Machinery, shop	10–30	O

Kind of Plant	Estimated Life, Years	Authority
Machinery, shop	12.5	L
Machinery, shop	14.25	N
Machinery, shop	20	R
Machinery, shop	20–50	P
Meters, electric service	12.5	B
Meters, electric service	15	I, A
Meters, electric switchboard	20	A
Meters, gas (consumers)	25	A
Meter cases, station (gas plant)	50	A
Meter drums, station (gas plant)	20	A
Meters, water	20	I
Meters, water	20–30	E
Motors	10–20	S
Motors	15–25	U
Motors	20	T
Motors, railway	10	G
Motors, railway	20	A, K
Motors, railway	30	C
Motors, railway, see Rolling Stock, Electrical Equipment.		
Overhead equipment (electric railway)	5	Q
Overhead spans, complete	20	R
Overhead, special work	12.5	R
Overhead, systems	10–20	U
Overhead, systems	13.3	T
Paving	2	K
Paving	10	L
Paving	10–25	P
Paving	11.7	M
Paving	12.5	N
Paving	20	I
Paving	20	K
Paving, block	40	R
Paving, brick	22.2	R
Paving, tracks in car houses	28.6	R
Pipe, black iron	10	I
Pipe, cast iron, small diameter	20–40	E
Pipe, cast iron, large diameter	50–75	E
Pipe, galvanized iron	10	I
Pipe, Matheson	30	I
Pipe, screw flange	20	I
Pipe-steel	25–50	E
Pipe, wood stave	20–30	E
Pipe, wood	25	I
Pipe, also see Gas Mains.		
Pipe, fittings	20	I
Piping and covering	5	K

Kind of Plant	Estimated Life, Years	Authority
Piping and covering	15	B
Piping and covering	15–20	P
Piping and covering	16.6	D
Piping and covering	20	A, O
Piping and covering	22.2–25	C
Piping and covering	28.6	Q
Piping, steam	13.3	T
Piping, steam	28.6	S
Piping, steam	30–40	U
Poles, iron	20	P
Poles, iron	40	A, O, Q
Poles, iron	50	R
Poles, steel	50	K
Poles, telephone	12–15	A
Poles, wood	12–15	O
Poles, wood	14.3	R
Poles, wood in concrete	20	A
Poles, wood in earth	12–18.2	A
Poles, wooden	10	G
Pole lines, exchange (telephone)	10.5	H
Pole lines, tool (telephone)	16	H
Power plant	12.5	L
Power plant	20	N
Power plant	50	Q
Power plant and wire telephone	8	A
Pumps	20	C, D, K, Q
Pumps	20–25	I
Pumps, small steam	15	A, B
Pumps, small steam	20	G
Pumps and auxiliary machinery	20–30	E
Pumps and auxiliaries	22.2	R
Purifiers, modern (gas plant)	50	A
Rental equipment, electric	15	I
Reservoirs	33.3–50	J
Reservoirs (except where subject to heavy deposit of silt)	50–100	E
Rolling stock, cars electric	13.3–16.7	T
Rolling stock, cars (including all equipment)	20	R
Rolling stock, bodies closed cars	20	C
Rolling stock, cars, bodies open cars	25	C
Rolling stock, bodies open trailer	25	C
Rolling stock, cars, bodies, trucks	12.5	L
Rolling stock, cars, bodies, trucks	15	M
Rolling stock, cars, bodies trucks	15–20	O
Rolling stock, cars, bodies, trucks	16.7	N
Rolling stock, cars, bodies, trucks	20	P
Rolling stock, car bodies and equipment	15	A

Kind of Plant	Estimated Life, Years	Authority
Rolling stock, electrical equipment...	3.3–4–8.5	Q
Rolling stock, equipment electrical...	11.75–15	P
Rolling stock, trucks...............	20	K
Rolling stock, trucks...............	30	C
Rolling stock, utility equipment.....	20	R
Rolling stock, cars, wooden freight...	27.5	V
Rotary converters..................	20	T
Rotary converters..................	20–25	U
Rotaries..........................	22.2	R
Service connection, electrical........	28.5	I
Service connections, water..........	20	I
Scrubbers (gas plant)...............	30	A
Signal apparatus....................	10	R
Signal apparatus, interlocking.......	20	I
Stacks, see Chimneys.		
Standpipe.........................	10	I
Standpipes........................	25–40	E
Stokers...........................	4	Q
Stokers, moving parts..............	5	C
Stokers, fixed parts................	20	C
Switchboards......................	15–20	P
Switchboards......................	20	T
Switchboards......................	20–50	U
Switchboards......................	22.2	R
Switchboards......................	50	O, Q, S
Switchboard and wiring.............	33.3	C
Switchboard and wiring.............	16.7	D
Switchboard and wiring.............	20	K
Switchboard and wiring, modern type.	20	A
Switchboard and wiring, obsolete type............................	15	A
Tanks, storage ammonia, wrought iron or steel......................	15	A
Teams and vehicles................	5–8	I
Teams and vehicles................	10	L, R
Teams and vehicles................	20	N
Telegraph (signal).................	10	R
Telephone equipment, central office..	8.25	H
Telephone equipment, sub-station (except installations but including sub-station, central office equipment, public brand exchanges, booths and special fittings, sub-license station apparatus).........	9	H
Telephone central office equipment including distributing frame.......	10	A
Telephone and telegraph lines.......	20	I
Telephone, subscribers' sets.........	10	A

Kind of Plant	Estimated Life, Years	Authority
Tools	5	I
Tools	7	A
Tools, roadway	5	I
Tools, shop	10	J, R
Tools, shop	12.5	L
Tools, shop	14.25	N
Tools, shop	10–30	O
Tools, shop	20–50	P
Track-ballast	33.3	I
Track, bonds	20	A, C
Track, fastenings and joints	33.3	I
Track fastenings	40	J
Track, main	11.7	M
Track, main	12.5	L, N
Track, main	12.9–13.9	P
Track, main	13.3	T
Track, straight	18.2	A
Track-rails	33.3	I
Track-rails	40	J
Track rails in city streets	22.2	R
Track-rails in country roads	28.6	R
Track-rails in private right of way	28.6	R
Track, rail joints	20	C
Track, special work	8.5	T
Track, special work	8–14–25	O
Track, special work	10	Q
Track, special work	11.1	R
Track, special work	11.75	A
Track, special work	12.9–13–9	P
Track, special work	25	I
Track, substructure:		
City streets	22.2	R
Country roads	14.3	R
Private right of way	10	R
Track, ties	8	J
Track, ties	13.3	I
Track, ties	20	C
Transformers	20	T
Transformers	20–33.3	U
Transformers	22.2	R
Transformers, station service	15	B
Transformers, station service	15–20	A
Transformers, station service	20	I
Transmission line material	33.3	I
Transmission line material	50	R
Turbo-generators	16.7	R
Turbines, steam	20	A
Turbines, steam	15	B

Kind of Plant	Estimated Life, Years	Authority
Turbines, steam	20	T
Turbines, steam	20–40	U
Turbines, water	30	A
Valves	16.6	I
Valves, gate (water)	40–50	E
Washers, cast iron (gas plant)	40	A
Water supply lines	25–33.3	I
Wells, tar and ammonia (gas plant)	50	A
Wharves	33.3	J
Wires	15–25	U
Wires	20	T
Wires	50	S
Wiring	7.1–10	P
Wire, copper	20	A
Wire, guard	10	R
Wire, guard	8–15	A
Wire, telephone aerial exchange bare copper	14	H
Wire, telephone aerial exchange bare iron	8.5	H
Wire, telephone aerial exchange insulated (including drop wires)	6.5	H
Wire, telephone aerial toll, copper	30	H
Wire, telephone aerial toll, iron	15	H
Wires, telephone interior block	8	H
Wire, trolley No. 0–1 min. headway	2	A
Wire, trolley No. 00–1 min. headway	2.5	A
Wire, trolley No. 000–1 min. headway	3	A
Wire, trolley	20	R
Wire, weatherproof	13.3	G
Wire, weatherproof	16	A
Wire, weatherproof iron	15	A

APPENDIX C

BIBLIOGRAPHY

Arranged By Chapters

The works listed under each chapter head below contain matter that bears directly or collaterally on one or more of the topics discussed in the chapter.

It is not expected that the student will look into every book in the list—far from it. For example, one or more chapters in any one of the first four books named below might be adequate to the student's present interest in a certain topic. The reason for naming four books having the same general purpose when one would suffice, is that the student will often find it far easier to lay hands on some one of the four books than on any particular one of the four. The insertion of other similar groups of titles in the Bibliography is similarly justified.

If to the reader there occurs an idea for the betterment or enrichment of this Bibliography, I hope he will give way to the generous impulse to communicate it to me.

CHAPTER I

The A B C's of Business. H. S. Mc Kee. The Macmillian Co., N. Y.
Outlines of Economics. R. T. Ely. The Macmillan Co., N. Y.
Principles of Economics. F. W. Taussig. The Macmillan Co., N. Y.
Principles of Political Economy. J. S. Mill. (Edited by J. Laughlin.) D. Appleton & Co., N. Y.
Principles of Economics. A. Marshall. Macmillan & Co., London.
Economics of Business. N. A. Brisco. The Macmillan Co., N. Y.
Principles of Business. C. W. Gerstenberg. Prentice-Hall, N. Y.
The Young Man in Civil Engineering. G. F. Swain. The Macmillan Co., N. Y.
The Young Man in Mechanical Engineering. L. P. Breckenridge. The Macmillan Co., N. Y.
The Young Man in Electrical Engineering. C. F. Scott. The Macmillan Co., N. Y.
Addresses to Engineering Students. (A collection.) J. A. L. Waddell. Waddell & Harrington, Kansas City, Mo.
Engineering Economics. (A series of lectures at University of Kansas, 1917.) J. A. L. Waddell. Distributed by Dean of School of Engineering, Univ. of Kansas, Lawrence.

Contracts, Specifications, and Engineering Relations. D. W. Mead. McGraw-Hill Book Co., N. Y.

CHAPTER II

(See below.)

CHAPTER III

The Mathematical Theory of Investment. E. B. Skinner. Ginn & Co., Boston.

Introduction to Actuarial Science. H. A. Finney. American Institute of Accountants, N. Y.

Theory of Finance. G. King. C. & E. Layton, London. (Spectator Pub. Co., N. Y.)

Institute of Actuaries' Textbook, Part I. Charles and Edwin Layton, London.

Mathematics of Finance. H. L. Rietz and A. R. Crathorne. Henry Holt & Co., N. Y.

Mathematics for the Accountant. E. R. Vinal. Biddle Business Publications, N. Y.

Engineer's Valuing Assistant. H. D. Hoskold. Longmans, Green & Co., London & N. Y. (Contains very extensive tables pertaining to interest and its applications.)

CHAPTERS II AND IV

Economics of Electrical Distribution. P. O. Reyneau. McGraw-Hill Book Co., N. Y.

Financial Engineering. O. B. Goldman. John Wiley & Sons, N. Y.

Commercial Economy in Steam and other Thermal Plants. R. H. Smith. A. Constable & Co., Ltd., London.

"Electric lighting conductors." *Electrical Review* (London), Feb. 28, 1885, p. 189.

"Kelvin's law of economy in conductors." Chap. VIII of Conductors for Electrical Distribution. F. A. C. Perrine. D. Van Nostrand Co., N. Y., 1903.

"The maximum distance to which power can be economically transmitted." R. D. Mershon. Am. Inst. of Elec. Engineers, 1904, Vol. 23, p. 759.

"Economic sizes for cast-iron force mains." W. L. Butcher. *Eng. Rec.*, May 13, 1905, p. 558.

"A solution of the problem of determining the economic size of pipe for high-pressure water-power installation." A. L. Adams. *Trans. Am. Soc. C. E.*, December, 1907, Vol. 59, pp. 173–194.

"Pipe lines for hydraulic power plants." A. Jobson. *Eng. Rec.*, Dec. 21, 1907, p. 673.

"To determine when repairs have grown so great as to justify renewal." Handbook of Cost Data, pp. 27–33. H. P. Gillette. McGraw-Hill Book Co., N. Y.

"Deduction of a new rational formula for the most economic length of each of a series of bridge spans. H. P. Gillette. *Engineering-Contracting*, Jan. 4, 1911, p. 21.

"Deduction of a general formula for determining the most economic size of pipe to carry pumped water." H. P. Gillette. *Engineering-Contracting,* Jan. 25, 1911, p. 113.

"A new method of determining economical pipe diameters." F. H. Carter. *Eng. Rec.,* Dec. 16, 1911, p. 722.

"Comparative value of cross-ties of different materials." A. S. Whinnery. *Railroad Gazette,* Nov. 11, 1904, p. 537.

"Economic limit of pavement repairs." G. H. Norton. *The Cornell Civil Engineer,* Jan. 1915, p. 123.

"The value of a hydraulic horse-power at the wheel." E. W. Retger. *The Cornell Civil Engineer,* Jan. 14, 1920, pp. 47–49.

"Present Worth Calculations." W. O. Pennell. *Jour. Assn. Eng. Soc.,* September, 1914.

CHAPTERS V–VIII

Handbook of Construction Cost. H. P. Gillette. McGraw-Hill Book Co., N. Y.

Handbook of Cost Data. H. P. Gillette. McGraw-Hill Book Co., N. Y.

Handbook of Mechanical and Electrical Cost Data. H. P. Gillette and R. T. Dana. McGraw-Hill Book Co., N. Y.

Handbook of Electrical Cost Data. H. A. Foster. D. Van Nostrand Co., N. Y.

Cost of Mining. J. R. Finlay. McGraw-Hill Book Co., N. Y.

Radford's Estimating and Contracting. W. A. Radford. The Radford Architectural Co., Chicago.

The Building Estimator's Reference Book. F. R. Walker. Frank R. Walker Co., Chicago.

Concrete Costs. F. W. Taylor and S. E. Thompson. John Wiley & Sons, N. Y.

Handbook of Clearing and Grubbing. H. P. Gillette. McGraw-Hill Book Co., N. Y.

Earthwork and its Cost. H. P. Gillette. McGraw-Hill Book Co., N. Y.

Handbook of Rock Excavation (Methods and Cost). H. P. Gillette. McGraw-Hill Book Co., N. Y.

Railroad Structures and Estimates. J. W. Orrock, John Wiley & Sons, N. Y.

Handbook of Construction Equipment. R. T. Dana. McGraw-Hill Book Co., N. Y.

Handbook of Building Construction. G. A. Hool and N. C. Johnson. McGraw-Hill Book Co., N. Y.

Railway Estimates. F. Lavis. McGraw-Hill Book Co., N. Y.

Economic Theory of Railway Location. A. M. Wellington. John Wiley & Sons, N. Y.

Engineering Office Systems and Methods. J. P. Davies. McGraw-Hill Book Co., N. Y.

(See also list of periodicals which publish current prices, below under chap. xv.)

CHAPTERS IX–XI

Private Corporations:

Modern Business Corporations. W. A. Wood. Bobbs-Merrill Co., Indianapolis.

Corporate Organization. T. Conyngton. The Ronald Press, N. Y.
Corporate Management. T. Conyngton. The Ronald Press, N. Y.
Manual of Corporate Management. T. C. Spelling. 3d ed. revised by C. Hillyer. Bender-Moss Co., San Francisco.
Corporation Laws of California. C. F. Curry. Supt. of State Printing, Sacramento, Calif.
Funds and Their Uses. F. A. Cleveland. D. Appleton & Co., N. Y.
Corporate Finance. W. H. Lough. Alexander Hamilton Institute, N. Y.
The Financial Policy of Corporations. (5 vols.) A. S. Dewing. The Ronald Press Co., N. Y.
Corporation Finance. Hastings Lyon. Houghton, Mifflin Co., N. Y.
Financing an Enterprise. H. R. Conyngton. The Ronald Press, N. Y.

Public Service Corporations:

A Treatise on the Law of Public Utilities Operating in Cities and Towns. O. L. Pond. The Bobbs-Merrill Co., Indianapolis.

Public Corporations:

Law of Municipal Corporations. J. F. Dillon. Little, Brown & Co., Boston.
A Treatise on the Law of Municipal Corporations. E. M. McQuillen. Callaghan & Co., Chicago.
Handbook of the Law of Municipal Corporations. R. W. Cooley. West Publishing Co., St. Paul, Minn.
Science of Finance. H. C. Adams. Henry Holt & Co., N. Y.
American City Government—A Survey of Newer Tendencies. C. A. Beard. The Century Co., N. Y.
The Law and the Practice of Municipal Home Rule. H. L. McBain. Columbia University Press, N. Y.
The Government of American Cities. W. B. Munro. The Macmillan Co., N. Y.
The Government of the United States, National, State and Local. W. B. Munro. The Macmillan Co., N. Y.
Municipal Government. F. J. Goodnow. The Century Co., N. Y.
History and Analysis of the Commission and City Manager Plans of City Government in the United States. T. S. Chang. University of Iowa, Iowa City.
Municipal Administration and Accounting. F. A. Cleveland. Longmans, Green & Co., N. Y.
Municipal Accounting. D. C. Eggleston. The Ronald Press Co., N. Y.

CHAPTER XII

Principles of Bond Investment. L. Chamberlain. Henry Holt & Co., N. Y.
Investment Bonds. F. Lownhaupt. G. P. Putnam's Sons., N. Y.
Investment Analysis. W. E. Lagerquist. Macmillan Co., N. Y.
Money and Investments. M. Rollins. Dana, Estes & Co., Boston.
The Business Man and His Bank. W. H. Kniffin. McGraw-Hill Book Co., N. Y.

Engineering Economics. (A series of lectures at University of Kansas, 1917.) J. A. L. Waddell. Distributed by Dean of School of Engineering, Univ. of Kan., Lawrence.
Elements of Business Law. E. W. Huffcut. Ginn & Co., N. Y.
Business Law for Engineers. C. F. Allen. McGraw-Hill Book Co., N. Y.
The Law of Operations Preliminary to Construction in Engineering and Architecture. J. C. Wait. John Wiley & Sons, N. Y.
Engineering and Architectural Jurisprudence. J. C. Wait. John Wiley & Sons, N. Y.
Contracts in Engineering. J. I. Tucker. McGraw-Hill Book Co., N. Y.
Contracts, Specifications, and Engineering Relations. D. W. Mead. McGraw-Hill Book Co., N. Y.

CHAPTERS XIII, XIV

Fundamentals of Accounting. W. M. Cole. Houghton, Mifflin & Co., N. Y.
Modern Accounting. H. R. Hatfield. D. A. Appleton & Co., N. Y.
Principles of Accounting. A. C. Hodge and J. O. McKinsey. University of Chicago Press, Chicago.
Principles of Accounting. W. A. Paton and R. A. Stevenson. Geo. Wahr, Ann Arbor, Mich.
Accounting (3 vol.) R. B. Kester. The Ronald Press, N. Y.
Fundamentals of Accounting. S. B. Koopman. The Ronald Press Co., N. Y.
Lecture Notes on Some of the Business Features of Engineering Practice. A. C. Humphreys. Stevens Institute of Technology, Hoboken, N. J.
Net Worth and the Balance Sheet. H. G. Stockwell. The Ronald Press Co., N. Y.
Statistics in Business. H. Secrist. McGraw-Hill Book Co., N. Y.
Business Statistics. M. T. Copeland. Harvard University, Cambridge, Mass.
Efficient Democracy. W. H. Allen. Dodd, Mead & Co., N. Y.
Factory Organization and Administration. H. Diemer. McGraw-Hill Book Co., N. Y.
Principles of Industrial Organization. D. S. Kimball. McGraw-Hill Book Co., N. Y.
Practical Accounting for General Contractors. H. D. Grant. McGraw-Hill Book Co., N. Y.
Cost Accounting. J. R. Wildman. The Accountancy Pub. Co., N. Y.
Business Costs. D. C. Eggleston and F. B. Robinson. D. Appleton & Co., N. Y.
Business Accounting. (5 vol.) H. D. Greeley. The Ronald Press Co., N. Y.
Cost Reports for Executives. B. A. Franklin. The Engineering Magazine Co., N. Y.
Practical Cost Keeping for Contractors. F. R. Walker. Frank R. Walker & Co., Chicago.
Construction Cost Keeping and Management. H. P. Gillette and R. T. Dana. McGraw-Hill Book Co., N. Y.

Manufacturing Cost and Accounts. A. H. Church. McGraw-Hill Book Co., N. Y.

CHAPTER XV

Interest Rate:

The Rate of Interest. I. Fisher. The Macmillan Co., N. Y.

Population:

Municipal Engineering Practice. A. P. Folwell. John Wiley & Sons, N. Y.

Vital Statistics. G. C. Whipple. John Wiley & Sons, N. Y.

"The Growth of population of American cities of medium size" and "The density of population in cities." E. Kuichling. *Report* on the Proposed Trunk Sewer for the East Side of the City of Rochester, N. Y., Apr. 29, 1889.

Earnings:

Electric Railway Economics. W. C. Gotshall. McGraw-Hill Book Co., N. Y.

Economics of Railroad Location. W. L. Webb. John Wiley & Sons, N. Y.

Economic Theory of Railway Location. A. M. Wellington. John Wiley & Sons, N. Y.

Design of Railway Location. C. C. Williams. John Wiley & Sons, N. Y.

"Modern methods of economical location." H. H. Edgerton, Jr. *Engineering and Contracting,* Feb. 18, 1914, pp. 229–232.

Prices and Wage Rates:

A Century of Prices. T. E. Burton and G. C. Selden. *The Magazine of Wall Street,* N. Y.

Price Changes and Business Prospects. L. P. Ayres. The Cleveland Trust Co., Cleveland, 1921.

Indices of General Business Conditions. W. M. Persons. Harvard Economic Service, Cambridge, Mass.

"Price levels in relation to value." C. F. Elmes. Paper read at the fortieth annual meeting of the Am. Soc. Mech. Eng. Reprints by Sanderson & Porter, Chicago, N. Y., San Francisco.

"Appraisals and rate making." C. F. Elmes. Paper read before Ill. Gas Assn., March 20, 1919. Chicago. Reprints by Sanderson & Porter, Chicago, N. Y., San Francisco.

"Quantitative analysis of all factors that affect average prices and a formula for predicting price changes." H. P. Gillette. *Engineering and Contracting,* April 7, 1920, pp. 381–402.

"Past and future price levels." *Engineering and Contracting,* Aug. 3, 1921, pp. 97–101.

Periodicals Which Publish Price Data:

American Contractor (weekly): building materials, lumber, sand, gravel, crushed stone, structural material.

American Lumberman (weekly): lumber.

American Machinist (weekly): metals.

American Metal Market (daily): iron, steel, junk, metals.

Annalist (weekly): iron, steel, metals, petroleum, rubber.

Architectural Forum (monthly): building materials.

Automobile Trade Journal (monthly): automobiles, metals, junk, rubber, rubber tires, tubes.

Bradstreet's Journal of Trade (weekly): coke, hay, straw, feed, iron, steel, junk, linseed oil, metals, petroleum.

Brass World and Platers' Guide (monthly): metals, chemicals.

Brookmire's Trade Bulletin (monthly): building materials, chemicals, coke, hay, straw, feed, rope, iron, steel, petroleum, linseed oil, rubber, lumber, turpentine, metals, paper.

Buildings Materials Prices (U. S. Dept. of Commerce. Div. of Building and Housing of Bureau of Standards. Washington, D. C.) (monthly): market prices of 24 kinds of materials in 45 cities of the U. S.

Building Supply News (bi-weekly): building materials, sand, gravel, crushed stone.

Chemical Color and Oil (daily): chemicals, linseed oil, oils, paints, resin.

Chemicals and Metallurgical Engineering (weekly): chemicals.

Commercio (monthly): grease, hay, straw, feed, rope, ink, linseed oil, lumber, oils, paints, paper, resin, rubber, turpentine.

Commerce and Finance (weekly): grease, iron, steel, metals.

Commercial and Financial Chronicle (weekly): grease, iron, steel, linseed oil, metals, petroleum, rubber, turpentine.

Commercial Car Journal (monthly): metals, junk, rubber.

Dun's Review (weekly): rope, iron, steel, lumber, metals, naval stores, paints, paper, petroleum, rubber, turpentine.

Electrical World (weekly): metals, electrical supplies, electrical appliances, junk.

Engineering and Mining Journal—Press (weekly): metals.

Engineering News-Record (weekly): construction materials, railway supplies, pipe, labor, lumber, sand, gravel, crushed stone, structural material, explosives.

Financial America (daily): grease, metals.

Financial Review (annual): building materials, chemicals, coal, coke, grease, hay, straw, feed, rope, iron, steel, linseed oil, metals, naval stores, paper, petroleum, rubber, turpentine.

The Foundry (semi-monthly): raw materials for foundries.

Index (weekly): a statistical service for the construction industry (published by Associated General Contractors of America, Washington, D. C.)

India Rubber World (monthly): chemicals, cotton fabrics, dyes, oils, rubber, linseed oil, resin, shellac.

Iron Age (weekly): metals, coke, iron, steel, junk, paints, structural material.

Iron Trade Review (weekly): metals.

Journal of Commerce and Commercial Bulletin (daily): chemicals, gasoline, naphthas, grease, hay, straw, feed, rope, iron, steel, junk, linseed oil, metals, paints, paper, petroleum, resin, rubber.

Journal of Industrial and Engineering Chemistry (monthly): chemicals, coal tar, colors, fertilizers, metals, oils.

Marine News (monthly): iron, steel, metals.
Metal Industry (monthly): chemicals, metals.
Metal Statistics (annual): iron, steel metals, structural materials.
Mineral Industry (annual): minerals.
Mining and Scientific Press (weekly): metals.
Monthly Crop Reporter (monthly): hay, straw, feed.
Monthly Summary of Foreign Commerce of the United States: building materials, chemicals, coal, gasoline, naphthas, hay, straw, feed, rope, iron, steel, naval stores, oils, paper, petroleum, rubber, structural materials, turpentine.
Motor Records (monthly): electrical supplies, electrical appliances, rubber tires, tubes.
National Petroleum News (weekly): gasoline, naphthas, linseed oil, oils, petroleum, turpentine.
National Provisioner (weekly): grease, linseed oil, oils.
New York Commercial (daily): chemicals, gasoline, naphthas, grease, hay, straw, feed, rope, iron, steel, junk, linseed oil, metals, naval stores, oils, petroleum, resin, rubber, shellac, structural material, turpentine.
Oil, Paint, and *Drug Reporter* (weekly): chemicals, drugs, dyes, naval stores, oils, paints, gasoline, naphthas, glass, glues, grease, metals, petroleum, resin, shellac, turpentine.
Paint, Oil and Chemical Review (weekly): chemicals, glues, linseed oil, naval stores, oils, paints, resin, shellac, steel, wool.
Paper (weekly): paper, pulp, rope.
Paper Trade Journal (weekly): rope, paper.
Real Estate Record and Builder's Guide (weekly): building material, glass, linseed oil, lumber, sand, gravel, structural material, turpentine.
Rock Products (bi-weekly): building materials, sand, gravel, crushed stone.
Steel and Metal Digest (monthly): coke, iron, steel, junk, metals, structural materials.
Street (weekly): iron, steel, metals.
Textile World (weekly): chemicals, cotton, dyestuffs, textile yarns, waste, paints.
Waste Trade Journal (weekly): iron, steel, junk, metals, rubber.

CHAPTERS XVI AND XVII

Public Utilities, Their Cost New and Depreciation. H. V. Hayes. D. Van Nostrand Co., N. Y.
Present Value and Fair Return. H. V. Hays. D. Van Nostrand Co., N. Y.
What is Fair? W. G. Raymond. John Wiley & Sons, N. Y.
65 L.11
Valuation of Public Service Corporations. (2 vol.) R. W. Whitten. The Banks Law Pub. Co., N. Y.
Valuation of Public Utility Properties. H. Floy. McGraw-Hill Book Co., N. Y.
Value for Rate Making. H. Floy. McGraw-Hill Book Co., N. Y.

Engineering Valuation of Public Utilities and Factories. H. A. Foster. D. Van Nostrand Co., N. Y.
Public Utility Rates. H. Barker. McGraw-Hill Book Co., N. Y.
Railroad Valuation by the Interstate Commerce Commission. H. B. Vanderblue. Harvard University Press.
The Valuation of American Timber Lands. K. W. Woodward. John Wiley & Sons, N. Y.
Oil Land Development and Valuation. R. P. McLaughlin. McGraw-Hill Book Co., N. Y.
Mathematics of Finance. H. R. Rietz and A. R. Crathorne. Henry Holt & Co., N. Y.
Principles of Depreciation. E. A. Saliers. The Ronald Press Co., N. Y.
"Price levels in relation to value." C. F. Elmes. Presented at fortieth annual meeting Am. Soc. M. E. Reprints by Sanderson & Porter, Chicago, N. Y., San Francisco.
"Appraisals and rate making." C. F. Elmes. Paper read before Ill. Gas. Assn. March 20, 1919. Reprints by Sanderson & Porter, Chicago, N. Y., San Francisco.
"Bibliography on valuation of public utilities." *Trans. Am. Soc. C. E.*, vol. lxxvi, Dec., 1913. Also *Bulletin* 159, Am. Ry. Eng. Assn.

CHAPTER XVIII

Method of Least Squares. M. Merriman. John Wiley & Sons, N. Y.

CHAPTER XIX

The Philosophy of Style. Herbert Spencer. D. Appleton & Co., N. Y.
Theory and Practice of Technical Writing. S. C. Earle. The Macmillan Co., N. Y.
A Manual for Writers. J. M. Manley and J. A. Powell. The University of Chicago Press, Chicago.
Technical Exposition. K. O. Thompson. Harper & Bros., N. Y.
Handbook of English for Engineers. W. O. Sypherd. Scott, Foresman & Co., Chicago.
Composition of Technical Papers. H. A. Watt. McGraw-Hill Book Co., N. Y.
Technical Writing. T. A. Rickard. John Wiley & Sons, N. Y.
Mechanics of Writing. E. C. Wooley. D. C. Heath & Co., Boston.
Manual of Style. The University of Chicago Press, Chicago.
Putnam's Word Book. L. A. Fleming. G. P. Putnams' Sons, N. Y.
Roget's Thesaurus. C. O. S. Mawson. T. Y. Crowell Co., N. Y.
Elements of Specification Writing. R. S. Kirby. John Wiley & Sons, N. Y.
"Technical Book Writing." J. A. L. Waddell. Reprinted from *Bulletin* of the Society for the Promotion of Engineering Education, vol. vii, No. 3, 1916.

APPENDIX D

QUESTIONS AND PROBLEMS

It is not expected that the instructor will assign to any one class all of the numerical problems appearing below under any chapter head; but rather that he will select from those offered such as best fit the class.

I shall be grateful for notice of any errors or inconsistencies discovered herein, and shall gladly receive new questions or problems, few or many, from anyone who will kindly contribute them for the improvement of this appendix.

CHAPTER I

1. Name some way of profitably employing capital, which does not involve a business unit of some kind.
2. Describe the series of "services" the resultant of which is (*a*) a slice of bread served in a restaurant; (*b*) a ride on a steam railroad; (*c*) any other common service.
3. Name a particular business unit, known to you, which is (*a*) an individual business; (*b*) a partnership; (*c*) a private corporation; (*d*) a public corporation.
4. Justify the statement, The true and legitimate purpose of a city government is the rendering of that service which offers the most advantageous employment of capital available.
5. Give a definition of the word "structure" as employed in the text.
6. Name an engineering structure not listed in the text.
7. State the steps likely to be taken in converting into an actual operating structure the idea that an electric lighting plant (or any other kind of engineering structure) would be profitable in a given locality.
8. In which steps (of question 7) would an engineer be called on to take part?
9. In what ways, other than through the medium of a business unit, can an engineer convert his professional knowledge into public benefit?
10. Give two reasons why the young engineer should have some knowledge of the operations of the non-engineering departments of the business unit which he serves.
11. What is Engineering Economics?
12. Name an engineering position in which some knowledge of Engineering Economics is not necessary for success.
13. Read (in some text book on the Elements of Economics) (*a*) on capital; (*b*) on business units. Hand in a 250-word summary of your reading on each topic.

CHAPTER II

1. Name types of investment opportunities advertised in the daily newspaper.
2. What is the object of analyzing a problem?
3. If two analyses, made respectively by two men acting each independently, are not carried along the same lines, is one of the analyses necessarily incorrect? State your reasons.
4. What qualities would you consider in choosing between a portable typewriter and a standard typewriter?
5. What qualities of educational institutions did you consider in reaching a decision to attend the institution you now attend? Distinguish between the reducible data and the irreducible data which you considered. Which irreducible datum carried most weight in your choice?
6. What irreducible data have influenced you in deciding whether or not to enter "student activities"?
7. Give an example to illustrate the third paragraph of §210.
8. What is the essential difference between first procedure given for solving Example (§214) and procedure of current practice? Why is the second procedure, rather than the first, used in practice?
9. Do you think the second sentence of the quotation of §224 is true? State your reason.
10. We can (1) loan $8000 on mortgage to a farmer, for 3 yr. at 7% per annum; or (2) engage in a trucking business for 3 yr. with the following outlays and returns: total cost of trucks, new, $12,000; trucks sell, after 3 yr. use, for $3000; annual cost of operation, upkeep, etc., $15,000; annual gross earnings, $20,000. Compare these two opportunities (*a*) on the lines followed in §§215–220; and (*b*) on the lines followed in §§221–227.

CHAPTER III

SIMPLE INTEREST

1. What interest is due on each interest day in case of note for $7800 for 2 yr., interest at 6% payable semi-annually? (Compute on 365 day basis. Check by Table H.)
2. A note for $1438.50 bearing 8% interest payable annually is dated Feb. 15, 1917. What is the accrued interest June 1, 1917? (Compute on 365 day basis, check by Table H.)
3. What is accrued interest on Sept. 25, 1918 on note for $250 at 5% payable annually dated July 14, 1918? (Compute on 365 day basis and check by Table H.)
4. What is the accrued interest on March 13, 1916, on note dated July 20, 1915 for $1000 bearing interest at 7% payable annually? (Compute on 360 day basis. Check by Table J.)
5. J. Smith gave note for $2000 on Jan. 1, 1911 at 6% payable annually. He paid principal and interest on June 3, 1911. What was the interest? (Compute on 360 day basis. Check by Table J.)

6. Y. Jones gives his note for \$1700 on Feb. 15, 1919 with interest at 4% payable annually. He pays principal and interest on March 11, 1919. What interest does he pay? (Compute on 360 day basis. Check by Table J.)

COMPOUND INTEREST

Amount or Accumulation

7. By arithmetic find the compound amount of \$1 at 3% per period: (*a*) at the end of the first period; (*b*) at the end of the second period; (*c*) of the third period. (*d*) With the help of logarithms find the compound amount at 3% per period for 10 periods, 20 periods, 30 periods, 100 periods. (*e*) Set all your amounts down in tabular form with number of periods in one column and the corresponding amount opposite each number in another column. Compare your table with Table A.

8. With the help of Table A solve the following problems: (*a*) Find the compound amount of \$300 at 4% compounded annually for 16 yr. (*b*) \$100 was deposited in bank, paying 2% compounded annually, 20 yr. ago. What is the amount now? (*c*) If the principal is \$200, and the rate 3% per annum, find the amount at the end of 12 yr. (*d*) Accumulate \$100, 10 yr., at 4% compounded annually. (*e*) \$200 runs 8 yr. at 8% compounded annually. Find the amount. (*f*) Find the amount at 6% at the end of 6 yrs. on \$300. (*g*) Find the amount of \$250 after running 30 yr., if the interest rate is 8% compounded every three months. (*h*) At 8% compounded quarterly, what is the amount of \$362.50 at the end of 26 yr.? (*i*) Accumulate \$412.05, 35 yr. at 8% compounded quarterly.

9. (*a*) Find the credit existing today in an account in a bank paying 6% (annually), in which \$300 was deposited 64 yr. ago. (*b*) A \$100 deposit earns interest at 4% compounded annually for 53 yr. What is the amount?

10. (*a*) Laying off years along the x-axis and amounts along the y-axis, plot the amount of 1 (unit of money) at 2½% for 10, 20, 30, 40, 50 yr. (Use these scales: 1 in. = 10 yr.; 1 in. = 1 (unit of money. The amounts may be taken from Table A.) Draw a smooth curve through the plotted points. (*b*) On the same axes plot a corresponding curve on the basis of 5% interest. (*c*) Describe these curves. Where do they change most rapidly, in the immediate future or in the distant future? (*d*) What is the effect of changing the rate of interest? When is this effect most noticeable, in the immediate future or the distant future? What bearing do these curves have on the question of building a structure now for use only in the distant future?

Present Worth, or Discount

11. (*a*) Using arithmetic find the present worth of \$1 due one period hence at 2%; due 2 periods hence; 3 periods hence. (*b*) Using logarithms, find the present worth at 2% of \$1 due 10 periods hence; 20, 30, 100 periods hence. (*c*) Set these values down in a table as in 7(*e*) above.

Compare your table with Table B. (*d*) What relation exists between each value of Table B and the corresponding value of Table A? If you had only Table B, how would you find compound amount? With Table A, how would you find present worth? Why are both these tables given?

12. With the help of Table B solve the following: (*a*) Find the present worth of $100 due 24 yr. hence at 4% compounded semi-annually. (*b*) Discount $300 at 3½% (compounded annually) for 28 yr. (*c*) Find the present worth of $500 due 15 yr. hence at 5% (compounding semi-annually). (*d*) Find the value of $425.20, discounted 28 yr. at 8% (compounding quarterly). (*e*) What was the equivalent 29 yr. ago of $342.70 today, at 8% (compounding quarterly)? (*f*) What does $528.30 become after being discounted 51 yr. at 4% (compounding semi-annually)?

13. (*a*) Find the value today of $358 due in 52 yr. (money is worth 3%). (*b*) What sum must be deposited in a bank paying 4% compounded annually to amount to $280 in 64 yr.? (*c*) Find the present worth of $358.80 due in 52 yr. at 3% (compounding annually). (*d*) Discount $280, 64 yr. at 4% (compounding annually). (*e*) Find the equivalent 76 yr. ago of $426, at 2½% (compounding annually).

14. (*a*) With the help of Table B, plot a curve of the present worths of $1 up to 50 yr. hence at 2½%. (Take ordinates every 10 yr.; abcissa scale, 1 in. = 10 yr.; ordinate scale, 1 in. = one dime.) (*b*) Double the rate and plot the resulting curve on the same axes. (*c*) Describe these curves. (*d*) What is the effect of increasing the rate? Where does a change in rate have the most effect?

15. (*a*) What would be the effect today, in per cent, of an error of 100% in the estimated salvage value of a concrete and steel bridge whose life is 50 yr., if money is worth 4%? (*b*) What difference would it make in the effect if the bridge were to last only 45 yr.? (*c*) What difference would it make if the rate on money were 6% instead of 4%?

Rate

16. By inspection of the tables find approximate answers to the following: (*a*) Find the rate at which $200 amounts to $273.70 in 8 yr. (annual compounding). (*b*) At what rate will $100 grow to $121.90 in 10 yr., interest compounded annually? (*c*) If $300, compounded annually for 6 yr., accumulates to $358.20, what is the rate of interest? (*d*) $155 is present value of account at a savings bank in which $100 was deposited 10 yr. ago. On the basis of semi-annual compounding, what rate does the bank pay? (*e*) After running 8 yr., a $200 deposit becomes $274.56. If the bank pays interest compounded semi-annually, what is the rate?

Time

17. By inspection of tables find approximate answers to the following: (*a*) Find the time in which $100 will amount to $116 at 4%. (*b*) The accumulated value, after a certain time, of $250 is $1134.50. What is the time if the interest rate is 5%? (*c*) $50 amounts to $369.60 at 5%. How long did it run? (*d*) How long will it take a sum to triple itself at 4%? (*e*) $100 at 4½% will amount to $185.20 in how long?

Equivalence

18. Make a time schedule, and enter thereon the following payments: $1 due today, $3 due two periods hence; $5 due one period ago; $2 due four periods ago; and $1 due three periods hence. (Assume money worth 2% per period.) (*a*) Discount all these payments to the due date of the earliest (using Table B); add results. (*b*) Accumulate them all to the date of the latest (using Table A), and add results. (*c*) Discount the payment $2 to the date of $1, and compare. (*d*) Reduce them all to the present date. (*e*) What is the right to receive these payments on their given dates worth today? What was it worth four periods ago? What will it be worth three periods hence?

19. Solve the following: (*a*) Given, $100 due on Jan. 1 of each year, 1901, 1910, 1918, 1921, 1925. Reduce all these to a single equivalent payment on Jan. 1, 1903. (*b*) Reduce to a single payment on Jan. 1, 1912. (*c*) Reduce to a single payment on Jan. 1, 1902.

Uniform Series of Payments

20. Using Table A, find the present equivalent of a series in which $1 has been deposited at the end of each year for the past 2 yr., 3 yr., 4 yr., 5 yr., 6 yr. (money at 4% per annum). Set these values down in a table as in previous similar exercises. Compare your table with Table C. What is the relation between values in Table C and the corresponding values in Table A?

21. I deposit $500 at the end of each year for 8 yr. in a savings bank paying 4% compounded annually. What is my credit at the bank after making the last deposit?

22. What is the equivalent, at the date of the last payment, of a series of 6 annual $200 payments, if money is worth 6%?

23. Plot a curve showing amount of 1 per annum (payable at the year end) for 50 yr. (*a*) Without interest; (*b*) with interest at 3%; (*c*) with interest at 6%. (Take values from Table C. Use these scales: 1 in. = 10 yr.; 1 in. = 10 units of money.) (*d*) Describe the resulting curves.

24. Using Table A or B, find the present equivalent of a series of payments in which $1 will be paid at the end of each year for 1 yr.; 2 yr.; 3 yr.; 5 yr. Interest at 4% per annum. Set your values down in a table as in previous similar exercises. Compare your table with Table D. What is the relation between a value in Table D and the corresponding value in Table C? Prove the relation.

25. If money is worth 4% per annum, what is the present equivalent of the following series of payments: $1200 at each year end for a 32 yr. period beginning now?

26. What sum must I deposit in a bank, paying 4% per annum, on Dec. 31 this year in order to draw from the bank on each succeeding Dec. 31 for the next 25 yr. the sum of $500?

27. I obtain the lease of a farm for 5 yr. beginning Jan. 1, 1922, at an annual rental of $2000 payable at each year end. If money is worth 8% per annum to me, what could I afford to offer, as a lump sum payment on Jan. 1, 1922, in lieu of the series of annual payments?

28. Plot a curve showing the present worth of 1 per annum (*a*) without interest; (*b*) with interest at 3%; (*c*) with interest at 6%. (Use these scales: horizontal, 1 in. = 10 yr.; vertical, 1 in. = 10 units of money. Points need be plotted only for 5, 10, etc., up to 50 yr. Values of ordinates may be taken from Table D.) (*d*) Describe the curves.

29. Using Table A find what uniform deposit must be made in a savings bank, paying 4% per annum, at each year end for each of the following periods in order that after making the final deposit the accumulated credit shall be $1: 1 yr.; 2 yr.; 3 yr.; 4 yr.; 5 yr.; 6 yr. Make a table of your values, and compare it with Table E.

30. Plot a curve showing the relation between (1) the size of the uniform year-end payments of a series which amounts to $1, and (2) the life of the series: (*a*) with interest at zero; (*b*) with interest at 4%; (*c*) with interest at 8%. (Use these scales: horizontal, 1 in. = 10 yr.; vertical, 1 in. = $0.10. Points need be plotted only for 5, 10, etc., up to 50 yr. Values for ordinates may be taken from Table E.) (*d*) Describe the resulting curves.

31. Using Table B and 5%, find the annual year-end payment of the series of payments which is equivalent to a present lump sum of $1, when the life of the series is 1 yr.; 2 yr.; 3 yr.; 4 yr.; 5 yr.; 6 yr. Tabulate your results. Compare your tables with Table F. What relation exists between values of Table F and corresponding values of Table E?

32. I borrow $10,000 to build a house, and agree to pay the lender a uniform sum at each year end for 14 yr., the uniform sum being so computed that the final payment will cancel the debt for principal and all interest at 8%. What is the annual payment?

33. Plot a curve showing the relation between (1) the year-end payment of that series of payments whose present worth, at the beginning of the life of the series, is $1, and (2) the life of the series with interest at (*a*) zero; (*b*) 4%; (*c*) 8%. (Scales, etc., as in Prob. **30** above.) (*d*) Describe the curves.

SINKING FUNDS AND ANNUITIES

34. (*a*) C spent, on an average, $2 a month for tobacco, between Jan. 1, 1905, and Jan. 1, 1914. If he had deposited each six months' tobacco money in a savings bank, paying 4% compounded semi-annually, beginning with July 1, 1905, and ending with Jan. 1, 1914, what would have been his credit at the bank on the latter date after making the deposit of that date? (*b*) If C made no withdrawals and no further deposits, what would his bank credit have been on July 1, 1917?

35. Jones at 40 plans to build up a 35 yr. sinking fund, by depositing at the end of each half-year in a savings bank, paying 4% compounded semi-annually, for the purpose of buying at the end of the life of the sinking fund a 20 yr. annuity of $1000 each year end. If the annuity is calculated on the basis of 5% compounded annually, what must Jones deposit semi-annually in the sinking fund?

36. A town issues 4½% bonds of $100,000 total par value, to run 12 yr. The town plans to pay the interest on the bonds regularly and create a sinking fund, by year-end deposits paying 4%, with which to redeem the bonds at maturity. What must be the annual deposit?

37. I buy a house and lot for $22,000 and 8 yr. later sell it for $14,000. What is the annual depreciation cost, calculated on the basis of (*a*) 8% interest? (*b*) 4%?

CHAPTER IV

ANNUAL DEPRECIATION COST

1. Find the annual depreciation cost in each of the following cases on basis of (1) 4% interest rate on depreciation fund; (2) on basis of 8%; and (3) on basis of 0%: (*a*) Structure costs $8000; life 15 yr.; scrap value $300; service period 16 yr. (*b*) Structure costs $8000; life 100 yr.; scrap value $300; service period 100 yr. (*c*) Structure costs $8000; life 100 yr.; scrap value $300; service perpetual.

2. A pumping engine costs $32,000. Its estimated life is 25 yr.; and at life end it is expected to sell for $500. At age 15 yr. it is expected to be salable at $8000. Interest on depreciation funds is 4%. What is the (equivalent uniform) annual depreciation cost if the period of service is taken as (*a*) 15 yr.? (*b*) 25 yr.? (*c*) 50 yr.? (*d*) 40 yr.? (*e*) 65 yr.? (*f*) perpetual?

3. A wooden bridge costs $2000; life 12 yr.; salvage at life end, zero; salvage at age 6 yr., $200. Interest on depreciation fund at 4½%. Find (equivalent uniform) annual depreciation cost for each of the following lengths of service period: (*a*) 6 yr.; (*b*) 12 yr.; (*c*) 24 yr.; (*d*) 18 yr.; (*e*) 30 yr.; (*f*) perpetual.

4. A structure costs $5500; life 8 yr.; salvage value at age 4 yr., $1000; at age 8 yr., $200. Assuming the rate of 6% on depreciation funds, find the (equivalent uniform) annual depreciation cost for each of the following periods of service: (*a*) 4 yr.; (*b*) 8 yr.; (*c*) 16 yr.; (*d*) 12 yr.; (*e*) 20 yr.; (*f*) perpetual.

5. (*A*) Distinguish between (1) actual annual depreciation, (2) average annual depreciation, (3) annual depreciation cost, and (4) equivalent uniform annual depreciation cost. (*B*) Under what conditions, in your opinion, is the use of average annual depreciation cost justified in the work of calculating annual service cost?

ANNUAL OPERATION COST

6. Calculate for the whole period named in each of the three following cases (1) the (equivalent uniform) annual cost of operation; (2) the average annual cost of operation; and (3) the per cent error in result of (2): (*a*) The estimated cost of operating a small plant is, for each of the 1st 3 yr., $3000; for the 4th yr., $3500; for the 5th yr., $4000.

Interest on deposits, 6%. (*b*) The annual cost of operating a pump is $8400 a year for the 1st 4 yr.; $10,000 a year for the next 2 yr. Interest at 5%. (*c*) The cost of operating a certain motor truck is as follows: 1st yr., $3000; 2nd yr., $3200; 3rd yr., $3400; 4th, 5th, and 6th yr., $3600 each. Interest at 7%.

7. Under what conditions would you expect annual operation cost to be constant?

CHOICE OF INVESTMENT

8. We can (1) loan $8000 on mortgage to a farmer for 3 yr. at 7% per annum; or (2) engage in a trucking business for 3 yr. with the following outlays and returns: Total cost of trucks, new, $12,000; trucks sell, after 3 yr. use, for $3000; annual cost of operation, upkeep, etc., $15,000; annual gross earnings, $20,000. Compare these two opportunities for employment of capital, making use of true annual depreciation cost calculated on the basis of 4%. Compare the results in this problem with the results in problem 10 under chap. II.

9. Compare these two bridges proposed for a logging road to a lumber camp of which the estimated life is 8 yr. Wooden bridge: first cost $550; life 12 yr.; salvage at age 8, zero; annual upkeep $5. Steel bridge: first cost $800; life 30 yr.; annual upkeep zero. Interest rate 5%; tax rate ½%. The wooden bridge can be made ready three months earlier than the steel.

10. For bleacher service, there were proposed the following two types of structure: wooden, and concrete on earth bank. Compare these two on the basis of the estimates given below. Wooden bleachers: first cost $30,000; life 15 yr.; salvage value, at end of life, $1000; insurance 0.7%; annual upkeep $300. Bleachers of concrete on earth bank: first cost $80,000; life 90 yr.; salvage, at end of life, zero; annual upkeep $50; insurance zero; interest rate 6%.

11. A mining company wishes to know which one of its stock sizes of pipe will be the more economical for a proposed pipe-line through which to pump water under the given conditions. The period of service is 5 yr. The following estimates were based on a length of 1 mi. and a flow of 1 cu. ft. per second. Six-inch pipe: first cost laid $5000; annual operation and upkeep $1740. Eight-inch pipe: first cost $7000; annual operation and upkeep $414. In both cases: salvage value, at age 5, 25%; tax rate 1%; interest rate 6%.

12. There were two feasible reservoir sites. One was in the main stream of the drainage area. The other site was in a tributary gulch into which water from the main stream could be led by tunnel and ditch. It was estimated that the main-stream reservoir would need to be cleared of silt every 2 yr., at a cost of $600 for labor and a loss of $200 revenue due to interrupted service. Similarly it was estimated that the gulch reservoir would need cleaning every fifth year at a cost of $600 for labor and a loss of $200 revenue. Main-stream reservoir: first cost $39,200; annual operation $500. Gulch reservoir: first cost $50,600; annual operation $600. In both cases: life 100 yr.; taxes ½%; interest rate 5%. In the case of the dam proposed for the main stream there was possibility of maximum flood greater than that assumed for the design of spillway, and the consequent destruction of the dam. Compare these two structures as investments on the basis of a 100 yr. service.

13. It was required to carry water from a reservoir to irrigable lands. There were two practicable routes: one, 6½ mi. long, would require tunneling throughout; the other, 13 mi. long, would require 7 mi. ditch, 1 mi. tunnel, and 5 mi. flume on a bench cut on the mountain side. These routes will be called "short line" and "long line," respectively. The long line required a flume the life of which was estimated at 10 yr., but the other parts of the line were considered to be everlasting. Short line: first cost $275,000; life infinite; salvage value zero; annual operation and upkeep $2000. Long line (ditch, tunnel, and bench): first cost $72,600; salvage value zero; annual operation and upkeep $4000. Long line (flume): first cost $57,500; life 10 yr.; salvage value, at end of life, $5000; annual operation and upkeep $4200. For all structures: taxes ½%; interest rate 5%. Compare the two proposals on the basis of perpetual service.

14. Compare the following plants proposed for generating electric current, on the basis of perpetual service and 4% interest. Non-condensing plant: first cost $42,500; life 25 yr.; annual operation and upkeep $19,900. Condensing plant: first cost $48,500; life 20 yr.; annual operation and upkeep $13,575. For either plant: salvage value, at end of life, zero; taxes and insurance together 1% per annum.

15. Compare these two pavements on the assumption of perpetual service and 5% interest. The estimated costs are for one square (100 sq. ft.). Sheet asphalt: first cost $25; salvage, at end of life, $16; annual upkeep $0.40. Asphaltic macadam: first cost $16; salvage, at end of life, $3; annual upkeep $0.20. In either case: life 20 yr.

16. Compare these two second-hand automobiles on the basis of a 2 yr. service and 4% interest. Automobile No. 1: first cost (at second-hand) $3480; salvage, after 2 yr. service, $1300; annual operation and upkeep $700. Automobile No. 2: first cost (at second-hand) $2300; salvage, after 2 yr. service, $1000; annual operation and upkeep $740. In either case: tax rate 1%.

17. Compare the following head frames, proposed for a mine, on the basis of a 10 yr. service and interest at 10%. Steel: first cost $2200; life 40 yr.; salvage, at age 10, zero. Wood: first cost $2000; life 10 yr.; salvage, at age 10, $200. In either case: annual upkeep $15. Of course the wood head-frame is liable to destruction by fire.

18. Compare these two steam-shovels, for steady work in a phosphate mine, on the basis of a 20 yr. service and 6% interest. Fifty-five-ton shovel: first cost $7000; life 20 yr.; salvage, at life end, $500; annual operation $8700; annual upkeep $800; annual output 130,000 cu. yd. Seventy-ton shovel: first cost $10,000; life 25 yr.; salvage, at age 20, $2000; annual operation $10,500; annual upkeep $1000; annual output 180,000 cu. yd. In either case: taxes and insurance together 1½%.

19. The question here is whether to renew a non-condensing steam engine generating set of 250 kv-a. capacity or replace it by a Diesel engine generating set of the same capacity. Make the comparison on the basis of a 15 yr. service and 5% interest. Non-condensing set: first cost $23,400; annual operation and maintenance $15,700. Diesel set: first cost $28,000; annual operation and maintenance $7,800. In either case: life 15 yrs.; salvage $1000. It would take 4 mo. longer to obtain the Diesel than to obtain the other set.

NOTE.—From the following problems all irreducible data are omitted for the reason that it is impracticable to describe such with sufficient particularity and completeness to enable the reader to form a satisfactory opinion as to the weight which should be given them. It is important, however, to bear in mind that nearly always, in practice, there are irreducible data which must be considered along with calculated quantities, to arrive at an economic choice.

20. On the basis of the following data, which of the following two structures should prove the better investment for a 12 yr. service? (1) Gas engine costs $4200; salvage at age 12, $800; annual operation cost, $3800. (2) Electric motor cost $3700; salvage at age 12, $400; annual operation cost, $3900. Assume interest on depreciation fund at 6% in both cases.

21. On the basis of 4% on depreciation funds, no salvage value in either case, and the following reducible data, which of the following should be the better investment for, say, a 40 yr. service? (1) Fuel-oil plant costs $27,000, has an estimated life of 8 yr., and an annual operation cost of $31,700. (2) Powdered coal plant, which costs $97,000, will last 10 yr., and cost annually $19,300 for operation.

22. These two plants are proposed for heating furnaces: (1) water-gas plant: first cost $96,000; life 10 yr.; annual operation cost $14,000. (2) Fuel-oil plant: first cost $27,000; life 8 yr.; annual operation cost $31,700. In either case a 40 yr. service is anticipated. Assume 4% on sinking fund.

23. Plans and estimates were made for (1) a proposed concrete arch and for (2) an alternative steel arch on concrete abutments. (1) For the concrete arch: first cost, $12,230; life 100 yr.; annual maintenance cost, zero. (2) The steel arch would cost $3000, last 50 yr., and require painting every 5th yr. at a cost of $100; the abutments would cost $8240 and last forever. The service is perpetual. Assuming interest at 4% on sinking funds, find which is the better of the two proposed investments.

CHAPTER V

1. Carry out the "land, labor, and capital" analysis of first cost. Why does this analysis not serve the purpose of this chapter?

2. Carry out the analysis begun in §503. Why does this analysis not serve the purpose of this chapter?

3. (*a*) Are steel and cement likely to cost more in Alaska than in Chicago? Why? (*b*) Is skilled labor likely to cost more in Alaska than in San Francisco? Why?

4. Give an original example of interest on capital tied up during construction. Explain how, in your example, this interest might be reduced to a minimum.

5. What is the essential difference between contingencies and irreducible data?
6. Give an original example of a case in which the first cost of a structure includes (*a*) outlays made on one or more other existing structures; (*b*) outlays made on one or more imagined structures.
7. Distinguish between first cost and operation cost. Give an example of a case in which it might be difficult or impossible to draw a sharp line between items of first cost and items of operation cost.

CHAPTER VI

1. Distinguish between direct cost and expense, or indirect cost.
2. Distinguish between production and construction. Wherein does the difference affect the nature and interrelations of the elements of first cost?
3. What is meant by "taking off" quantities?
4. Distinguish between a man-hour and a unit of achievement.
5. Make a complete analysis of cost to a contractor of furnishing all labor and material for the construction of 500 ft. of 6 in. sewer. Exhibit the analysis by diagram of the kind used throughout this book. Do not evaluate any of the quantities. Show the characteristics of the sewer by dimensioned sketches.

CHAPTER VII

1. What are the three controlling factors in the work of making an estimate of first cost?
2. Name the chief steps in the procedure of making a detailed estimate of first cost.
3. Characterize each of the five short methods of estimating first cost. Which one of the five is not merely an abbreviation of the typical general procedure?
4. How many different so-called methods of estimating might be used in estimating the first cost of a complex structure?
5. Make a detailed analysis of cost of the garage shown in the illustration on p. 289, and exhibit the analysis in diagrammatic form. Do not evaluate any of the elements of cost.

CHAPTER VIII

1. Name five kinds of sources of data for estimating. Can you name any kind of source of cost data, not mentioned in this chapter?
2. Justify the methods described in this chapter for bringing prices up to date. What conditions would prevent the satisfactory working of these methods?
3. Give an original example to illustrate the importance of the estimator's knowing the circumstances of past work from which he obtains costs.
4. What do you consider to be the most important of the "aids in estimating"?
5 Fix upon a site for the garage described by drawings on p. 289. Take off the quantities, following your diagram of analysis of cost of the

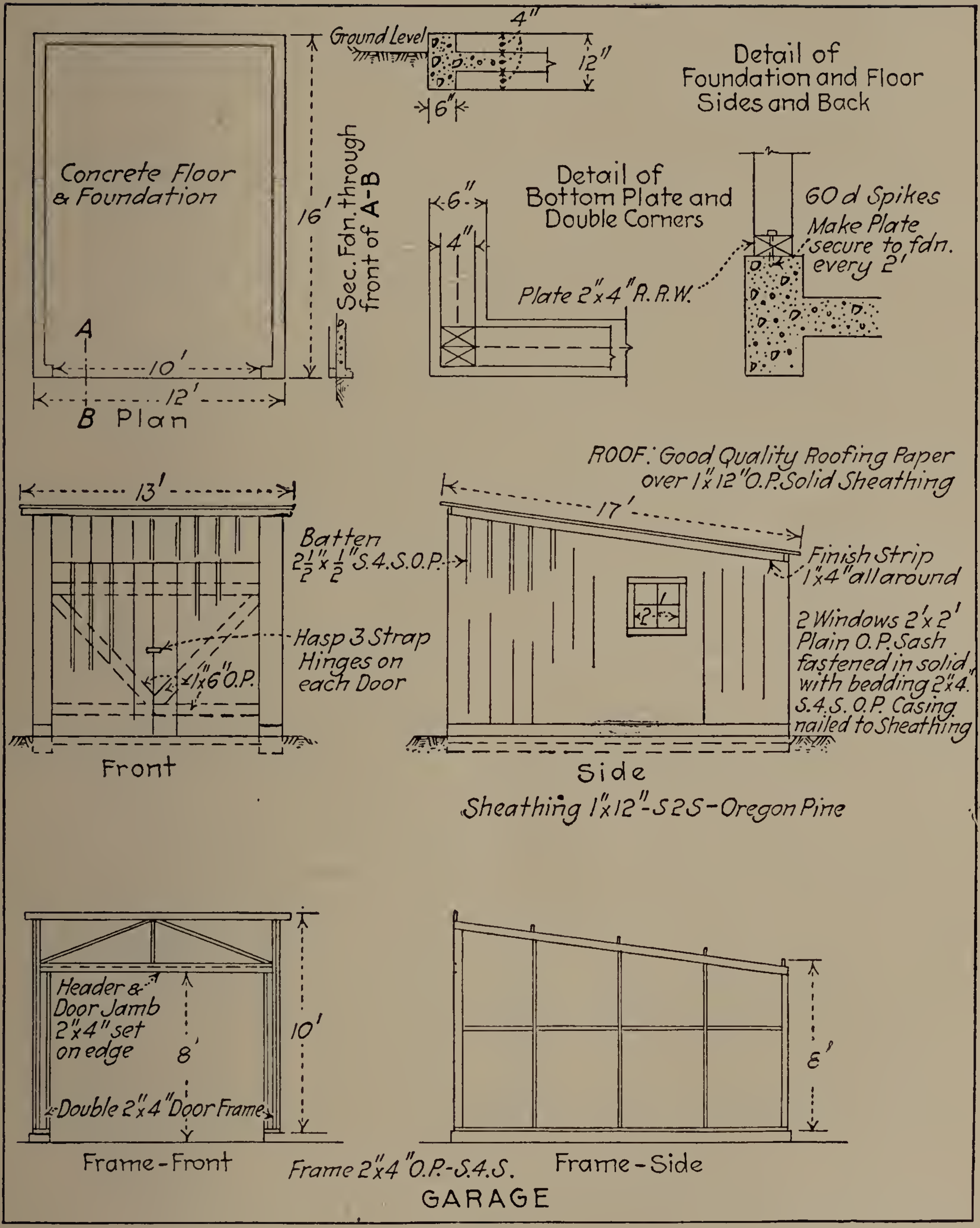

Ground Level
4"
12"
6"
Detail of Foundation and Floor Sides and Back
Concrete Floor & Foundation
16'
Sec. Fdn. through front of A-B
A
10'
12'
B Plan
Detail of Bottom Plate and Double Corners
6"
4"
Plate 2"x4" R.R.W.
60 d Spikes
Make Plate secure to fdn. every 2'
13'
Batten 2½"x½" S.4.S.O.P.
Hasp 3 Strap Hinges on each Door
1"x6" O.P.
Front
ROOF: Good Quality Roofing Paper over 1"x12" O.P. Solid Sheathing
17'
Finish Strip 1"x4" all around
2'
2 Windows 2'x2' Plain O.P. Sash fastened in solid, with bedding 2"x4" S.4.S. O.P. Casing nailed to Sheathing
Side
Sheathing 1"x12"-S2S-Oregon Pine
Header & Door Jamb 2"x4" set on edge
8'
10'
Double 2"x4" Door Frame
Frame-Front
Frame 2"x4" O.P.-S.4.S.
Frame-Side
8'
GARAGE

Fig. 9.

Specifications to supplement the drawing above: Concrete mix is to be 2:4:7. Frame has one stud in back. Header and door jamb consist of two pieces, outside piece set on edge. All lumber to be No. 1 S4S unless otherwise indicated. Three ply prepared tarred felt will satisfy requirement of good quality roofing paper.

structure. Evaluate the cost elements as closely as practicable. Calculate the cost of the garage. Hand in your calculations.

6. *Trestle vs. Fill.*—A railroad or highway may be carried across a valley and stream by means of either a trestle or a fill (embankment) pierced by a culvert or broken by a bridge for the passage of the stream. From this fact arises the problem of choosing between the trestle and the fill, for a given set of conditions.

Before the problem can be solved, it is necessary to establish the conditions for a specific case; and for this, the following steps are required.

(*A*) Provide a profile of a valley not less than 80 ft. deep, on which is drawn the grade line for a railroad, in such wise that the maximum height of grade line above valley bottom, is not less than 80 ft. If you have such a profile, use it. If you are acquainted with the ground represented by the profile, so much the better. If you have no such profile, the instructor will put you in the way of making one.

(*B*) Make pencil drawings of a trestle sufficient to show all parts and dimensions of one bent for each of two or more stories of height, and of one full span of the floor system and the longitudinal bracing. Copy these drawings from some publication, unless you are supplied with blue prints of some existing trestle.

(*C*) Draw cross-section of the proposed fill, to show road bed and side slopes. This may be copied or drawn from published data.

(*D*) Make dimensioned sketch drawings of the proposed culvert. One of these drawings will show a longitudinal section of the culvert site. The size of the culvert may be fixed upon in any of the following four ways, at your option: (1) Adopt arbitrarily the drawings of a culvert which you find in some publication. In this case copy the drawings in so far as may be necessary to show all dimensions which will be required in the calculation of quantities. (2) (*a*) Adopt arbitrarily a tributary drainage area; (*b*) from this find the necessary area of the culvert opening; and then (*c*) adapt some published culvert plans to your case. (3) If you have a map showing the drainage, from it determine the drainage area; and then proceed as in (2) (*b*) and (*c*) above. Or, (4) if practicable, make a rapid survey of the actual tributary drainage area; and then proceed as in (2) (*b*) and (*c*) above.

(*E*) Other assumptions you will make from time to time, as you find it necessary to do so.

You are now prepared to analyze the problem. After the analysis comes the solution, finally the report (see chap. XIX).

7. *Tunnel vs. Open Cut.*—A tunnel is made, in most cases, in a certain place and of a certain length to take the place of a corresponding open cut, because the tunnel, in that particular place, is the cheaper in the long run. Very shallow cuts are cheaper than tunnels per lineal foot; but since the cost per lineal foot of cut increases much faster than the depth of cut, and the cost per lineal foot of ordinary tunnel is little affected by the depth and nearly constant for an increasing length, there is some depth for which the cost per lineal foot is the same for tunnel as for open cut and beyond which the tunnel is the cheaper. The problem here is to determine the economic minimum depth of tunnel, or the economic maximum depth of open cut, under a given set of conditions.

It is necessary first to establish the conditions for a specific case; and for this the following steps are required.

(*A*) Provide a profile showing a railroad projected through a hill not less than 200 ft. high, with reference to the grade line. If you have not such a profile, copy one, or construct one from a topographic map. Confer with the instructor in regard to this.

(*B*) Classify the material along the profile. If practicable this will be done by observation; otherwise, by assumption, after reading a reference or two. Indicate the classification on the profile.

(*C*) Assume, or adopt after reading, the roadbed width and side slopes of open cut. Make a drawing of the cross-section showing all dimensions.

(*D*) Assume (or adopt after reading) the characteristics of the tunnel: (1) cross-section, with or without lining; (2) gradient; (3) portals. Make pencil drawings from published plans selected by you.

Analyze and solve the problem, and write a report (see chap. XIX).

8. *Choice of Highway Diversion.*—A proposed railroad crosses an existing highway at about grade. On one side of the highway there is to be a fill, and on the other side a cut. A grade crossing is to be avoided, and it can be avoided by diverting the highway to pass either under the railroad through an opening in the fill, or over the railroad on a bridge spanning the cut. The problem is to choose between the two feasible diversions.

The following steps must be taken in order to establish conditions for a specific case. (*A*) Provide a topographic map showing railroad, actual or proposed, crossing a highway at or near grade. (This map may be taken ready made, or it may be drawn from notes of a situation survey made by you. Confer with the instructor.)

(*B*) The highway locations will be controlled by the following: (1) The minimum headroom permissible at each proposed crossing. (2) The maximum permissible highway gradient. (3) The minimum width of right of way, and of roadway. (4) The minimum radius of highway curvature. (5) The minimum permissible forward view. (6) Drainage.

(*C*) Plan a satisfactory location for each of the proposed diversions. (1) Pencil these locations with soft sharp pencil, showing (*a*) rights of way required; (*b*) station numbering along the diversions; (*c*) a reference letter at each end of each diversion and at the point where the diversion crosses the railroad; (*d*) any structures required at proposed crossings. (2) Construct from the map a profile of each diversion, and place on each profile the corresponding reference letters taken from the map. (3) Make ink tracings of the original map and profiles (do not trace in ink any of the lines or lettering pertaining to the proposed diversions). (4) Trace with sharp soft pencil, the proposed diversions and all marks and lettering pertaining to them.

Make the lines heavy enough to show clearly on the blueprints. (5) Blueprint the tracings. (*b*) On the blueprints retrace in yellow all marks pertaining to one of the proposed diversions and in orange or vermilion those pertaining to the other diversion.

(*D*) Make sketches to scale of any structures required at the proposed crossings. Place enough dimensions on the sketches to enable you to take off approximate quantities.

Which of the two highway diversions planned, is the better? This is the problem, now before you. Analyze and solve the problem, and write a report (see chap. XIX).

CHAPTER IX

USE OF BORROWED CAPITAL

1. Do you think the subject of business units is appropriately included in a treatise on Engineering Economics? Give your reasons.
2. What is the object of using borrowed capital in conducting a business? Why should engineering students be interested in principles pertaining to the use of borrowed capital?
3. How would you go about determining how much borrowed capital to use in a business? What factors would you have to consider?
4. Explain why each additional borrowing of capital by a business unit is likely to be at a higher rate than the one proceeding?

THE INDIVIDUAL BUSINESS AND THE PARTNERSHIP

5. What are the essential features of a partnership? Contrast them with the corresponding features of a sole proprietorship.
6. What advantages may a sole proprietor gain, and what lose, when he takes in one or more partners and thus transforms the individual business into a partnership?
7. Name four events the occurrence of any one of which will dissolve a partnership.
8. What relation, if any, necessarily exists between the size of a partner's capital contribution and the degree of control possessed by him?
9. What is the limit of personal liability of a partner for the debts of the partnership business?
10. How are partnership profits apportioned among the partners? How are the assets of a partnership business apportioned among the partners when the partnership is dissolved?

CHAPTER X

1. Name four characteristics of a private corporation which do not pertain to a partnership.
2. Do all private corporations in the United States have the same powers?
3. Do all private corporations created in the same state have the same powers?
4. Describe the powers that all private corporations have.
5. By what governmental agencies have the great majority of the private corporations of our country been created?

6. What are the two chief steps in the typical legal procedure for the creation of a private corporation?
7. Are all of the steps of the legal procedure for the creation of a private corporation the same in all of the states?
8. What is generally understood by the term "charter"?
9. Where must one look for those points, of organizations and management of a private corporation, which are not set forth in the articles of incorporation?
10. What steps must be taken by the stockholders and what by the board directors to complete the organization of the private corporation, before the corporation can transact business?
11. Distinguish between the capital stock and the total assets of a corporation. Between a share of stock and a stock certificate. Between income, net income, and dividends.
12. Among officers, directors, holders of common stock, holders of preferred stock, and bondholders, how is control apportioned? How is income apportioned? How is risk apportioned?
13. Outline an organization for the management of a large corporation.
14. What means are used by the private corporation to raise capital?
15. Give your own definition of "bond" as an instrument used for financing a corporation.
16. Suggest a reason for the practice of financing a corporation partly by preferred stock or partly by bonds instead of entirely by common stock.
17. Why is an engineer concerned with corporate affairs? What do you consider to be his first duty to the corporation of which he is an employee?
18. What relation can you see between corporation financing and engineering?

CHAPTER XI

PUBLIC SERVICE CORPORATION

1. How would you distinguish the public service corporation from the other private corporations?
2. What powers are held by some public service corporations that are not held by other private corporations?
3. Why should a city grant a monopoly to a gas company? Why not instead have two or more competing companies?

PUBLIC CORPORATION

4. Name four classes of public corporations.
5. Whence are the powers of a public corporation derived?
6. Have all public corporations the same powers?
7. Name a power that is held by public corporations and by but few public service corporations.
8. What means of raising money are employed by both private and public corporations? What means does each use that is not used by the other?

9. Compare organization for control and management of public corporations with corresponding organization of private corporations.
10. Make an organization chart of a city that is not run on the city-manager plan. Of one that is run on the city-manager plan.
11. In what ways does the public corporation obtain the greater part of its income?
12. What is the engineer's professional interest (*a*) in the income of a public corporation, (*b*) in the governmental machinery of the public corporation?

PROMOTION

13. At what stages of promotion of an engineering structure may the engineer become professionally concerned with it?
14. On reviewing §107, do you find that the promoter is as necessary as the engineer to the creation of engineering enterprises?
15. What are the essential qualifications of a first rate promoter? In what steps of his work will he most need each of these qualifications?
16. Suggest the persons or groups of persons together with their motives, who are likely to promote each of the following: (*a*) establishing a manufacturing business in a town; (*b*) a city's taking over the street railway system; (*c*) a city's building an additional bridge across the river; (*d*) establishing a city market; (*e*) rebuilding the state asylum for the blind; (*f*) improving the city water supply; (*g*) enlarging the city water supply; (*h*) establishing a municipal hospital.

CHAPTER XII

1. What qualities are taken into consideration when comparing one investment opportunity with another? Which of these qualities are taken into the calculation of yield?
2. Why do not all capitalists pick out the same investment as the most desirable?
3. What makes it difficult to form an opinion as to the relative desirabilities of two investments?
4. Give an example of the effect of the "individuality" of a business unit on one or more qualities of an investment offered by the unit.
5. Discuss concisely the effect of "trend of conditions" on the qualities of an offered investment. The trends of what conditions touch the work of the engineer the more closely?
6. Make a comparison quality by quality of some railway company's particular issue of bonds and its common stock.

CHAPTER XIII

1. What are the characteristics of a balance sheet? Of an income sheet?
2. Characterize a "supplementary statement" which supports (*a*) a balance sheet; (*b*) an income sheet.
3. Why is it that the total of liabilities exhibited on a balance sheet can be taken at its face value, while judgment must be passed on the exhibited total of assets?

4. Whence come the facts which the balance sheet and the income sheet set forth?
5. Characterize an "account."
6. Which is the "debit" side of an account? Does this convention arise from accident or from design?
7. How many accounts may be affected by a single transaction?
8. What plan is suggested in the text as an aid to classification as to debit and credit?
9. Characterize a ledger.
10. Why must the ledger balance?
11. The facts exhibited in the ledger are drawn from what sources?
12. What are your reasons for thinking that the student of engineering should (or should not) take systematic training in book-keeping and accounting?
13. Rewrite Doe's income sheet to make it better show the real status of his contracting business.

CHAPTER XIV

1. You are building 1000 ft. of 6 in. sewer. What facts must be noted on the daily record in order that you may know from day to day just how you stand on the job?
2. Suggest a way of finding out daily the average number of shingles laid per man-hour; the per cent of waste in laying 1 in. × 4 in. T & G (*i.e.*, tongue and groove) flooring; and the average number of man-hours required to load a cubic yard of gravel.
3. What is cost keeping?
4. What do you consider to be the controling principles in devising a cost-keeping system.
5. Analyze the labor cost of laying sewer pipe, and using this analysis as a basis, make an original design for a time card to be turned in at the end of each day by the foreman of the gang.
6. Design a form for a summary of a week's cards designed in 5, above.
7. Explain the utility of designating operations, parts, etc., by letters and numbers in some systematic fashion.
8. Which ratios, of those which can be formed between quantities shown on Doe's balance sheet, do you consider to have significance? What does each of the significant ratios tell?
9. What are the telling ratios of Doe's income sheet? What does each signify?
10. What significance has "turnover" in construction or production?
11. What ratios do you think worth obtaining, and plotting, on a job of building a 6 in. sewer?
12. To what uses would you as contractor put the plotted curves?
13. If a steam power plant and a hydro-electric power plant are equally well managed, should we expect to find that they have the same operating ratio? State reasons for your answers.

CHAPTER XV

1. Discuss and illustrate with original examples, forecasting (*a*) annual output of a machine; (*b*) life of a structure; (*c*) period of service; (*d*)

interest rate on borrowed capital; (*e*) interest rate on sinking funds. (*f*) What is the engineer's professional concern with future interest rates?

2. (*a*) Name two of the cruder methods of forecasting population. (*b*) What method is preferred by the Census Bureau?
3. Using the population figures of 1910, 1900, 1890, 1880, and 1870, determine what should have been the population in 1920 of a town or city in which you are interested or for which you can readily find the necessary data, by means of (*a*) the first graphical method; and (*b*) the third graphical method. Compare the two results with each other and with the population as given by the 1920 census.
4. What steps would you take in forecasting the earnings of a proposed electric lighting enterprise for a specified town in Mexico?

CHAPTER XVI

1. What is salvage value as defined in this chapter?
2. Form an equation expressing the relation between first cost, scrap value, depreciation, and wearing value.
3. What conditions have an influence on salvage value?
4. What is market price?
5. Name the ways in which a private enterprise might be taken over by a public corporation.
6. Name the factors some or all of which are taken by court or commission as a basis for reaching a judgment as to the "fair value" of a private enterprise to be taken over by a city.
7. What weight is given each of the factors?
8. Describe two ways of ascertaining the original cost of an old plant. Do both ways call for the service of the engineer?
9. How would you fix upon the unit prices to be used in finding "cost of reproduction new?"
10. What two ways are used for determining depreciation?
11. Which of the basic "values" would you, as a disinterested party, use in fixing a fair price to be paid by a firm to one of its members for an operating plant owned by him?
12. For what purposes, other than transfer of ownership, are valuations undertaken by government?
13. What basic "values" are logically excluded from consideration when the purpose of the valuation is to determine reasonable charges or rates?
14. What professional part does engineer play in valuation cases?

CHAPTER XVII

1. What is a depreciation formula?
2. What is the basis of the so-called (*a*) straight-line formula, (*b*) sinking-fund formula, (*c*) Matheson formula, (*d*) Gillette formula, (*e*) equal profit ratios formula?
3. Which of the depreciation formulas of the text are most commonly used by accountants in calculating depreciation reserves (consult any book on accounting).

CHAPTER XVIII

1. Define: simple estimate; compound estimate; absolute error; relative error; limit of error.
2. Discuss compensation of errors, and the reasonable effect of this principle on choice of procedure in estimating first cost of structure.
3. Is it unreasonable, in making economic comparisons of proposed structures for a period of service which would necessitate renewals, to assume that each renewal cost will be the same as the corresponding original cost? Justify your answer.
4. Is it inconsistent, when making calculations for economic selection, to take the trouble to make a closely approximate estimate of original cost to be combined with a renewal cost arbitrarily made equal to the original cost? State your reasons.

CHAPTER XIX

1. What preparation have you had for the work of writing a report?
2. Characterize a perfect method of giving an answer to a question.
3. Why is it usually more difficult to make a satisfactory report in writing than by word of mouth?
4. What is the chief purpose of a report?
5. A written report should contain how much information? The form of presentation of the information should stand what two tests?
6. Give in your own words brief specifications for writing a report.
7. Write a brief criticism of chap. XIX, pointing out the ways and the places in which the composition fails to conform to §§1905 and 1914.
8. Write a brief criticism of an old report of your own composition—a report on a laboratory experiment, for example.
9. Write a brief criticism of any available report of an engineer.
10. Write a brief critical review of any text on technical writing.

Table VII.—Formulas for Equivalent Uniform Annual Costs, Income, Yield, and Profit

No.		Number of years in the period of service				
		L (one life)	A (less than one life)	n (covering one or more whole lives of L yr. each)	n' (covering one or more whole lives of L yr. each plus a fractional life of A yr.)	∞ (an infinite number of years)
		(1)	(2)	(3)	(4)	(5)
VIIa	Annual DEPRECIATION (AMORTIZATION) cost (equivalent uniform)	$D_L = (C - C_L)d_L$ (When $L = \infty$, $D_L = 0$)	$D_A = (C - C_A)d_A$	$D_n = (C - C_L)d_L$	$D_n' = [(C - C_L)d_L w_n + (C - C_A)d_A(w_n' - w_n)]a_n'$	$D_\infty = (C - C_L)d_L$ (When $L = \infty$, $D_\infty = 0$)
VIIb	Annual OPERATION cost (equivalent uniform)	$O = (O_1p + O_2p^2 + \ . \ . \ . \ + O_Lp^L)a_L$	$O = (O_1p + O_2p^2 + \ . \ . \ . \ + O_Ap^A)a_A$	$O = (O_1p + O_2p^2 + \ . \ . \ . \ + O_np^n)a_n$	$O = (O_1p + O_2p^2 + \ . \ . \ . \ + O_n'p^{n'})a_n'$	$O = (O_1p + O_2p^2 + \ . \ . \ . \ + O_Lp^L)a_L$
VIIc	Annual SERVICE cost exclusive of interest on capital (equivalent uniform)	$V_L = D_L + O$	$V_A = D_A + O$	$V_n = D_n + O$	$V_n' = D_n' + O$	$V_\infty = D_\infty + O$
VIId	Annual SERVICE cost inclusive of interest on capital (equivalent uniform)	$V'_L = D_L + Ce + O$	$V'_A = D_A + Ce + O$	$V'_n = D_n + Ce + O$	$V'_n{}' = D_n' + Ce + O$	$V'_\infty = D_\infty + Ce + O$
VIIe	Annual INCOME (equivalent uniform)	$R = (R_1p + R_2p^2 + \ . \ . \ . \ + R_Lp^L)a_L$	$R = (R_1p + R_2p^2 + \ . \ . \ . \ + R_Ap^A)a_A$	$R = (R_1p + R_2p^2 \ . \ . \ . \ + R_np^n)a_n$	$R = (R_1p + R_2p^2 + \ . \ . \ . \ + R_n'p^{n'})a_n'$	$R = (R_1p + R_2p^2 + \ . \ . \ . \ + R_Lp^L)a_L$
VIIf	Annual YIELD (dollars)	$Y_L = R - V_L$	$Y_A = R - V_A$	$Y_n = R - V_n$	$Y_n' = R - V_n'$	$Y_\infty = R - V_\infty$
VIIg	Annual YIELD (per cent)	$y_L = 100Y_L/C$	$y_A = 100Y_A/C$	$y_n = 100Y_n/C$	$y_n' = 100Y_n'/C$	$y_\infty = 100Y_\infty/C$
VIIh	Annual PROFIT (per cent)	$f_L = y_L - e$	$f_A = y_A - e$	$f_n = y_n - e$	$f_n' = y_n' - e$	$f_\infty = y_\infty - e$

Notation in Table VII

A = age of structure in years.
a_A = annuity produced for A yr. by \$1 principal (Table F).
a_L = annuity produced for L yr. by \$1 principal (Table F).
a_n = annuity produced for n yr. by \$1 principal (Table F).
a_n' = annuity produced for n' yr. by \$1 principal (Table F).
C = capital investment (first cost).
C_A = salvage value at age A yr.
C_L = salvage value at end of life L yr.
d_A = annual deposit in sinking fund to redeem \$1 in A yr. (Table E).
d_L = annual deposit in sinking fund to redeem \$1 in L yr. (Table E).
e = effective rate of interest paid or allowed for use of capital $= (1 + i/h)^h - 1$.
h = number of interest periods in 1 yr.
i = rate per annum of interest paid or allowed for use of capital.
L = life of structure in years.
n = number of years in one or more whole lives of L yr. each.
$n' = n + A = n$ plus a fractional life.
$O_1, O_2, O_3, \ . \ . \ .$ = annual operation cost during the first, second, third, . . . years respectively.
$R_1, R_2, R_3, \ . \ . \ .$ = annual income during first, second, third, . . . years respectively.
w_n = present worth of \$1 per year for n yr. (Table D).
w_n' = present worth of \$1 per year for n' yr. (Table D).

(To face p. 298.)

INDEX

NOTE.—References are to page numbers.

A

A = age of structure, in years
a_n = end-of-year annuity from principal 1 for n yr.
  = forward-spreading factor, 33
  table of values, 249
Abbreviated procedure, estimating, 71–75
Absolute error, 199
Abstract, report, 215
Accounts, 140
  classification, cost keeping, 156
  credit side, 141
  debit and credit, classification as to, 144
  debit side, 141
  two or more, one transaction, 142
Accumulate, 24
Accumulating factor, compound interest, 25
  table of values, 239
Accumulating series, 32
  table of values, 243
Achievement, unit, 62
Acts, series, precede service, 3
Aids, estimating, 83
Amortization, annual cost, 35
  equivalent uniform, formulas, *facing* 298
Amortization fund, 35 (*See* also Sinking fund).
  interest rate, 165
Amortize, 34
Amount, compound interest, 23
  table of values, 239
Amount, series, 30 (*See* also Uniform series of payments, amount).
  table of values, 243
Amount, simple interest, 20
Amount, sinking fund, 34
  table of values, 243
Analysis, assets, 132
  credit granting, 130
  diagram, 19 (*See* Diagram of analysis).
  first cost, preliminary, 44
  general, 6
  investment problem, 6 (*See* also Problem, investment, analysis).
  liabilities, 132
  operation cost, 54
Animals, domestic, first cost item, 46
Annual, amortization cost, 35 (*See* also Amortization, annual cost).
  depreciation cost, 13 (*See* also Depreciation cost, annual).
  operation cost, 54 (*See* also Operation cost).
  payments, uniform, 30 (*See* also Uniform series of payments).
  rate, interest, 23
Annuity, 34, 35
  formulas, 231–237
  investment, 35
  life, 35
  present worth, table of values, 245
  principal, 35
  sinking fund and, 35
  yield, 35
    table of values, 249
Appendix A, specimen incorporation papers, 219
  B, tables of formulas and values, 228
  C, bibliography, 269
  D, questions and problems, 278
  report, contents, 215
Arithmetic of investment, 42
Articles, incorporation, 95
  partnership agreement, 91
  specimen, 219
Articulate structure, 174

Artificial person, 94
Assets, 130
  current, 131
  elements, 133
  fixed, 131
  liquid, 132, 133
  net, 130 (*See* also Net assets).
  ratio, liabilities to, 158
  value, uncertain, 133
Auditor, private corporation, 102
Axiom, report writing, 208

B

Backward-spreading factor, 33
  table of values, 247
Balance, ledger, 146
  sheet, 129
    Doe's, 134
    net income and, 135
Bibliography, 269
Board, directors, elected, 99
  elects officers, 100
  powers, duties, 100
Bond, coupon, 104
  discount, 48
  registered, 104
Bonds, as investment, 124
  classification, 104
  issued by private corporation, 103
  risk, 124
Borrowed capital, use, 86
Bradford, Kimball & Co., articles of incorporation, 219
  certificate of incorporation, 221
  by-laws, 222
Bringing old cost data up to date, 80
  data indispensable, 82
Burden, 57
Bureau of the Census, estimating population, 166, 167
Business, enterprise, 2
  individual, 2, 89 (*See* also Individual business).
  reputation, credit basis, 139
  statistics, 129
    comparisons, 158
    cost keeping, 149 (*See* also Cost keeping).
    financial condition and, 129
Business statistics, financial condition and, 129
  statistics, financial condition and statements, 129
    persons, interested, 129
  units, 2, 86
    individual business, 2, 89
    individuality, 125
    investor and, 122
    legitimate purpose, 2
    private corporation, 2, 94
    promotion, 115
    public corporation, 2, 110
      service corporation, 108
    sole proprietor, 2, 89
    types, 2
      investor and, 123
By-laws, private corporation, 96
  specimen, 222

C

C = first cost, or investment
C_A = salvage value at age A yr.
C_L = salvage value at end of life, L yr.
Calculation, yield, 13, 17
Capital, 1
  acquirement, private corporation, 102
    public corporation, 113
  borrowed, use, 86
  interest, 39 (*See* also Rate of interest).
  working, 55
Capitalist, 1
  ideal of, 121
Capitalize, 31
Capitalized, cost, 25, 31, 34
  value, 25, 31, 34
Card, daily report, 154
Census, Bureau of the, estimating population, 167
Certificate, incorporation, 95
  specimen, 221
Certificate, stock, 97
Chart, organization, 102
Charter, private corporation, 96
  public corporation, 111
Chief engineer, private corporation, 102

Choice, in general, 8
Classification, bond, 104
  debit and credit, 144
  item, cost keeping accounts, 156
Colors, key, in cost keeping, 157
Commercial value, fair price factor, 176, 178
Commission, fair price fixed by, 175
Common stock, 103, 105
  investment, 123, 124
Comparison, business statistics, 158
  general, 8
  investments, 10, 40, 41
    qualities, table, 10
  payments, 34
  standard, 160
  unit, 161
Compensation of errors, 201
Compound, estimates, 199
  amount, table of values, 239
  interest, 22
    accumulation factor, 25
    discounting factor, 25
    elements, 22
    formulas, 229
    graph, 27
    law, 23
    process, 23
    table of values, 239
Conditions, economic, 127
  financial, 127
  investment and, 122
  miscellaneous, 127
  political, 126
  salvage value and, 172
  social, 127
  trend, 126, 127
Construction and production, 59
Contents, report, 211, 217
Contingencies, 51
Control, contingent, 89
  estimating, 66
  individual business, 89
  partnership, 92
  preferred stock, 124
  private corporation, 99
  public corporation, 112
  stockholders', 99, 124
Corporation, lending to, 123
Corporation, private, 2, 94 (*See* also Private corporation).
  public, 2, 110 (*See* also Public corporation).
    service, 108 (*See* also Public service corporation).
  quasi-public, 108
Cost, annual amortization, 35 (*See* also Amortization, annual cost).
  capitalized, 25, 31, 34
  data, bringing old up to date, 80 (*See* also Bringing old cost data up to date).
    cost keeping and, 85
    circumstances and, 82
  depreciation, 13, 17, 35, 36 (*See* also Depreciation cost, annual).
  direct, 58, 59
  error, forecasting, present effect, 203
  finding, 152
  first, 13, 17, 41, 44, 56 (*See* also First cost).
  indirect, 59
  keeping, 149, 152
    estimator and, 85
    materials record, 157
    system, 153
  labor, 45, 62 (*See* also Labor cost).
  materials, 60 (*See* also Materials cost).
  operation, 54 (*See* also Operation cost).
  original, 37
  plant, per unit of product, 64
  product, analysis diagram of unit, 151
  records of labor, 154, 155
  renewal, 37
  replacement, 37
  reproduction new, fair price factor, 176, 178
  service, 13, 17, 19, 41 (*See* also Service cost).
  structure, 69, 72
  unit, 63 (*See* also Unit cost).
    product, 64, 151

Counsel, general, private corporation, 102
Coupon bond, 104
Court, fair price fixed by, 175
Creation of private corporation, 94
Credit, to, 141
  debit and, classification as to, 144
  granting, analysis, 130
  reputation and, 139
Creditor, and business statistics, 129
Cumulative voting, 99
Current, asset, 131
  liability, 131

D

d_n = annual year-end deposit to accumulate 1 in n yr.
  = backward-spreading factor, 33
  table of values, 247
Daily report card, 154
Data, classes, report, 213
  cost, 80 (*See* also Cost data).
  irreducible, 9, 15 (*See* also Irreducible data).
  reducible, 9, 15 (*See* also Reducible data).
Date, interest, 20
Days in year, 21
  table, 250
Debit, to, 141
  credit and, classification, 144
Deposit, sinking-fund, 34
  table of values, 247
Depreciation, defined, 171
  annual, and annual depreciation cost, 37
  cost, annual, 13, 17, 35, 36
    equivalent uniform, 38
    formulas, *facing* 298
  fair price factor, 176, 178
  formulas, 180
    comparison, 191
    derivation, 180
    equal profit ratios, 186
    example of use, 191
    Gillette, 183
    Matheson, 182
    sinking-fund, 181
Depreciation, formulas straight-line, 180
  formulas, unit cost, 185
  fund, 35 (*See* also Sinking fund).
    interest rate, 165
Diagram of analysis, introduced, 19
  cost, bar, 67
    first, 44, 45, 52, 59
    highway, 68
    labor, 62, 63, 64, 69, 151
    materials, 60, 64, 69, 151
    structure, 69, 72
    unit product, 64, 69, 151
  estimating, 68, 76
  investment problem, 19, 41, 121, 122
  utility, 76
Digest, report, 215
Direct cost, 58, 59
Directors, board, 99 (*See* also Board of directors).
Discount, 25
  bond, 48
Discounting factor, compound interest, 25
  table of values, 241
Discounting series, 32
  table of values, 245
Dissolution, partnership, 91
  private corporation, 98
District, public corporation, 112
Dividends, 98
  passed, 100
Doe, John, contractor, introduced, 129

E

e = effective rate, interest, 24
Earnings, forecasting, 168
Economics, engineering, 5 (*See* also Engineering economics).
Effective rate, interest, 24
Election of, board of directors, 99
  officers, by board, 100
Eminent domain, power, 112
Engineer, and, business, 4
  business statistics, 129
  public corporation, 114
  service creation, 3

Engineer, valuation, 179
Engineer, chief, private corporation, 102
Engineering, economics, 5
  fundamental problem, 5
Engineering knowledge, public benefit, 3, 4
  reports, 207
Enterprise, business, 2
Equal profit ratios depreciation formulas, 186
Equation of value, 29
Equity, trading on the, 86 (*See* also Trading on the equity).
Equivalence, 28
Equivalent uniform annual,—*See* Depreciation; Income; Operation; Service).
Errors in estimates, 199
  absolute, 199, 200
  compensation, 201
  future cost, present effect, 203
  limit, 199
    formulas, 200
  probable, 201
    formulas, 202
  relative, 199, 200
Estimate, compound, 199
  error, 199 (*See* also Errors in estimates).
  sheet, 70
  simple, 199
Estimating, abbreviated procedure, 71–75
  aids, 83
  analysis diagram, 68
  controlled by, 66
  earnings, 168
  errors in, 199
    limiting, 199
    probable, 201
  field, 42
  interest rate, 164
  life, structure, 162
  methods, 68, 71–75
  output, 162
  period of service, 163
  population, 166 (*See* also Population).
Estimating, prices, 169
  procedure, 66, 68, 71–75
  rate of interest, 164
  service period, 163
  short cuts, 71–75
  visualization and, 66
  wage rates, 170
  working rules, 77, 202
Events, political, and investments, 126
Evidence of genuineness, in report, 216
Examples, original, 278–297
Exercises, original, 278–297
Expenditure, justifiable present, 25, 31
Expense, 57, 58, 59
Extensions, estimate sheet, 70

F

Factor, accumulation, 25, 32 (*See* also Accumulation factor).
  backward-spreading, 33 (*See* also Backward-spreading factor).
  discount, 25 (*See* also Discount factor).
  forward-spreading, 33 (*See* also Forward-spreading factor).
  series-accumulating, 32 (*See* also Series-accumulating factor).
    -discounting, 32 (*See* also Series-discounting factor).
Fair, price, factors, 176
  fixed by, court or commission, 175
    third party chosen by buyer and seller, 175
  value, 171 (*See* also Fair price).
Field, arithmetic of investment, 42
  estimating, 42
  financial judgment, 43
  irreducible data, 42
  rendering service, 1
Financial, condition, business statistics show, 129
  engineer and, 107
  index, 158
  private corporation, 106
Financial judgment, field, 43

Financial judgment, relations, investor, business unit, 122
  statements, 129
  status, 129 (*See* also Financial, condition).
First cost, analysis diagram of, 44, 45, 52, 59
  interest element, 48
  practical analysis, construction, 58
    production, 56
  preliminary analysis, 44
  structure, includes outlays on other structures, 52, 53
Fixed, assets, 131
  liabilities, 131
Forecasting, cost, error, 203
  earnings, 168
  interest rate, 164
  life of structure, 162
  miscellaneous, 162
  output, 162
  period of service, 163
  population, 166
  prices, 169
  rate of interest, 164
  service period, 163
  wage rates, 170
Foremen, 101
Form, weekly labor summary, 155
Formulas, annual operation cost, 39, and *facing* 298
  annuities, 231–237
  depreciation, 180
    cost, equivalent uniform annual, *facing* 298
  error, limits, 200
    probable, 201, 202
  salvage value, 180
  simple interest, 21
  sinking funds, 231–237
  tables of, *see list*, 228
Forward-spreading factor, 33
  table of values, 249
Fund, amortization, depreciation, redemption, renewal, replacement, 35
  sinking, 34 (*See* also Sinking fund).

G

General, counsel, 102
  manager, 101
Genuineness, evidence of, in report, 216
Gesture, in reporting, 209
Gillette depreciation formula, 183
Graph, compound interest law, 27
Graphical forecasting, population, 167

H

h = number of interest periods in 1 yr., 24

I

i = annual rate, interest, 21, 23
Ideal investment, 121
Income, analysis diagram of net, 136, 138
  equivalent uniform annual, 40
    formulas, *facing* 298
  net, from balance sheets, 135
    private corporation, 98
  private corporation, 97
  sheet, 136
    Doe's, 137
  tax, 126
Incorporation, articles, 95
  specimen, 219
Incorporation, certificate, 95
  specimen, 221
Incorporation, procedure, 95
Incorporators, private corporation, 95
Index, financial status, 158
Indirect cost, 59
Individual business, 2, 89
  control, 89
  management, 90
  promotion, 119
  risk, 91
Individuality, business unit, 125
Inheritance tax, 126
Installment, 35

Insurance, bond discount like, 48
  item of first cost, 47
Interest, 20
  capital, 39
    tied-up, 48
    working, 55
  checking account, 49
  compound, 22 (*See* also Compound interest).
  date, 20
  formulas, 21, 228, 229
  period, 21
  rate, 21 (*See* also Rate, Interest).
  simple, 20 (*See* also Simple interest).
  tied-up capital, 48
    item of first cost, 48
    minimizing, 50
  working capital, 55
Interlocking records, 157
Investment, annuity, 35
  arithmetic, 42
  bond, 124
  calculations, 13, 17, 42
  common stock, 124
  data, irreducible, reducible, 9, 15, 121
  ideal, 121
  irreducible data, 9, 15, 121
  opportunities, type, 125
  partnership, 123
  preferred stock, 124
  problem, 6 (*See* also Problem, investment).
  promissory notes, 124
  qualities, conditions affecting, 122
    ideal, 121, 122
    table of comparison, 10
  reducible data, 9, 15, 121
  speculation and, 125
  yield calculation, 13, 17, 41
Investor, relations to business unit, 122
Irreducible data, 9, 15, 121
  analysis diagram, 122
  field, in investment problem, 42
  investment, 9, 15, 121
  weights, 8

J

Judgment, financial, field of, 43
Justifiable present expenditure, 25, 31

K

Keys, number, letter, color, in cost keeping, 157
Kimball, 219 (*See* also Bradford, Kimball & Co.).

L

L = life of structure, in years
Labor, cost, 62
  analysis diagram, 62, 63, 64, 69, 151
  item of first cost, 45
  records, daily, weekly, 154, 155
Labor, payment plans, 63
Law, compound interest, graphs of, 27
Lawyer, and legal problem, 86
Ledger, 145
  balance, 146
  data sources, 146
Legal problem, and lawyer, 86
Lending to, corporation, 123
  partnership, 123
  sole proprietor, 123
Letters, key, in cost keeping, 157
Liabilities, 130
  assets, ratio, 158
  current, 131
  fixed, 131
  value certain, 133
Liability, partner, 92
  private corporation, 98
  sole proprietor, 89
  stockholders, 98
Life, annuity, 35
  series of payments, 31
  sinking fund, 34
  structure, forecast of, 162
    original, 37
    renewal, 37
    table of values, 259

Limit of error, in compound estimates, 199
  formulas for, 200
Limited partnership, 92
Liquid assets, 133
Liquidate, 131

M

Management, individual business, 90
  private corporation, 99
  public corporation, 112
Manager, and business statistics, 129
  private corporation, 101
Man-hours, 62
  per unit of achievement, 79
    conditions and, 79
    from mental moving picture, 80
Manufactured parts, first cost item, 46
Massive structure, 174
Material things, first cost item, 46
Materials cost, analysis diagram, 60
  item of first cost, 46
  record, in cost keeping, 157
Matheson, depreciation formulas, 182
Member, private corporation, 97
Methods, estimating, 68, 71
  graphical, forecasting population, 167, 168
Miscellaneous forecasts, 162
Money, time value of, 20
Monopoly, 108

N

Net, assets, 130
  analysis diagram, 132
Net income, analysis diagram, 136, 138
  balance sheets and, 135
  private corporation, 98
Nominal rate, interest, 24 (*See* also Rate, Interest).
Notes, as investment, 124
  promissory, 124
  private corporation, 103
Numbers, key, in cost keeping, 157

O

Officers, private corporation, 100
Operating ratio, 159
Operation cost, equivalent uniform annual, 39
  formulas, *facing* 298
Operation, preliminary analysis, 54
Opinion, reasoned from premises, in report, 214
Opportunity, investment, 1, 5, 6 (*See* also Problem, Investment).
Oral report, 209
Organization, chart, 102
  private corporation, 94
  public corporation, 112
Original cost, 37
  fair price factor, 176
Output, forecast, 162
Overhead, 57
Owner, and business statistics, 129
Ownership, private corporation, 97

P

$p = i/s = s^{-1}$ = present worth of 1 due 1 yr. hence, 25
  = discount factor, 25
  table of values, 241
Partnership, 91
  articles of agreement, 91
  control, 92
  dissolution, 91
  investment, 123
  lending to, 123
  liability, 92
  limited, 92
  promotion, 119
  risk, 93
Par value, 97
Passed dividends, 100
Payments, comparison, 34
  series, 30 (*See* also Uniform series of payments).
Period, interest, 21
  service, forecasting, 163
Person, artificial, 94

Piece-work, 62
Plans and specification, 61, 64
Plant cost, per unit of product, 64
Political events affect investment, 126
Population, demand and, 109
  estimating methods, Bureau of Census, 167
    graphic, 167, 168
  law of growth, 166
Power, eminent domain, 112
  tax levying, 112
Powers, private corporation, 94
  public corporation, 112
Preferred stock, 105
  holder, control, 124
    risk, 124
Preliminary analysis, first cost, 44
Premises, reasoning from, to opinion, in report, 214
Preparation of report, procedure, 216
Present justifiable expenditure, 25, 31
  worth, compound interest, 25
    table of values, 241
  series, 31, 32
    table of values, 245
President, private corporation, 100
Price, fair, 171 (*See* also Fair price).
Prices, ascertained how, 78, 169, 170
  periodicals which publish current, 274
Principal, 20
  annuity, 35
Principles, general, report writing, 210
Private corporation, 2, 94
  articles of incorporation, 95
  auditor, 102
  bonds, 103
  by-laws, 96
    specimen, 222
  capital, ways of raising, 102
    stock, 97
  certificate of incorporation, 95
  characteristics, 94
  charter, 96
  chief engineer, 102
Private corporation, control, 99, 112
  creation, 94
  dissolution, 98
  financing, 102
  general, counsel, 102
    manager, 101
  income, 97
  incorporation procedure, 95
  incorporators, 95
  lending to, 123
  liability, 98
  management, 99, 112
  net income, 98
  notes, 103
  officers, 100
  organization, 94
    chart, 102
  ownership, 97
  powers, 94
  president, 100
  promotion, 116
  risk, 106
  secretary, 102
  stock, capital, 97
  treasurer, 102
  vice-president, 101
Probable error, 201, 202
Problem, investment, 6
  analysis diagram, 19, 41, 121, 122
  general case, two solutions, 9, 12
  review and preview, 41
  special cases, 16–18
  summary, 40
Problem, legal, and lawyer, 86
  statement, in report, 213
Procedure, estimating, 66
  abbreviated, 71–75
  detail, typical, 68
Procedure, incorporation, 95
  report writing, 216
Process, problem solving, description in report, 214
Product, unit cost, 63, 64, 151
Production and construction, 59
Profit, equivalent uniform annual, formulas, *facing* 298
Promissory notes, 103
  investment, 124
Promoter, 116

Promotion, 115
  individual business, 119
  partnership, 119
  private corporation, 116
  public corporation, 119
Proprietor, sole, 2, 89 (*See* also Sole proprietor).
Pro-rating, 57, 58
  bases, 57
Public, corporation, 2, 110
  acquires property of public service corporation, 110
  business unit, 2, 110
  capital, ways of raising, 113
  charter, 111
  district, 112
  engineer and, 114
  financing, 113
  lending to, 123
  organization, 112
  powers, 112
  promotion, 119
  purpose, 112
Public benefit, engineering knowledge and, 3, 4
Public service corporation, 108
  distinguished from other private corporations, 108
  lending to, 123
  monopoly, 108
  property acquired by municipality, 110
  regulation, 109
Public works, 114

Q

Qualities, investment, 122
  conditions affecting, 122
  table of comparison, 10
Quantities, taking off, 61
Quasi-public corporation, 108
Questions and problems for assignment, 278–297

R

Rate, interest, 21
  amortization fund, 165
Rate, interest, annual, 23
  depreciation fund, 165
  effective, 24
  forecasting, 164
  nominal, 21, 24
  risk, 164
  sinking funds, 165
  turnover, 159
Rates, valuation for fixing, 179
  wage, 78, 170, 274, 275
Ratio, assets to liabilities, 158
  operating, 159
Real estate, item of first cost, 46
Reasonable regulation, 109
Reasoning, premises to opinion, in report, 214
Records, cost, labor, 154, 155
  materials, 157
Records, interlocking, 157
Redemption fund, 35
Reducible data, 9, 15
Registered bond, 104
Regulation, 109
  investment and, 126
  reasonable, 109
Relations, investor to business unit, 122
Relative error, 199
Renewal, 37
  cost, 37
  fund, 35
  life, 37
Rent, first cost item, 47
Replacement, 37
  cost, 37
  fund, 35
  life, 37
Report, abstract, 215
  answer to question, 207
  appendices, 215
  arrangement, 211
  axiom for writing, 208
  card, daily, 154
  client and, 210
  connectives, 212
  contents, 211
  data, classes of, 213
  digest, 215
  engineering, 207

Report, genuineness, evidence, 216
  opinion, 214
  oral, 209
  problem statement, 213
  procedure for preparing, 216
  process of solution, 214
  quality test, 218
  results, 214
  specifications, 213
  statement forms, 214
  summary, 214, 217
  tabular forms, 214
  test of quality, 218
  transition, part to part, 212
  writing, axiom, 208
    general principles, 210
    procedure, 216
    specifications, 213
Reproduction new, cost, fair price factor, 176, 178
Reputation, credit basis, 139
Result, stated in report, 214
Rights, first cost item, 47
Risk, bonds, 124
  individual business, 91
  interest rate and, 164
  partnership, 93
  preferred stock, 124
  private corporation, 106
  stockholder, 124
Rule, working, for estimating, 77, 202

S

s = compound amount of 1 for 1 yr., 24
  = accumulation factor
  table of values, 239
Salvage value, analysis diagram, 172
  conditions and, 172
  defined, 171
  formulas, 180
  original, 37
  renewal, 37
Schedule, time, 26
Scrap values, 172
Secretary, private corporation, 102
Securities, valuation basis of issue, 179
Series, accumulating factor, 32
  table of values, 243
Series, acts, precede service, 3
  amount, 30, 32 (*See* also Uniform series of payments).
  discounting facting, 32
    table of values, 245
  payments, 30 (*See* also Uniform series of payments).
  present worth, 31, 32 (*See* also Present worth of series).
Service, annual cost, analysis diagram, 19, 41
  average, 13, 17
  equivalent uniform, formulas, *facing* 298
Service, period, forecasting, 163
  prerequisites, 3
  rendering, field, 1
  resultant, series of acts, 3
Services, example, 1
Share, stock, 97
Sheet, balance, 129 (*See* also Balance sheet).
  estimate, 70
  income, 136 (*See* also Income sheet).
Simple, estimate, 199
  interest, table of formulas, 228, 229
    table of values, 360-day year, 256
      365-day year, 252
Sinking fund, 34 (*Same* as Amortization fund, Depreciation fund, etc.).
  amount, table of values, 243
  annuity and, 35
  deposit, 34
    table of values, 247
  depreciation formula, 181
  formulas, 231–237
  interest rate, 165
  life, 34
  tables of formulas, *see list*, 228
    values, *see list*, 228
Social changes and investment, 127
Sole proprietor, 2, 89
  lending to, 123

Sole proprietor, liability, 89
  setting up, 123
Special structure, 174
Specifications, for report, 213
  plans and, 60, 64
Specimen, articles of incorporation, 219
  by-laws, 222
  certificate of incorporation, 221
Speculation and investment, 125
Standard, of comparison, 160
  structure, 174
Statement, financial, 129
  form, reports, 214
  problem, reports, 213
  supplementary, 139
  tabular form, report, 214
Statistics, business, 129 (*See* also Business statistics).
Status, financial, 129 (*See* also Financial condition).
Stock, capital, 97
  certificate, 97
  common, 103, 105, 123, 124
  new issue, 106
  preferred, 105, 124
  share, 97
Stocks and bonds, market value, fair price factor, 176, 178
Stockholder, 97
  control, 99, 124
  liability, 98
  risk, 106, 124
Straight-line depreciation formulas, 180
Structure, articulate, 174
  cost, analysis diagram, 69, 72
  defined, 2
  engineering, 2
  life, forecasting, 162
    table of values, 259
  massive, 174
  original, 37
  special, 174
  standard, 174
Summary, analysis of investment problem, 40
  form, weekly labor, 155
  in report, 214
Summary, report writing, 217
Superintendent, 101
Supplementary statements, 139
Supplies, first cost item, 46
System, cost keeping, 153
  taking off quantities, 70

T

Table, calculation of yield, 13, 17
  comparative, investment qualities, 10, 15, 16, 17
  per cent ratios, parts to whole, 73
Tables, formulas, *see list*, 228
  values, *see list*, 228
Tabular statement, report, 214
Taking off quantities, 61
  system, 70
Tariff and investment, 126
Taxation, valuation basis, 179
Taxes, first cost item, 47
  income, 126
  inheritance, 126
  investment and, 126
  operation cost item, 55
  power of levying, 112
Test, quality, report, 218
Time, compound interest, 26
  schedule, 26
  value of money, 20
Trading on the equity, 86
  incentive, 89
  risk, 89
Transaction affects two or more accounts, 142
Transition, part to part, report, 212
Transportation, first cost item, 47
  operation cost item, 55
Treasurer, private corporation, 102
Trend of conditions, investment and, 126
Turnover, rate, 159
Types, business unit, 2, 123
  investment opportunity, 125

U

Uniform series of payments, 30
  accumulating factor, 32
    table of values, 243

Uniform series of payments, amount, 30, 32
  amount, table of values, 243
  backward-spreading factor, 33
    table of values, 247
  discounting factor, 32
    table of values, 245
  forward-spreading factor, 33
    table of values, 249
  life, 30–34
  present worth, 31, 32
    table of values, 245
  tables of, formulas, *see list*, 228
    values, *see list*, 228
Unit, achievement, 62
  business, 2, 86 (*See* also Business units).
  comparison, 161
  cost, depreciation formula, 185
    product, analysis diagram, 64, 151
  material, 61

V

Valuation, 171, 180
  basis, securities issue, 179
  engineer and, 179
  rate fixing, 179
  taxation, 179
Value, assets, uncertain, 133
  capitalized, 25, 31, 34
  commercial, fair price factor, 176, 178
  equation of, 29
  liabilities, 133
  par, 97
  salvage, 171 (*See* also Salvage value).
  scrap, 172
  time, of money, 20
  wearing, 172
Values, tables of, *see list*, 228
Vice-president, corporation, 101
Visualization, estimating and, 66
Voting, cumulative, 99

W

w_n = present worth of 1 per year-end for n yr.
  = series-discounting factor, 32
  table of values, 245
Waddell, J. A. L., Dr., 83
Wage rates, ascertained how, 78, 170
  periodicals which publish current, 274
Waste, 61
Wearing value, 172
Weekly labor summary for, 155
Weights, irreducible data, 8
Wellington, A. M., 14, 107
Working, capital, interest, 55
  rule, estimating, 77, 202
Worth, present, 25 (*See* also Present worth).
Written report, 210

Y

Yield, annuity, 35
  table of values, 249
Yield, calculation, 13, 17
  equivalent uniform annual, formulas, *facing* 298

Z

z_n = amount of 1 per year-end for n yr.
  = series-accumulating factor, 32
  table of values, 243

Fish

Eng'g economics

M. I. T. LIBRARY

This book is due on the last date stamped below.

MAY 7 1929

~~E.E.Ref.~~

APR 22 1957

Subject to fine if kept beyond date due

MAY 10 1957

AUG 2 1965

L25-10M-1 June'28

MIT LIBRARIES
3 9080 02762 5836

Massachusetts
Institute of Technology

VAIL LIBRARY

SIGN THIS CARD AND LEAVE IT with the Assistant in Charge.
NO BOOK shall be taken from the room EXCEPT WHEN REGISTERED in this manner.

RETURN this book to the DESK.

Form L-32 10,000-5-Jan. '17